Optimal multi-year operation of a water supply system under uncertainty: robust methods

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Abstract The Robust Optimization (RO) methodology (Ben-Tal *et al.*, 2009) is applied to optimize the operation of a water supply system (WSS) which supplies water from aquifers with uncertain recharge and desalination plants through a network to consumers. The objective is to minimize the total cost of multiyear operation, while satisfying operational and physical constraints. The RO methodology optimizes the uncertain problem by requesting that the uncertain parameters reside within a user-defined uncertainty set. The static ("here and now") version of RO is called Robust Counterpart (RC), in which the original problem is converted into a deterministic equivalent problem. A generic RC model for optimal operation of a WSS is developed and demonstrated. The policies obtained by the RO methodology, each requiring a different reliability, are compared with other decision making approaches.

Key words water supply systems; optimal operation; uncertain recharge; robust counterpart; robust optimization

INTRODUCTION

Optimal management of Water Supply Systems (WSS) has been studied extensively and resulted in a large number of optimization models and techniques. The parameters of early models were assumed perfectly known, leading to deterministic models. The results obtained by such models usually perform poorly when implemented in the real world, when the problem parameters are revealed and are different from those assumed in the deterministic model. A variety of stochastic methodologies have subsequently been developed, including stochastic dynamic programming (Faber & Stedinger, 2001), implicit stochastic optimization (Lund & Ferreira, 1996), scenario-based optimization (Kracman *et al.*, 2006) and chance constraints (Lansey *et al.*, 1989). However, in these methodologies the uncertain data are assumed to have perfectly known Probability Density Function (PDF), which is not always the case in reality.

This paper considers the Robust Counterpart (RC) approach (Ben-Tal *et al.*, 2009), a novel methodology for optimization under uncertainty, in which the uncertainty is not described by a PDF. It is viewed as deterministic but known to reside within a defined uncertainty set. Hence, instead of immunizing the solution in a probabilistic sense, the decision maker searches for a solution that is optimal for all possible realizations within a defined uncertainty set. The RC approach has been applied to a variety of optimization problems (see Ben-Tal *et al.*, 2009) such as portfolio models, inventory theory, process scheduling and network models.

THE ROBUST COUNTERPART (RC) APPROACH

The RC approach is a min–max oriented methodology (Ben-Tal *et al.*, 2009) that seeks robust feasible/optimal "here and now" decisions which are determined at the beginning of the time horizon, before the uncertain data are revealed. This version of the RC approach is termed "static problem". Robust feasible decisions treat the uncertain constraints as hard constraints which have to be satisfied for all the realizations within the given uncertainty set, while robust optimal means optimizing the guaranteed value (for minimization it is the largest value) of the objective function over the uncertainty set.

To illustrate the method, consider the following LP subject to data uncertainty: (we assume without any loss of generality, that the data uncertainty affects only the elements of the left hand side matrix coefficients, since the objective function can be transformed to a constraint and if the right hand side (RHS) is uncertain then we can introduce new variables with a fixed value of 1):

$$\min_{x}\left\{c^T x : a_{i1}^T x_1 + a_{i2}^T x_2 \le 0, \forall i\right\} \tag{1}$$

where a_{i1} is vector of certain parameters and a_{i2} is vector of uncertain parameters.

The RC of problem (1) is:

$$\min_{x}\left\{c^T x : a_{i1}^T x_1 + a_{i2}^T x_2 \le 0,\ \forall i, \forall a_{i2} \in U_i\right\} \tag{2}$$

U_i is a user-defined uncertainty set. Worst case oriented methodologies can lead to overly conservative solutions such as Soyster's (1973) approach which considers interval uncertainties in LP, where every uncertain parameter takes is at its worst value in the uncertainty set. To address over-conservativeness the RC methodology introduces ellipsoidal uncertainty sets to reflect the fact that the coefficients of the constraints are not expected to be simultaneously at their worst values. The ellipsoidal uncertainty set is defined as affine mapping of a ball of radius θ:

$$U_i = \left\{a_{i2} : \hat{a}_{i2} + \Delta\varsigma, \|\varsigma\| \le \theta\right\} \tag{3}$$

where $\hat{a}_{i2}$ is the nominal value, Δ is a mapping matrix, and the parameter θ is a subjective value chosen by the decision maker to reflect his attitude towards risk. Ben-Tal *et al.* (2009) show that the RC of this LP is:

$$\begin{aligned} & a_{i1}^T x_1 + a_{i2}^T x_2 \le 0 \quad \forall a_{i2} \in \left\{\hat{a}_{i2} + \Delta\varsigma, \|\varsigma\| \le \theta\right\} \Leftrightarrow \max_{\|\varsigma\|\le\theta}\left[a_{i1}^T x_1 + \hat{a}_{i2}^T x_2 + (\Delta\varsigma)^T x_2\right] \le 0 \\ & \Leftrightarrow \\ & a_{i1}^T x_1 + \hat{a}_{i2}^T x_2 + \theta\left\|x_2^T \Delta\right\| \le 0 \end{aligned} \tag{4}$$

which is a convex tractable optimization problem that can be solved by polynomial time *interior point* algorithms. A special case is when the only uncertainty is on the RHS; in this case we obtain a linear RC of the form:

$$a_{i1}^T x_1 + \hat{a}_{i2}^T x_2 + \theta\|\Delta\| \le 0 \tag{5}$$

The RC solution immunizes the resulting optimal decision against deviations from the nominal value, as long as they remain within the set $\mathrm{Ball}_\theta = \left\{\|\varsigma\| \le \theta\right\}$ which is prescribed by selection of θ. Points that lie outside this domain are supposed to have very low probability, as they represent simultaneous extreme values of all uncertain variables.

RC MANAGEMENT MODEL OF WATER SUPPLY SYSTEMS (WSS)

WSS management models vary according to the time horizon covered and time steps, the level of spatial detail, and the physical laws (e.g. hydraulics) that are included. Models range from highly aggregate versions of an entire water system to much more detailed models in space and time (Shamir, 1971). Management models of a large-scale water supply system for seasonal to annual to multi-year operation can be captured in a model of medium aggregation (Fisher *et al.*, 2002; Draper *et al.*, 2003, 2004; Jenkins *et al.*, 2004; Watkins *et al.*, 2004; Zaide, 2006).

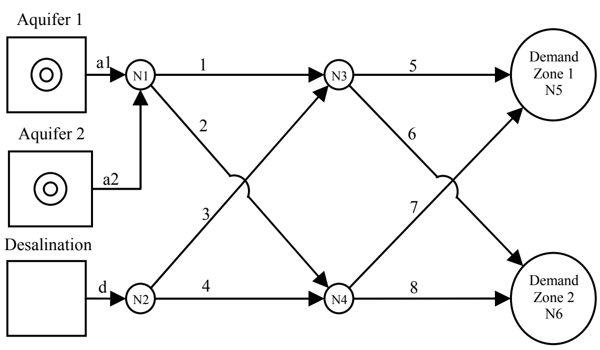

Fig. 1 Water supply system.

In the present paper we consider an optimization model with a medium aggregation level (Fig. 1): water is taken from sources, which include aquifers, reservoirs and desalination plants, and conveyed through a conveyance system to consumers who require certain quantities of water. The time horizon covers several years, with an annual time step. The operation is subject to constraints on water levels in the aquifers, annual carrying capacities of the conveyance pipes and production capacities of the desalination plants. The objective is to operate the system with minimum total cost of desalination and conveyance, plus a penalty/reward related to the final state of the aquifers at the end of the planning horizon, to represent sustainability. The annual replenishment series into the aquifers are uncertain, while the desalination plants are certain but more expensive than extracted groundwater.

Objective function

The objective is to operate the system with minimum total cost over the operation horizon T_f years. The objective is:

$$\sum_{t=1}^{T_f}[\underbrace{\sum_{d} des_{d,t}\cdot Q_{d,t}}_{\text{Desalination}}+\underbrace{\sum_{l} C_{l,t}\cdot Q_{l,t}}_{\text{Conveyance}}]+\underbrace{\sum_{a=1}^{a_f}[(\hat{h}_a-h_{a,T_f})\cdot E_a]}_{\text{Penalty/Reward}}\rightarrow \min \tag{6}$$

Water quantities are in ($\mathrm{MCM/year}$), elevations in (m), and costs in ($\mathrm{M\$/MCM}$); a, d, l and t denote aquifer, desalination, link and year, respectively, $des_{d,t}$ is the cost of desalinated water, $Q_{d,t}$ is the annual quantity of water desalinated, $C_{l,t}$ is the unit cost of transportation and $Q_{l,t}$ the annual transport in the link, $h_{a,t}$ is the water level in the aquifer at the end of year t, $\hat{h}_a$ the desired final water level, and E_a is the penalty per metre of deviation from this level (M$/m).

Network continuity constraints

The following linear equation system insures water balance in the network nodes:

$$G\cdot Q_t = S_t \tag{7}$$

where G is the graph matrix of the network; $Q_t=[Q_{natural,t},Q_{desalination,t},Q_{links,t}]^T$; $S_t=[0,Q_{demand,t}]^T$; $Q_{natural,t}$ is the vector of elements $Q_{a,t}\ \forall a$; $Q_{desalination,t}$ is the vector combining the elements $Q_{d,t}\ \forall d$; $Q_{links,t}$ is the vector combining the elements $Q_{l,t}\ \forall l$; $Q_{demand,t}$ is the vector combining the elements $Q_{z,t}\ \forall z$ where $Q_{z,t}$ denotes demand in year t in demand zone z.

Hydrological constraints in the aquifers

The hydrological water balance insures that the change in aquifer storage equals the difference between the recharge and withdrawal during the year:

$$h_{a,t}=h_{a,0}+\left(\sum_{i=1}^{t}R_{a,i}-\sum_{i=1}^{t}Q_{a,i}\right)/\,SA_a \tag{8}$$

where a,t denote aquifer and year; $Q_{a,t}$ is the extraction amount; $R_{a,t}$ is the (uncertain) recharge; SA_a (MCM/m) is the storativity multiplied by area.

Operation bounds

Bounds on water levels in the aquifer reflect both policy and physical/operational limits; bounds on links discharge, represent limited conveyance capacity and fixed direction flow; bounds on aquifers water extraction which represent hydrological and hydraulic considerations and fixed direction flow; bounds on desalinated water, represent plants capacity contract condition or fixed direction flow.

$$\begin{aligned} h_{a,t}^{\min}\le h_{a,t}\le h_{a,t}^{\max} \quad &; \quad 0\le Q_{l,t}\le Q_{l,t}^{\max} \\ 0\le Q_{a,t}\le Q_{a,t}^{\max} \quad &; \quad Q_{d,t}^{\min}\le Q_{d,t}\le Q_{d,t}^{\max} \end{aligned} \tag{9}$$

where $(\ )^{\min}$ is minimum allowed value; $(\ )^{\max}$ is maximum allowed value.

Construction of the uncertainty set

The resulting mathematical model is LP with uncertainty is in the recharge $R_{a,t}\ \forall a \forall t$ represented in the uncertain column vector $R=[R_{a=1..a_f,t=1},\cdots,R_{a=1..a_f,t=T_f}]^T$. The uncertainty set construction relies only on a given estimated average and covariance matrix of the recharge vector, without the need for further stochastic information. To construct an ellipsoidal uncertainty set for the recharge we assume that the annual recharge values are independent random variables, where the yearly recharge vector of the aquifers is $R'=R_{a=1..a_f,t'}$ in each year t' is correlated with covariance matrix $\Sigma_{R'}$ and expectation vector $\mu_{R'}$, indicating positive correlation between the recharge of different aquifers. Each row in $\Sigma_{R'}$ and $\mu_{R'}$ corresponds to an aquifer $a=1..a_f$. The annual recharges are assumed independent from year to year so the recharge data are repeated for the entire horizon. Hence, the expectation vector of the overall recharge is $\mu_R=[\mu_{R'},...,\mu_{R'}]^T$ and covariance matrix Σ_R is a diagonal block matrix with main diagonal elements $\Sigma_{R'}$. Consider the linear transformation of the stochastic vector R:

$$R=\mu_R+\Delta\cdot\xi \tag{10}$$

which implies $\mu_R=\mu_R+\Delta\cdot\mu_\xi$ and $\Sigma_R=\Delta\cdot\Sigma_\xi\cdot\Delta^{\mathrm{T}}$ then if we set $\mu_\xi=0$ and $\Sigma_\xi=I$ and we have to maintain the covariance of R we need to imply $\Sigma_R=\Delta\cdot\Delta^{\mathrm{T}}$. By replacing the stochastic vector ξ with the perturbation vector ς that varies in the perturbation set $\mathrm{Ball}_\theta=\{\|\varsigma\|\le\theta\}$, we obtain the ellipsoidal uncertainty set U of the uncertain vector R as $U=\{R:\mu_R+\Delta\varsigma,\|\varsigma\|\le\theta\}$. For more details which justifies replacing ξ by ς, we refer the reader to Ben-Tal *et al.* (2009).

The parameter θ determines the range of values of the uncertain R against which the optimal policy is immunized, i.e. remains feasible. A large value means immunization against more extreme values of R. $\theta = 0$ implies that only the expected value of R is taken into consideration, and any deviation of its actual value from the expectation could results in constraint violation. The matrix $\Delta=\Sigma_R^{0.5}$ can be obtained by Cholesky decomposition. Each row in Δ corresponds to year t and aquifer a and implies $\sigma_a=\|\Delta_{t,a}\|$, where σ_a is the standard deviation of recharge in aquifer a which remains constant over the years.

The RC model is constructed by extracting the state variable $h_{a,t}$ from the uncertain equation (8) and creating the robust version of the constraints. The resulting RC is LP, since no decision variables appear in the norms, and the uncertainty appears only on the RHS.

$$\begin{array}{ll}
K\rightarrow \min & \\
\text{Subjecte to} & \\
\sum_{t=1}^{T_f}\sum_a \frac{E_a\cdot Q_{a,t}}{SA_a}-\mu_R^T D_{SA}^{-1}v+\theta\left\|v^T D_{SA}^{-1}\Delta\right\|+\sum_{t=1}^{T_f}\sum_d des_{d,t}Q_{d,t}+\sum_{t=1}^{T_f}\sum_l C_{l,t}Q_{l,t}+P_0-K\le 0 & \\
h_{a,0}+\frac{t\cdot\mu_{R'_a}}{SA_a}+\theta\frac{\sqrt{t}\sigma_a}{SA_a}-\frac{1}{SA_a}\sum_{i=1}^{t}Q_{a,i}-h_{a,t}^{\max}\le 0 & \forall a\forall t\\
-h_{a,0}-\frac{t\cdot\mu_{R'_a}}{SA_a}+\theta\frac{\sqrt{t}\sigma_a}{SA_a}+\frac{1}{SA_a}\sum_{i=1}^{t}Q_{a,i}+h_{a,t}^{\min}\le 0 & \forall a\forall t\\
G\cdot Q_t=S_t & \forall t\\
Q_{d,t}^{\min}\le Q_{d,t}\le Q_{d,t}^{\max} & \forall d\forall t\\
0\le Q_{a,t}\le Q_{a,t}^{\max} & \forall a\forall t\\
0\le Q_{l,t}\le Q_{l,t}^{\max} & \forall a\forall t
\end{array} \tag{11}$$

where $v=[E_{a=1..a_f},\cdots,E_{a=1..a_f}]^T$; $D_{SA}=\mathbf{diag}([SA_{a=1..a_f},\cdots,SA_{a=1..a_f}])$.

APPLICATION

A small hypothetical water supply system (Fig. 1) is used for demonstration. The system is fed from two aquifers and a desalination plant to supply two customers over a 10-year horizon, for which a minimum total operation cost is sought. The annual costs of transportation in the links are {0.1, 0.05} (M$/MCM) for odd and even links, respectively, and the desalination cost is 1 (M$/MCM). The same costs hold for later years $t = 2..T_f$ and are capitalized to the present value with a 5% discount rate. The depletion penalty at the final stage is 0.3 (M$/m) for being below the prescribed value. Both aquifers have identical properties: SA = 0.8 (MCM/m), $h_0 = 75$ (m), $\hat{h} = 30$ (m), $h_{\min} = 0$ (m) and $h_{\max} = 500$ (m) (an arbitrary high value, to insure no spill and thus simplify the demonstration). All water quantities have the same bounds: 0–100 (MCM/year). The annual recharges are i.i.d. with a joint uniform discrete distribution {30, 40, 50} for aquifer 1 and {35, 40, 60} for aquifer 2, that remains the same for all 10 years. This distribution have mean vector {40, 48.33} (MCM/year) and covariance matrix $V_{R'}$ and uncertainty set U :

$$V_{R'} = \begin{pmatrix} 66.67 & 83.33 \\ 83.33 & 105.56 \end{pmatrix},\ U = \left\{ R' : \begin{pmatrix} 40 \\ 48.33 \end{pmatrix} + \begin{pmatrix} 8.17 & 0 \\ 10.21 & 1.18 \end{pmatrix} \begin{pmatrix} \varsigma_1 \\ \varsigma_2 \end{pmatrix}, \|\varsigma\| \le \theta \right\} \tag{12}$$

The annual demand in the first year is 80 (MCM/year) in each demand zone, and it increases by 5% in each subsequent year. The cost of deficit in supply to the customers is 3 (M$/MCM).

RC solution and simulation results

We compare five management policies: three Robust Policies (RP1, RP2 and RP3) which are obtained from the RC model with different values of $\theta = \{1, 2, 3\}$, a Nominal Policy (NP, θ = 0) which is essentially a deterministic solution with the average recharge, and a Conservative Policy (CP) which is obtained with the worst case realization, namely minimum recharge in all years. Each of these policies determines “here and now” decisions which are implemented at the beginning of the planning horizon before the uncertainty is revealed.

Figure 2 compares the annual amount of desalinated water in each of these polices. The CP results in constant desalination of 120 (MCM/year), which is the full capacity of the desalination plant. The NP results in taking as much as possible from the aquifers in the first stages while recognizing that the demand is increasing beyond the desalination capacity, which leads to storing aquifer water to close the gap between the demand and the supply capacity at later stages. The conservativeness of the CP over all other policies is apparent. The robust polices RP1, RP2 and RP3 require less desalinated water than the CP, indicating that these policies are not myopic; in other words, they take advantage of the variability of recharge over time. Compared to the NP a robust policy takes more desalinated water in the first stages, resulting in higher water level in the aquifers, which insures manoeuvrability of the aquifer within its operational limits in later years. The degree of conservativeness of the robust policies is noticeable: an RP with smaller θ results in less desalination but lower reliability/immunization and higher penalties, where NP (which is RP with $\theta = 0$) is the lower bound.

The performance of each policy is examined by simulation, which shows the trade-off between the amount of desalination and reliability: lower desalination result in lower reliability. The simulation is run 1000 times with random samples, each with $a_f T_f = 20$ recharge values drawn from the discrete distribution of the recharge. The results for NP and RP3 are shown in Figs 3 and 4: the final water levels in the two aquifers, the total cost and the penalized cost. The feasibility of policies RP1, RP2, RP3 and NP is obviously not guaranteed for all possible realizations of the recharge sequence, as seen by some excursions of the level to negative values, even in RP3, which covers the largest uncertainty set. However, as seen in Fig. 4 for RP3 these are very few; they are fewer as θ increases. In CP there are obviously no infeasibilities, as it considers the worst case, namely the lowest value of the recharge.

Since some of the generated samples can result in the reservoirs/aquifers becoming empty in some year it is necessary to take this into consideration in two respects: (a) continuing the path of the reservoir/aquifer beyond this point, and (b) penalizing the policy for failing to meet the specified operational limits. The two aspects are handled as follows: when the reservoir goes dry it is set to empty as the initial state for the next year:

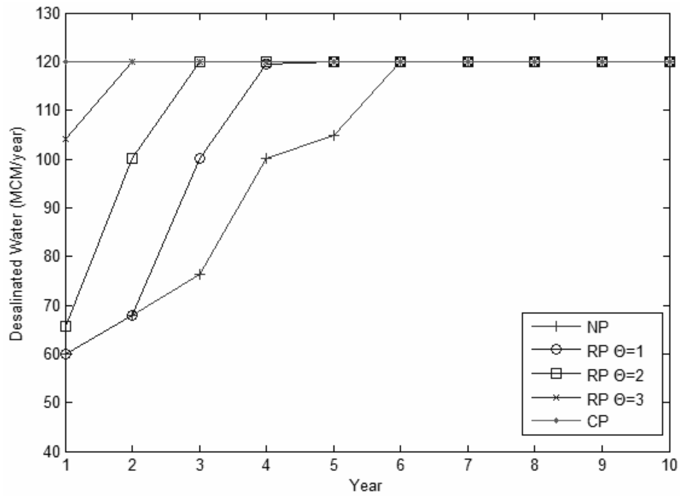

Fig. 2 Annual desalination amounts for the five policies.

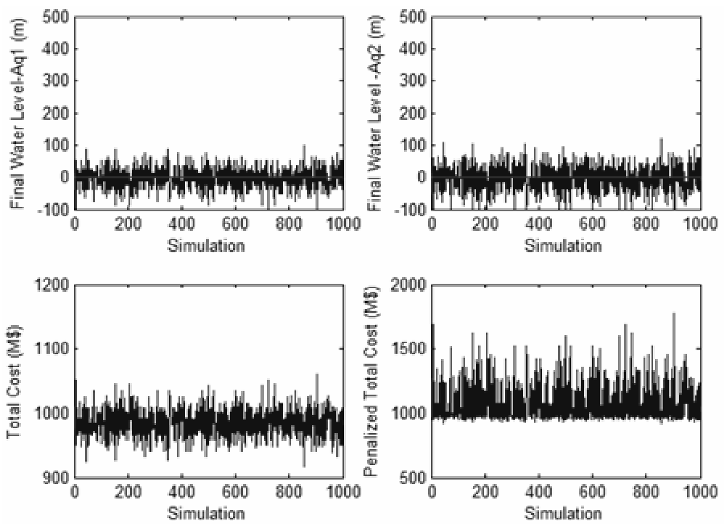

Fig. 3 Simulation results for NP.

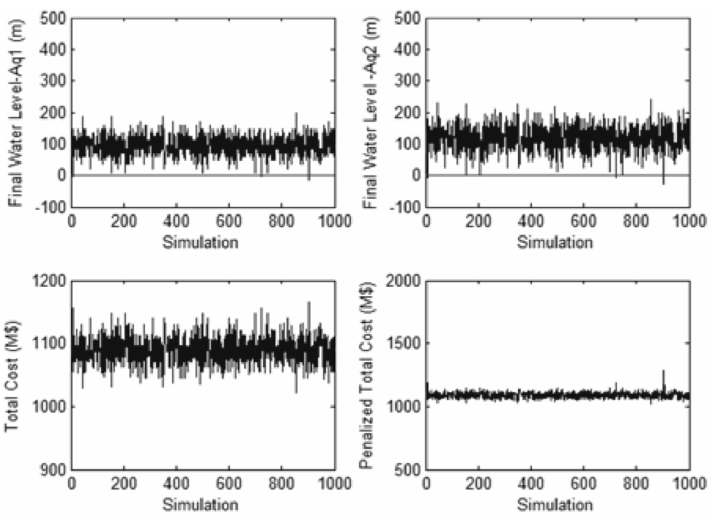

Fig. 4 Simulation results for RP3.

$$h_{a,t+1} = \max(h_{a,t}^{\min}, h_{a,t}) + \left(R_{a,t+1} - Q_{a,t+1}\right) / SA_a \quad \forall a \forall t \tag{13}$$

and a penalty term is added:

$$\max(h_{a,t}^{\min} - h_{a,t}, 0) \cdot DC_t / SA_a \qquad \forall a \forall t \tag{14}$$

where DC_t is the deficit cost, which can reflect demand shortage cost or yield loss. This is used only in evaluating the optimal solution by simulation and does not appear in the optimization models. The Penalized Cost (PenC) which is the operation cost plus the sum of penalties over the 10 years for all simulations appears in Figs 3 and 4.

The NP results in almost 50% of the samples deviating from the limits in both aquifers at the final stage, while in RP3 there are only four deviations over all simulations. Almost 10% of the samples in the NP exceed the worst cost of RP3. Moreover, a very large difference in the cost variability is exposed.

Table 1 reports the empirical maximum, minimum, average and standard deviation of the total cost and PenC for each policy, along with the reliability defined as the fraction of simulations which maintain feasibility in both aquifers in all years. The constant value of the cost standard deviation in Table 1 indicates that all policies were run on the same sample of the recharge.

Table 1 Simulation results.

Policy	Cost (M$)				PenC (M$)				Reliability %
	min	max	mean	std	min	max	mean	std	
NP	916.60	1060.98	984.54	21.27	916.60	1778.32	1074.89	143.71	48.6
RP1	948.43	1092.81	1016.38	21.27	948.43	1613.35	1035.52	74.01	81.4
RP2	983.28	1127.66	1051.22	21.27	983.28	1451.40	1053.66	34.07	97.7
RP3	1021.09	1165.46	1089.03	21.27	1021.09	1289.22	1089.22	22.29	99.7
CP	1101.62	1246.00	1169.56	21.27	1101.62	1246.00	1169.56	21.27	100

The cost of the NP ranges between 916 and 1061 M$ while the cost of RP3 ranges between 1021 and 1166 M$. The NP yields infeasibilities in 51.4% of the samples while RP3 has only 0.3% infeasibilities. Accounting for the cost of infeasibility shows clear preference of RP3 over NP. RP3 immunizes the NP from a reliability of 48.6–99.7% with only 10.6% increase in the mean cost. RP3 immunizes the NP with price of robustness (mean cost increment) of 2.05 M$ for each 1% reliability, while the CP immunize it with price of robustness of 3.6 M$ for each 1% reliability. Comparing CP with RP3 shows clear preference of RP3 since the CP immunizes RP3 by getting rid of the last remaining 0.3% unreliability with an associated cost of 80.5 M$, or 268 M$ for each 1% reliability.

Selecting the size of the uncertainty set against which the resulting policy is immunized (i.e. setting the value of θ) is clearly a multi-objective decision, but some clear choices can be revealed in this example. Figure 5 shows the trade-off between reliability and mean cost, for all policies. The trade-off is characterized by a mild slope of the last segment connecting RP3 with CP, which indicates that a large increment in the mean cost is needed in order to obtain a small increment in reliability. The question to be asked is whether it is justified to add this large cost to immunize against rare events of the recharge. The CP does not violate any constraint over all realizations of the recharge; hence the cost and Penalized Cost are identical. In Table 1 the mean PenC of RP3 is 80.34 M$ less than the mean cost of CP; in contrast the CP maximum cost is 43.22 M$ less than the maximum PenC of RP3. However, further analysis of the cost distribution shows that only one sample in RP3 would exceed the worst cost of the CP (1246 M$) while 695 samples in RP3 are below the best cost of the CP (1101.6 M$). This result shows that implementation of the CP would increase the mean cost by 80.34 M$ while the only gain is reduction of 43.22 M$ in the cost's upper bound, which is rarely realized. Policies RP2 and RP3 are indeed robust, their standard deviations of the PenC are less by a factor of 4.2–6.4 than the NP standard deviation, indicating that these robust policies lead to stable policies without large variability in the associated costs; this can be viewed as preference over other alternatives.

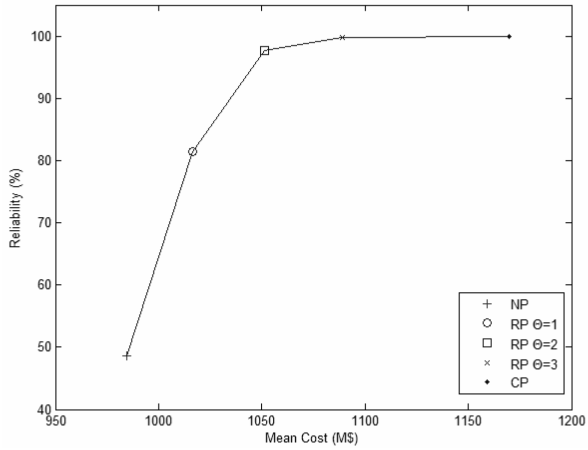

Fig. 5 Trade-off between mean cost and reliability.

CONCLUSION

These results demonstrate the advantage of being able to replace the stochastic behaviour of the uncertainty by specifying a user-defined set within which the resulting policies are immunized, as well as being able to show the trade-off between reliability and mean expected cost.

We have also applied the methodology and its dynamic variant Folding RC (FRC) to the small WSS as a test-bed, and to a central part of the Israeli National Water System, which has three aquifers, three desalination plants, nine consumer zones, and 14 network nodes. The results are very competitive with those obtained by stochastic and deterministic (CP and NP) methods. We continue to develop and test a variety of RO-based methodologies, including Adjustable Robust Counterpart (ARC) and Affine Adjustable Robust Counterpart (AARC) (Ben-Tal *et al.*, 2009).

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