Propagation of input errors: implications for model simulations and risk analysis

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Abstract Hydrological models are widely used in water resources management in Australia. Unpinning these models is streamflow data, which is commonly used as inputs, for calibration of parameters and, for verification of model performance. One of the lesser investigated issues in modelling and uncertainty analysis is how the choice of error models impacts on simulations, and how this propagates into decision-making where simulations are used to determine the volume, frequency and reliability of flows. We used an analysis of the deviations in gaugings from flow gauges in the Namoi River catchment, to derive empirically-based error models for the data. The error models were used to generate uncertainty in tributary and residual inflows in the Namoi River Integrated Quality and Quantity Model (IQQM). Several scenarios were run, including empirically-derived best-fit, empirically-derived Gaussian and standard Gaussian error models, with reference to a baseline simulation where the data are assumed to be error free. Analysis of end-of-system flows showed that there was no conclusive difference in the effect of the error models; however, this was likely to be due to the addition of random rather than auto-correlated errors, which arise from fitting of rating curves to gaugings. This study highlights the need for further investigation into rating curve uncertainty, error autocorrelation and sampling of error models.

Key words uncertainty; river model; rating curve; Namoi River

INTRODUCTION

Many uncertainty analysis methods in hydrological modelling require the specification of error models (see for example, Resfgaard *et al.*, 2007). These error models apply to a number of variables, including input data, parameters and output data, and define the probability distribution of errors in the data or parameters. Frequently, the error models are assumed to have a Gaussian distribution, with a mean of zero and standard deviation of one, though other probability distributions are also considered such as log-normal and rectangular. The error models are used to conduct uncertainty analyses to quantify 95% (2 σ) confidence limits for the uncertainty of the variable(s) concerned, or the total uncertainty of the hydrological model.

In many cases independent knowledge of, or validation of, error models is not undertaken, and recent work has highlighted this as a critical need (Di Baldassarre & Montanari, 2009; Renard *et al.*, 2010). This may be in part due to the quality and quantity of data available, or due to the implicit assumption that standard error model distributions are correct, or due to the widespread use of Monte Carlo simulation techniques, which theoretically predict that the model will converge to the true or most likely result (Ogilvie, 1984). However, an invalidated error model is itself subject to uncertainty, and the implications of using an erroneous error model in uncertainty analysis of model simulations needs to be considered, especially by decision-makers who rely on such information for risk assessments.

In addition to records of stage height, gauging stations also contain substantive other information which could be used to inform error models of flows. An example of such information is the percentage deviation in the rating curve from each gauging. In this paper we explore the concept further, using an Integrated Quality and Quantity Model (IQQM) developed for the Namoi River, Australia. The Namoi IQQM has been widely used in water resource assessments in the Murray-Darling Basin (e.g. CSIRO, 2007), though uncertainty of the model remains unquantified. Our aims are to: (1) use gauge information to define empirically-based error model distributions for flow data which is input into the model as tributary and residual inflows; (2) compare the results of an uncertainty analysis using these empirical error models, with standard Gaussian error models; and (3) evaluate the implications of using empirical *versus* assumed error models on the uncertainty of end-of-system flow predictions and how this could impact decision-makers.

METHODS

Namoi catchment and IQQM

The Namoi River is one of several major regulated river systems draining the Murray-Darling Basin in inland southeastern Australia. It is located at approximately 31°S and 150°E and flows in a northwesterly direction from headwaters in the highlands, to the Barwon River at Walgett. The catchment area is around 40 000 km^2 (CSIRO, 2007). The river is fed by six main tributaries: Macdonald River (upper Namoi River), Peel River, Manilla River, Mooki River, Coxs Creek and Baradine Creek, and at the lower end of the system, the river splits into multiple anabranches and effluent streams. Flows are regulated by two large storages in the upper catchment, Split Rock Dam (397 GL) and Keepit Dam (425 GL). There are also numerous minor dams, re-regulating structures and on-river storages such as Mollee Weir.

The catchment has an extensive network of streamgauges. The earliest gauges were established on the Namoi River in the 1890s, though many did not commence until after the 1950s, coinciding with construction of the major dams and significant irrigated agricultural development in the catchment. The gauges vary in their infrastructure and instrumentation, but most consist of 15-min automatic stage recorders. The data, including stage height, gaugings, rating curves and cross-sections is stored in a HYDSTRA database owned by the NSW Office of Water. The stability of each gauge control, affecting the reliability of rating curves, varies significantly with control type and sediment movement through the systems. Artificial control structures include weirs, culverts and gravel crossings, though for many gauges the control is natural bedrock or channel alluvium.

An IQQM of the Namoi River and major tributaries was first developed in the late 1990s by the New South Wales government, and has since been used extensively for water resource assessments of the catchment. IQQM is a suite of integrated component models (e.g. crop water models, reservoir models, channel routing models, etc.) that simulates flows at a daily time-step, including hydrological gains, losses, extractions, management rules and flow routing through a river system. It has a node-link structure, with inputs of climate and streamflow time series data, fixed parameters and other boundary conditions such as maximum storage volumes. The Namoi IQQM is divided into 17 reaches determined by the location of reliable streamflow gauges on the river. Each reach is calibrated separately and includes variables such as tributary inflows, residual or ungauged inflows, lumped irrigation diversions, lumped stock and domestic diversions, town water supply, groundwater losses and gains, effluent flows and lumped losses. Full descriptions of the Namoi model, data and calibration are provided in CSIRO (2007).

Inflow nodes, error models and error sampling

The focus of this study is uncertainty of the tributary and residual inflow nodes in each reach (see list in Table 1). For simplicity, any uncertainty of the other model variables, including uncertainty of the flow data used for calibration of the model, was assumed as zero. The gauges that were used to estimate the tributary and residual inflows in the original model set-up were analysed accordingly to determine and compare different empirical error models for the streamflow data.

To define the error models for each gauge, we used the percentage deviations in the rating curves from the gaugings (calculated for each gauging as –[gauging – rating curve]/rating curve × 100). These data are auto-generated by HYDSTRA and can be used to indicate rating curve uncertainty from the distribution, and minimum and maximum values of the deviations (Fig. 1). For example, some gauges show a high proportion of deviations greater or less than 10%, especially at low stage / low discharge indicating a substantial component of rating uncertainty in the streamflow data. In reaches 1, 4 and 6, residual inflows were estimated using flow data from two gauges, so the percentage deviations from the relevant gauges were combined.

Table 1 Gauges used to provide streamflow data for tributary and residual inflows in the Namoi IQQM.

Inflow node	Gauge	Deviations in gaugings		
		No.	Min, max (%)	Best–fit distribution[a]
Upper Manilla River	419043 Manilla R @ downstream Split Rock Dam	363	–85, 100	Cauchy and Johnson SU
	419053 Manilla R @ Black Springs	249	–180, 65	Cauchy
Macdonald River	419005 Namoi R @ North Cuerindi	639	–707, 79	Cauchy
Halls Creek	419029 Halls Crk @ Ukolan	217	–226, 100	Cauchy
Peel River	419006 Peel R @ Carrol Gap	783	–490, 100	None (Cauchy)
Mooki River	419027 Mooki R @ Breeza	381	–184, 100	Cauchy
Coxs Creek	419032 Coxs Crk @ Boggabri	158	–57, 100	None (3P log–logistic)
Maules Creek	419051 Maules Crk @ Avoca East	221	–88, 32	Cauchy, 3P log–logistic, 4P burr
Baradine Creek	419072 Baradine Crk @ Kienbri	125	–48, 91	None (3P log–logistic)
Reach 1 residual	Combined 419053 & 419029	466	–226, 100	Cauchy
Reach 2 residual	419029 Halls Crk @ Ukolan	As above	As above	As above
Reach 3 residual	419029 Halls Crk @ Ukolan	As above	As above	As above
Reach 4 residual	Combined 419027 & 419006	1164	–490, 100	None (Cauchy)
Reach 5 residual	419032 Coxs Crk @ Boggabri	As above	As above	As above
Reach 6 residual	Combined 419051 & 419032	379	–88, 100	None (Cauchy)
Reach 7 residual	419072 Baradine Crk @ Kienbri	As above	As above	As above
Reach 8 residual	419032 Coxs Crk @ Boggabri	As above	As above	As above
Reach 9 residual	419072 Baradine Crk @ Kienbri	As above	As above	As above
Reach 10 & 11	n/a	n/a	n/a	n/a
Reach 12 residual	419072 Baradine Crk @ Kienbri	As above	As above	As above
Reach 13 residual	419072 Baradine Crk @ Kienbri	As above	As above	As above
Reach 14–17	419072 Baradine Crk @ Kienbri	As above	As above	As above

[a]3P and 4P indicates 3 parameters and 4 parameters, respectively.

Best-fit distributions of the deviations in gaugings from each gauge were determined using the Kolmogorov-Smirnov, Anderson-Darling and Chi-squared goodness-of-fit tests. The distributions were accepted if at least two of the three tests were significant at the 95% level. Where none of the tests were significant (gauges 419006, 419032, 419072, combined 419027-006, combined 419051-032) the closest fitting distribution was selected (indicated in brackets in Table 1). For comparison, normal distributions were also fitted to the deviations for each gauge.

Replicate versions of the Namoi IQQM were created to run the following scenarios:

1. Original IQQM with no error models on the inflow nodes.
2. Modified IQQM with empirical best-fit error distributions on all inflow nodes; upper and lower limits of the distribution set as the minimum and maximum deviations.
3. Modified IQQM with empirical normal error distributions on all inflow nodes; upper and lower limits of the distribution set as the minimum and maximum deviations.
4. Modified IQQM with standard normal distribution (mean = 0 and standard deviation = 1) on all inflow nodes. Upper and lower limits of the distribution set as infinity.

The models were run for the default period of 1 February 1895 to 30 June 2006. The IQQM adds random errors as a percentage of flow onto the time series in each node using the parameters of the distribution and a random number generator. Total end-of-system flows were compared under each scenario for the total run period, annually and daily.

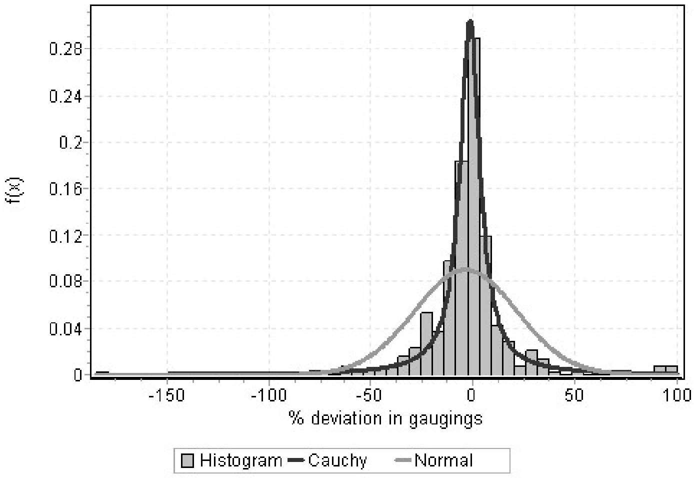

Fig. 1 Histogram of the percentage deviations in gaugings for 419027 Mooki R @ Breeza, showing the best-fit distribution of Cauchy, and the fitted normal distribution.

Table 2 Comparison of end-of-system flows over the total run period 1 January 1895–30 June 2006.

	Error free	Empirical best-fit	Empirical normal	Standard normal
Total (ML)	66 440 869	66 318 439	67 155 416	66 452 567
Mean (ML d^{-1})	1632	1629	1649	1632
Std dev. (ML d^{-1})	7299	7276	7362	7295

RESULTS

The difference in end-of-system flows over the 110-year run period is negligible, regardless of which, if any, error model is used (Table 2). This was a somewhat unexpected result, and we suggest that it is probably due to the addition of random errors to the flow time series rather than auto-correlated errors, which are more likely. The routing on links was removed and the models were run again to check whether the routing was also a factor, but similar results were obtained. The results suggest that any differences in daily flows with random errors are averaged out through the model over longer time periods, compared with a model that is error free.

A comparison of end-of-system flows for each year revealed some differences between the error models, with the empirically derived normal error distributions resulting in predicted flows that were slightly higher (average +2%) than the error free model and other models (Fig. 2). The flows ranged from 90 to 110% of error free annual flow; however, it is probable that this range could become much larger using auto-correlated rather than random errors. A check of the years with the 10 highest and 10 lowest values was made to determine whether there were any correlations with wetter or drier conditions. For all the error models, these mostly coincide with drier, lower flow years, such as 2002.

At a daily time-scale, the addition of error models shows an impact on simulated flows, though there are no clear trends in how these differ with the choice of error distributions (Fig. 3). Similar checks by removing all routing were also undertaken, which revealed a much greater discrepancy between the models, though no consistent trends were evident over time (Fig. 3(b)). The results suggest that storage in the reaches plays a key role in dampening the effects of additive random errors over short time-scales. As a result, errors are cancelled out rather than persist.

DISCUSSION

This study aimed to evaluate the impact and implications of using empirically-based error models *versus* assumed standard error models in uncertainty analysis of a river system model. To date, this

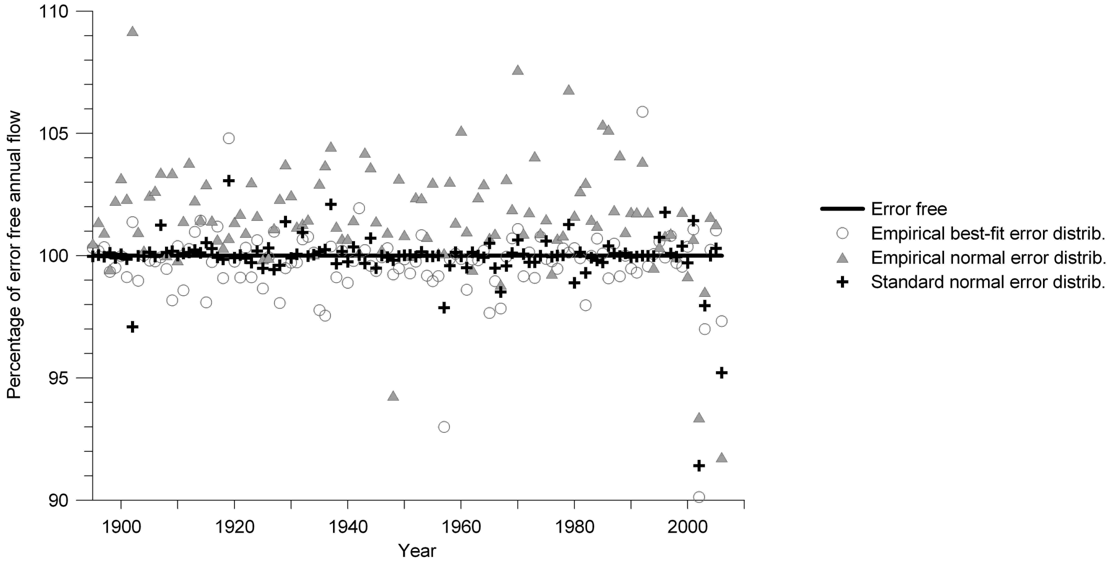

Fig. 2 Comparison of annual end-of-system flows, 1895–2006.

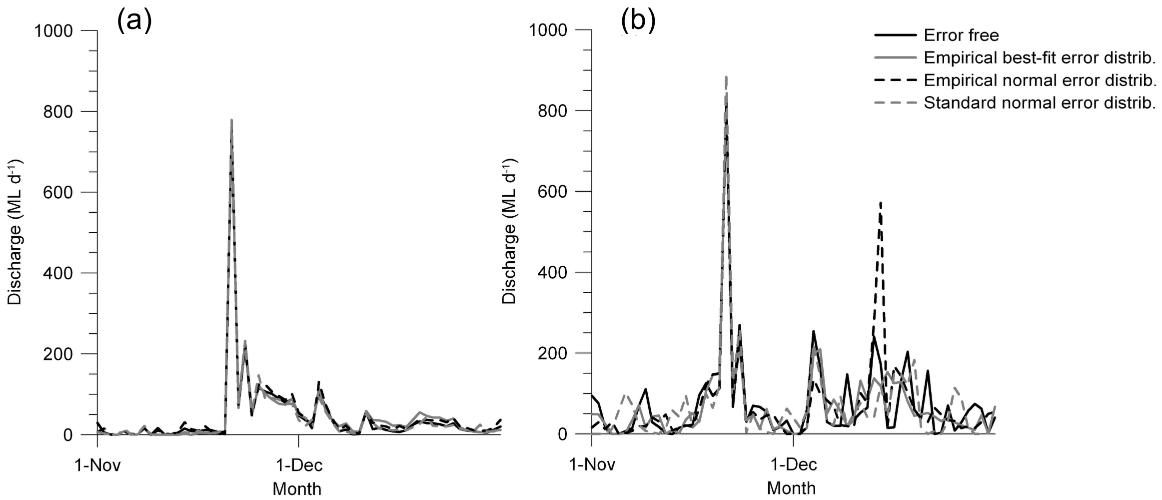

Fig. 3 Comparison of daily end-of-system flows, November–December 2003. (a) IQQM with channel routing on links; (b) IQQM with all routing removed.

type of analysis has been confined to simpler rainfall–runoff models with a focus on rainfall uncertainty, and rarely have empirical data been used to provide baseline information (e.g. Kavetski *et al.*, 2006; Thyer *et al.*, 2009; McMillan *et al.*, 2010).

While we found no convincing evidence in favour of using empirically-derived error models over standard Gaussian models, the results highlight a key outcome: that the choice of error sampling can have a substantial impact on uncertainty propagation. Here we added random errors to the inflow nodes in the Namoi IQQM, and over longer time periods (decades) these were shown to be averaged out at the end-of-system, regardless of the error distribution used, whereas on any day the errors can actually be quite significant. These findings have two important implications:

First, a major source of uncertainty in the flow data is through rating curve uncertainty which generates auto-correlated rather than random errors in the time-series; hence, further work on error sampling and serial correlation of errors in river models is required. Rating curve uncertainty is particularly relevant in the Namoi catchment and others, where many of the gauges have natural controls and experience continuous changes in channel geometry, thus affecting the stage–discharge relationship over time. For these gauges, the rating curves are periodically modified to reflect changing conditions, but despite extensive efforts, it proves an ongoing challenge to capture the exact timing and pattern of rating curve shifts based on the existing and historical frequency of gaugings. Nonetheless, the gaugings and rating curves contain additional useful information that can be better used for predicting shifts and autocorrelation of errors. These should be explored and included in methods addressing uncertainty in streamflow data where required.

Second, the analysis of empirically-derived error models for inflows provided some important insights into the reliability of the Namoi IQQM and whether the model is fit-for-purpose. The Namoi IQQM has been widely used for water resources decision-making in the Murray-Darling Basin, particularly through simulations of the total volume, frequency and reliability of end-of-system flows under different climatic, management and development scenarios (e.g. CSIRO, 2007; MDBA, 2010). Our analysis of error models based on the deviations in gaugings indicates that at a model scale, the errors on the streamflow data from each gauge may be relatively small. The results also imply that the reliability of model simulations increases with run time (i.e. from daily to decadal time-scales) as a result of dampening and compensation of errors, though for individual days or years the errors can still be quite large.

CONCLUSIONS

Empirical data provide a fundamental source of information for uncertainty assessments and risk analysis of hydrological models. At the river model scale and over longer decadal run times, additive random errors on inflows were averaged out providing no clear guidance on the choice of error distributions at this stage. However, further work incorporating rating curve uncertainty, error autocorrelation and error sampling is required.

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