Application of a particle swarm optimization to the tank model

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Abstract A major risk concerning the modelling of hydrological processes with conceptual models is the optimization of the parameters because they cannot be directly measured in the field, mainly in nonlinear models. Several optimization methods have been tested in the past during the calibration of such models, but it is difficult to ensure that the final values are not trapped in a local minimum. Thus, the difficulties involved in calibration of hydrological models have been partly attributable to the lack of robust optimization tools. This paper presents the essential concepts and application to optimize the main parameters in a conceptual hydrological model, with a global optimization method known as Repulsive Particle Swarm (RPS), which is a variant of the Particle Swarm Optimization (PSO) method. The hydrological model that was chosen is the tank model, whose basic principle consists of representing the river basin as a set of tanks in which the outflows of each tank are proportional to the water height from the respective outlets. The tank model is nonlinear and mathematics is nearly useless for nonlinear problems. Therefore, mathematics could not be used for the tank model calibration, and consequently, the RPS technique seems to be suitable for such a task. The optimization technique was tested with the field data from Ishite River Dam, which is the reservoir that supplies water to the city of Matsuyama, Japan. On the basis of these results, the parameter values are given, which could serve as an initial estimate for other similar Japanese watersheds.

Key words hydrological modelling; optimization; RPS; tank model

INTRODUCTION

Optimization is a common problem in many fields of science. The optimization of a mathematical function corresponds to the search of its maximum or minimum value. These functions could also have a set of restrictions for the variables to be optimized. Many techniques have been proposed in order to find these values. However, most of these traditional techniques are not very efficient for solving nonlinear optimization problems.

The major problem concerning the use of conceptual models in hydrological prediction is the need for parameters which cannot be directly measured in the field, mainly in nonlinear models. It is difficult to ensure that the final values for the parameters are not a result of either a local minimum or another trap. Therefore, more robust algorithms are required to estimate the parameter's final values. Particle Swarm Optimization (PSO) is a population-based stochastic optimization technique, inspired by social behaviour of bird-flocking or fish-schooling. It shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA) (Santos *et al.*, 2003). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. The detailed information will be given in the following sections. Compared to GA, the advantages of PSO are that PSO is easy to implement and there are fewer parameters to adjust. PSO has been successfully applied in many areas: function optimization, artificial neural network training, fuzzy system control, and other areas where GA can be applied. The objective of this work is to use Repulsive Particle Swarm (RPS) method of optimization, which is one of the variants of PSO, for application with the tank model, which is a hydrological model whose basic principle consists of representing the river basin as a set of tanks in which the outflows of each tank are proportional to the water height from the respective outlets for the reservoir of Ishite dam that supplies water to the city of Matsuyama, Japan. The paper presents the general details of the hydrological modelling with the RPS method.

REPULSIVE PARTICLE SWARM

The Repulsive Particle Swarm (RPS) proposed by Urfalioglu (2004) is a method of optimization that is a variant of the particle swarm optimization (PSO). It is particularly effective in finding out the global optimum in very complex search spaces; although, it may be slower on certain types of optimization problems.

The Particle Swarm Optimization (PSO) was invented by Eberhart & Kennedy (1995) inspired by simulating the behaviour of birds. This method is an instance of a successful application of the philosophy of bounded rationality and decentralized decision-making to solve the global optimization problems (Simon, 1982). It is observed that a swarm of birds or insects or a school of fish searches for food, protection, etc. in a very typical manner. If one of the members of the swarm sees a desirable path to go, the rest of the swarm will follow quickly. Every member of the swarm searches for the best in its locality and learns from its own experience. Additionally, each member learns from the others, especially from the best performer among them. Even human beings show a tendency to learn from their own experience, their immediate neighbours and the ideal performers. The particle swarm method of optimization mimics the said behaviour. Every individual of the swarm is considered as a particle in a multidimensional space that has a position and a velocity. The particles fly through hyperspace and remember the best position that they have seen. Members of a swarm communicate the good positions to each other and adjust their own position and velocity based on these good positions. There are two main ways this communication is done: (i) "swarm best" that is known to all, (ii) "local bests" are known in neighbourhoods of particles. Updating the position and velocity is done at each iteration as follows:

$$v_{i+1} = \omega v_i + c_1 r_1 (\hat{x}_i - x_i) + c_2 r_2 (\hat{x}_{gi} - x_i) \tag{1}$$

$$x_{i+1} = x_i + v_{i+1} \tag{2}$$

where x is the position and v is the velocity of the individual particle, the subscripts i and $i + 1$ stand for the recent and the next (future) iterations, respectively; w is the inertial constant, good values are usually slightly less than 1; c_1 and c_2 are constants that say how much the particle is directed towards good positions, good values are usually right around 1. Further, r_1 and r_2 are random values in the range [0, 1], $\hat{x}_i$ is the best that the particle has attained in the past, $\hat{x}_g$ is the global best seen by the swarm. This can be replaced by $\hat{x}_L$, the local best, if neighbourhoods are being used.

The traditional RPS gives little scope of local search to the particles. They are guided by their past experience and the communication received from the others in the swarm. We have modified the traditional RPS method by endowing stronger (wider) local search ability to each particle. Each particle flies in its local surrounding and searches for a better solution. The domain of its search is controlled by a new parameter (*nstep*). This local search has no preference to gradients in any direction and closely resembles tunnelling. This added exploration capability of the particles brings the RPS method closer to what is observed in real life. However, in some cases, a moderately wide search (e.g. *nstep* = 9) works better. It has been said that each particle learns from its "chosen" inmates in the swarm. Now, at the one extreme is to learn from the best performer in the entire swarm. This is how the particles in the original PS method learn. However, such learning is not natural. It is neither expected nor even feasible that an individual knows of the best performer in the population and interacts with all others in the swarm, and therefore relies only on the possibility of a limited interaction and limited knowledge that any individual can possess and acquire. Then, our particles do not know the "best" in the swarm. Nevertheless, they interact with some chosen inmates that belong to the swarm. Now, the issue is: how does the particle choose its inmates? One of the possibilities is that it chooses the inmates nearer to it. But, since our particle explores the locality by itself, it is likely that it would not benefit much from the inmates closer to it. Other relevant topologies are (the celebrated) ring topology, ring topology hybridized with random topology, star topology, von Neumann topology, etc.

Let us visualize the possibilities of choosing (a predetermined number of) inmates randomly from among the members of the swarm. This is much closer to reality in the human world. When

we are exposed to the mass media, we experience this. Alternatively, we may visualize our particles visiting a public place (e.g. railway platform, church, etc.) where it (he) meets people coming from different places. Here, geographical distance of an individual from the others is not important. Important is how the experiences of others are communicated to us. There are many sources of such information, each one being selective in what it broadcasts and each of us selective in what we attend to and, therefore, receive. This selectiveness at both ends transcends the geographical boundaries and each one of us is practically exposed to randomized information. Of course, two individuals may have a few common sources of information.

We have used these arguments in the scheme of dissemination of others' experiences to each individual particle. Presently, we have assumed that each particle chooses a pre-assigned number of inmates (randomly) from among the members of the swarm. However, this number may be randomized to lie between two pre-assigned limits.

TANK MODEL

Many models have been developed to simulate rainfall–runoff processes, and one of them is known as the "tank model". The model has been modified and some mathematical tools have been adapted in order to improve its efficiency (Lee & Singh, 1999). A tank model is a conceptual representation of a basin hydrological process. It simulates the wetness of the several soil layers by tanks arranged vertically in series, and each one adapted with one or more outlets to account for water flow and filtration to lower layers.

Precipitation is put into the top tank, and evaporation is subtracted from the top tank. If there is no water in the top tank, evaporation is subtracted from the second tank; if there is no water in both the top and the second tank, evaporation is subtracted from the third tank. The outputs from the side outlets are the calculated runoffs. The output from the top tank is considered as surface runoff, output from the second tank as sub-base runoff and output from the third tank as baseflow. Thus, the outflow or seepage from each tank is assumed to be proportional to the water height from the whole position of discharge or seepage. Water depth of the tank is assumed to be the storage in the basin. In this paper, the tank model was implemented with three tanks. Figure 1 shows how the tank model was implemented.
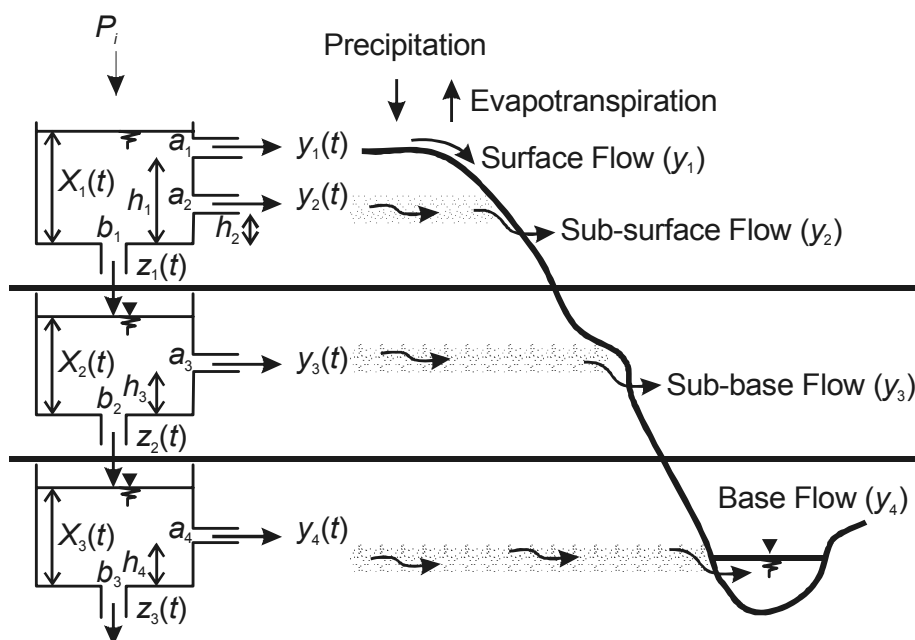

Fig. 1 Schematic design of a tank model with three tanks.

The parameters to be optimized are the runoff parameters a_1, a_2, a_3 and a_4; the infiltration parameters b_1, b_2 and b_3; and the height of the runoff outlets h_1, h_2, h_3 and h_4. These quantities and the tank model are defined by the following expressions:

$$y_1(t) = a_1[X_1(t) - h_1] \tag{3}$$

$$y_2(t) = a_2[X_1(t) - h_2] \tag{4}$$

$$y_3(t) = a_3[X_2(t) - h_3] \tag{5}$$

$$y_4(t) = a_4[X_3(t) - h_4] \tag{6}$$

$$z_1(t) = b_1 X_1(t) \tag{7}$$

$$z_2(t) = b_2 X_2(t) \tag{8}$$

$$z_3(t) = b_3 X_3(t) \tag{9}$$

$$Q(t) = y_1(t) + y_2(t) + y_3(t) + y_4(t) \tag{10}$$

$$X_1(t) = X_1(t-1) + P(t) - y_1(t) - y_2(t) - z_1(t) \tag{11}$$

$$X_2(t) = X_2(t-1) + z_1(t) - y_3(t) - z_2(t) \tag{12}$$

$$X_3(t) = X_3(t-1) + z_2(t) - y_4(t) - z_3(t) \tag{13}$$

where t is the day index; $y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$ are the runoffs from outlets at day t; $z_1(t)$, $z_2(t)$ and $z_3(t)$ are the values of infiltration of each tank at day t; $X_1(t)$, $X_2(t)$ and $X_3(t)$ are the storages in depth at day t; $Q(t)$ is the total runoff at day t; and $P(t)$ is the precipitation at day t.

FIELD DATA

The Ishite River basin is a sub-basin of Shigenobu River basin in Matsuyama city located in Shikoku Island, Japan (Fig. 2). The basin is 72.5 km², the river is 11 km long, and most of the basin is covered by pine forest. The daily rainfall and runoff data from January 1992 to December 2003 at Ishite River Dam were used.
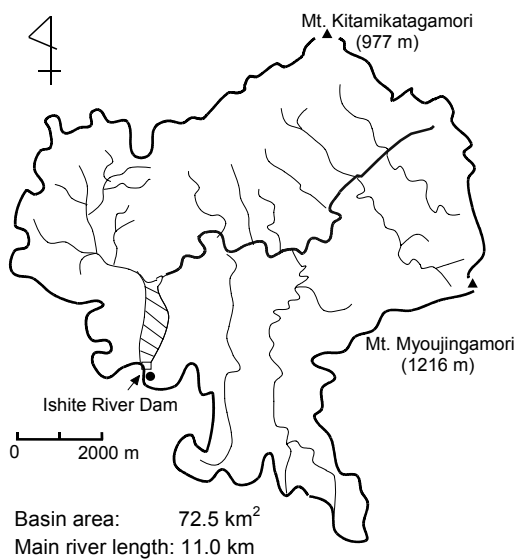
Fig. 2 Map of the Ishite River basin.

The annual mean precipitation depth is between 1300 and 1500 mm, and the rainy season is from the middle of June to the middle of July, with the typhoon season being from August to October. The selected period of observed data for daily rainfall and runoff was 1992 to 1993 for the calibration process, while the years from 1994 up to 2003 were used to validate the calibrated tank model.

APPLICATION AND RESULTS

Setting of the RPS parameters

The RPS method contains some probabilistic and deterministic components which are controlled by some algorithmic parameters. For the method to perform optimally, these parameters must be chosen carefully. The following set of parameters was used in the present work: $N = 100$, $NN = 25$, $MX = 100$, $NSTEP = 15$, $ITRN = 500$, $ITOP = 3$ and $NSIGMA = 1$. The parameter N is the population size, in most of the cases $N = 30$ works well, but its value may be increased to 50 or 100. The parameter NN is the size of randomly chosen neighbours, which ranges from 15 to 25 (but sufficiently less than N) is a good choice. The parameter MX is the maximal size of decision variables. In $f(x_1, x_2,..., x_m)$, m should be less than or equal to MX. The parameter $ITRN$ is the number of iterations, which may depend on the problem. Commonly, the range from 200 (at least) to 500 iterations may be good enough. The $ITOP$ sets the topology, i.e. if less than or equal to 1, it means ring topology; $ITOP$ equal to 2 means ring and random topology; $ITOP$ larger than or equal to 3 means random topology. $NSIGMA$ sets the existence or not of chaotic perturbation, i.e. if $NSIGMA$ is equal to 0, it means no chaotic perturbation, and $NSIGMA$ equal to 1 means chaotic perturbation. In certain cases the one or the other specification works better. Different specifications of parameters may suit different types of functions or dimensions – one has to do some trial and error.

Optimization of the rainfall–runoff model

There are 11 parameters in the tank model to be determined by optimization, which are a_1, a_2, a_3, a_4, b_1, b_2, b_3, h_1, h_2, h_3 and h_4. The ranges for each parameter are presented in Table 1. The initial storages for each tank were set as $X_1 = 0.00$ mm, $X_2 = 0.00$ mm and $X_3 = 100.00$ mm.

Table 1 Ranges for the tank model parameters.

Limits	Parameters										
	a_1	a_2	a_3	a_4	b_1	b_2	b_3	h_1	h_2	h_3	h_4
Lower	0.001	0.001	0.01	0.01	0.15	0.01	0.0	10	10	10	10
Upper	0.1	0.1	0.3	0.7	0.9	0.04	0.3	90	120	120	120

The following objective function F, to be minimized, was chosen:

$$F = \sum_{t=1}^{n} \frac{|Q_o - Q_c|}{Q_o} \tag{14}$$

where Q_o and Q_c are the observed and calculated runoff (mm), respectively, and n is the number of days in the data set. As stated earlier, the period used for the calibration was from January 1992 to December 1993, while the period from January 1994 to December 2003 was used to validate the calibrated tank model.

The correlation (r) and bias (B) statistical indexes were used as criteria for evaluating the model performance. The correlation computes the variability of a number of predictions around the true value. Different from correlation, the bias is a measure of systematic error and thus it

calculates the degree to which the estimation is consistently below or above the actual value. High correlation alone does not mean high accuracy. For example, a significant constant bias in the estimations would provide the highest correlation ($r = 1$) but poor accuracy. As a result, the accuracy of estimations is better analysed by using both bias and correlation. The perfect fit between observed and predicted values, which is unlikely to happen, would have $r = 1$ and $B = 0$. Salas (1993) provides the equations to calculate these indexes.

The RPS method found the following parameter values $a_1 = 0.062$, $a_2 = 0.098$, $a_3 = 0.033$, $a_4 = 0.011$, $b_1 = 0.168$, $b_2 = 0.040$, $b_3 = 0.007$, $h_1 = 11.879$ mm, $h_2 = 40.919$ mm, $h_3 = 65.172$ mm and $h_4 = 10.107$ mm. Figure 3 shows the comparison between observed and calculated reservoir inflows for this calibration data set ($r = 0.80$ and $B = -0.50$ mm). The optimized parameter values are used to validate the tank model using the period January 1994 to December 2003, $r = 0.65$ and $B = -0.14$ mm, as shown in Fig. 4. These figures and indexes (high correlations and low biases) reveal that the calibrated tank model is very efficient for estimating reservoir inflows.
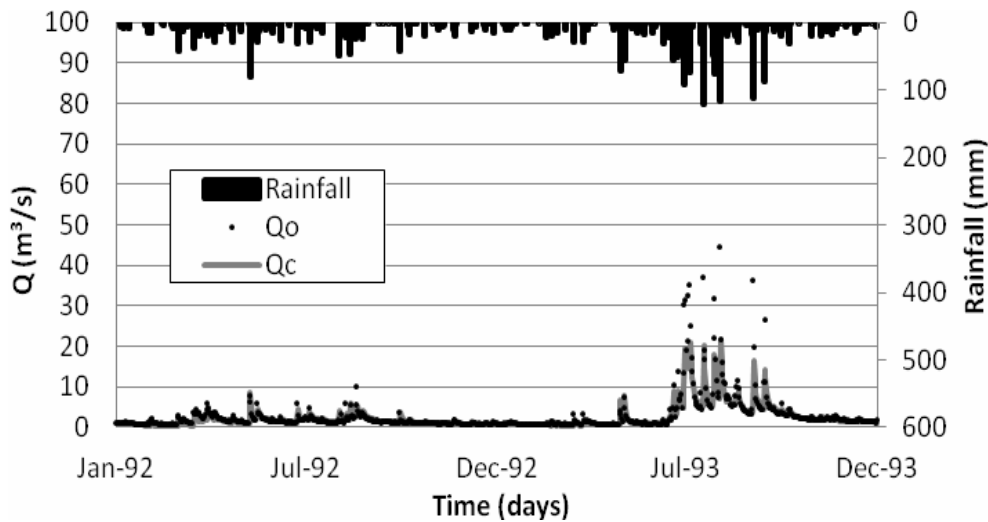

Fig. 3 Hyetograph, and observed (Q_o) and calculated (Q_c) inflows, January 1992–December 1993 (calibration).
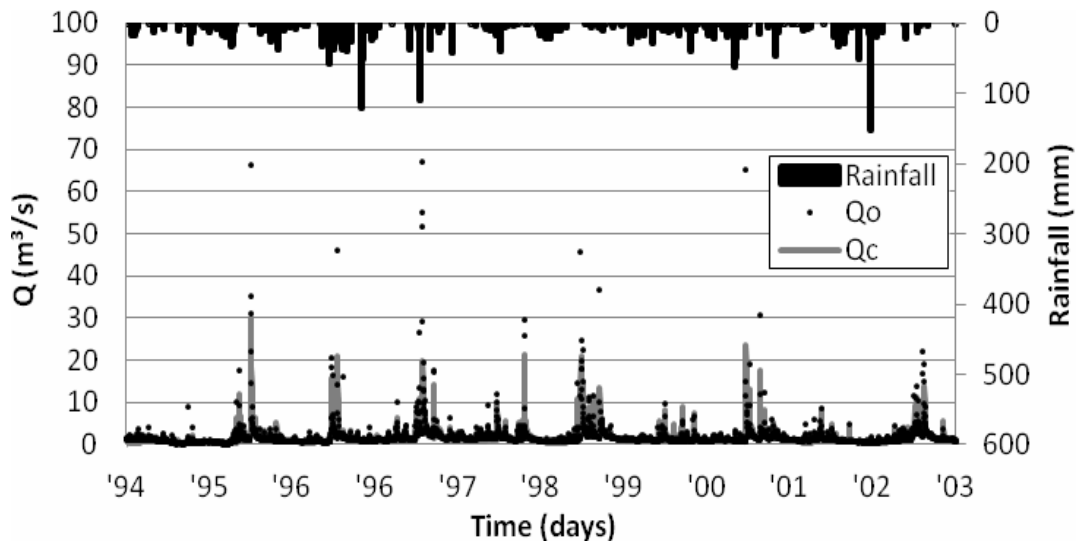

Fig. 4 Hyetograph, and observed (Q_o) and calculated (Q_c) inflows, January 1994–December 2003 (validation).

CONCLUSIONS

A conceptual hydrological model, named the tank model, was used to simulate the daily runoff in Ishite River basin, Matsuyama city, Japan. The main conclusions are as follows: (1) the tank model was shown to be useful for simulation in such a basin; (2) the RPS method was proved to be robust to optimize 11 parameters; (3) in order to perform optimally, the probabilistic and deterministic components in the RPS method were chosen carefully as $N = 100$, $NN = 40$, $MX = 100$, $NSTEP = 15$, $ITRN = 500$, $ITOP = 3$ and $NSIGMA = 1$; and (4) the optimized parameter values are as follows: $a_1 = 0.062$, $a_2 = 0.098$, $a_3 = 0.033$, $a_4 = 0.011$, $b_1 = 0.168$, $b_2 = 0.040$, $b_3 = 0.007$, $h_1 = 11.879$ mm, $h_2 = 40.919$ mm, $h_3 = 65.172$ mm and $h_4 = 10.107$ mm, which could be representative for the area.

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