Understanding and getting started with physically based snowmelt models

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Introduction

A previous article in this series (Link et al. 2019) contrasted empirical temperature index models and physically based energy balance models of snowmelt. This article is a primer on the principles and use of physically based snowmelt models. Many such models exist, ranging from simple single-layer representations of snowpack processes in climate and weather prediction models (e.g. Douville et al. 1995) to complex multi-layer models used for avalanche risk forecasting (e.g. Vionnet et al. 2012). They are all physically based in the sense that they are built around equations accounting for physical principles of energy and mass conservation, but representations of energy and mass transfer processes within these models may be physical, empirical, highly simplified or simply neglected. Physically based models have a much wider range of applications than temperature index models, and they can also be used for snowmelt runoff modelling when the necessary inputs are available. Models of this type are essential for land surface schemes that provide surface energy and mass flux inputs for the atmosphere in climate models.

Model concepts and results will be illustrated here using the Factorial Snow Model (FSM, Essery 2015). A distinctive feature of FSM is that it contains representations of snow processes that can be switched on or off independently. These factors controlling snow accumulation and melt can be selected in every possible combination (a factorial experiment design, hence the model name), giving 32 different model configurations. The Fortran code for FSM, a quickstart guide and a range of input data are freely available from links at the end of this article, allowing the user to repeat the examples and attempt their own experiments.

Model terminology

Before getting into the physics, let's establish some widely-used model terminology, using the components of FSM as an example. Models predict the evolution of ***state variables*** in time (e.g. the mass of snow on the ground), they contain internal ***parameters*** (e.g. the albedo of fresh snow) and they require external ***driving variables*** (e.g. the amount of snowfall).

State variables describe the state of a physical system. FSM divides a snowpack vertically into a varying number of layers, depending on snow depth (more sophisticated models preserve layers deposited by different snowfall events), underlain by a fixed number of soil layers. The state variables of FSM are listed in Table 1 (more sophisticated models may have additional state variables, for example to represent the sizes and shapes of snow grains in layers). State variables are often governed by prognostic equations, meaning that the value of a variable at the beginning of a forward step in time (a ***timestep***) is required as an initial condition for prediction of its value at the end of the timestep. State variables therefore have to be stored in computer memory from one timestep to the next.

Diagnostic variables are calculated from the state variables at each timestep and so do not need to be held in memory. Mass and energy fluxes are common examples, e.g. heat flux between snow layers is a diagnostic variable that depends on differences between the prognostic temperatures of the layers and their thermal conductivity, which may be a fixed value or a diagnostic variable depending on snow density.

Table 1. *State variables in FSM*

variable	units	description
N_{snow}	–	number of snow layers
α_s	–	snow albedo
Δz_n	m	thickness of snow layer n
I_n	kg m^{-2}	ice content of snow layer n
W_n	kg m^{-2}	liquid water content of snow layer n
T_0	K	snow surface temperature
T_n	K	temperature of snow layer n
T_{soil}	K	temperatures of soil layers

Parameters are constant quantities that characterize a system. Some parameters may vary spatially in distributed models. Ideally, parameters should be measurable quantities, but selection of parameter values often has to be based on the influence that they have on other variables in model evaluation. This process of ***model calibration*** is common practice for adjusting empirical hydrological models to local conditions, but it is less often undertaken with physically based models that have to be applied globally. The configurations of FSM have between 8 and 14 parameters, depending on their complexity, and all of the FSM parameters can be varied when the model is run for sensitivity or calibration studies. In addition to parameters, models contain other quantities that are never varied; some of these are physical constants (e.g. the Stefan-Boltzmann constant) , and others are treated as such for simplicity. For example, the latent heat of sublimation (the amount of energy required to convert a specific mass of ice into water vapour) has a weak temperature dependence but is invariably taken as a constant in models.

Driving variables are external factors that influence a system. Continuous timeseries of driving variables are required as inputs to models, so any gaps in measurements of variables have to be filled before use. Table 2 lists the driving variables required by FSM. Other models may have additional requirements; for example, a distributed model allowing horizontal redistribution of snow by wind will need wind direction as an input. Minimum requirements for an empirical snowmelt model would be just precipitation and air temperature. Whereas empirical models are often run with daily timesteps, physically based models require substantially shorter timesteps to adequately represent daily melt-freeze cycles.

Table 2. *Driving variables required by FSM*

variable	units	description
$LW_{\downarrow}$	W m^{-2}	incoming longwave radiation
$SW_{\downarrow}$	W m^{-2}	incoming shortwave radiation
R_f	kg m^{-2} s^{-1}	rainfall rate
S_f	kg m^{-2} s^{-1}	snowfall rate
P_s	Pa	air pressure
RH	%	relative humidity
T_a	K	air temperature
U_a	m s^{-1}	wind speed

Distinctions between state variables, diagnostic variables, parameters and driving variables are not rigid and depend on choices made by model developers. For example, snow albedo might be treated as a prognostic variable that decreases as snow ages, it might be a diagnostic function of snow temperature, it might be a fixed parameter in a simple model, or it might not be required if net radiation is provided as a driving variable instead of incoming shortwave radiation.

Testing of model performance and calibration require ***evaluation data***. Measurements of any quantity that corresponds with a model state variable or diagnostic can be used for evaluation. Driving and evaluation data periods have to overlap but, unlike driving data, evaluation data do not need to be continuous. For example, intermittent manual measurements of snow mass on the ground are often used for evaluating snow models. Low errors for a model after calibration for a set of evaluation data are no guarantee that the model can make accurate predictions for other periods or locations. For this reason, the data should be divided into calibration and evaluation sets.

Whenever a measurement of a quantity predicted by a model is available, an error can be calculated by taking the difference between the measured and predicted values. When many measurements are available for different times or different quantities, statistical metrics are required to summarize the errors. Many different metrics have been used for model evaluation, and there are lively debates in the hydrological literature about which metric is most relevant and what values have to be achieved for a model to be regarded as "good". The choice of metric is less critical for the simple signal of snow mass increasing and then decreasing over a seasonal cycle. However, model deficiencies, parameter uncertainty, driving data errors and evaluation data errors all contribute to differences between model predictions and measurements, and they may be impossible to disentangle (Günther et al. 2019).

Energy and mass balances

Energy and mass balances for the snow surface and internal snow layers in FSM are illustrated in Figure 1. For snow layer n with areal heat capacity c_n, conservation of energy is expressed by

$$\frac{d}{dt}\left(c_n T_n - L_f I_n\right) = G_{n-1} - G_n, \tag{1}$$

where L_f is the latent heat released per kg of liquid water freezing in the snow. G_{n-1} and G_n are vertical heat fluxes into the top of the snow layer and out of the bottom; they may be conducted fluxes due to temperature gradients or advected fluxes due to liquid water moving within the snow. To calculate the heat flux at the bottom of a snowpack, the temperature of the underlying surface has to be known from measurements or a coupled soil thermodynamics model. The net heat flux at the surface of the snow is given in FSM by a surface energy balance equation

$$G_0 = (1-\alpha_s)SW_\downarrow + \varepsilon(LW_\downarrow - \sigma T_0^4) - H - L_s E - L_f M, \tag{2}$$

where ε is the thermal emissivity of snow (assumed to be 1 in FSM), L_s is the latent heat of sublimation and M is the surface melt rate. Net radiation

$$R_n = (1-\alpha_s)SW_\downarrow + \varepsilon(LW_\downarrow - \sigma T_0^4) \tag{3}$$

is absorbed by the snow surface. Sensible heat flux H and moisture flux E to the atmosphere are calculated as functions of air temperature, humidity, wind speed and surface characteristics. Using these functions, solving Equation 2 gives the surface temperature and melt rate. Some fluxes are neglected in FSM that may be included in other models; for example, shortwave radiation is assumed to be absorbed and reflected right at the surface (in reality, shortwave radiation penetrates some distance into snow) and heat advected by rain falling on snow is neglected (although latent heat released if the rain water freezes is included in Equation 1).

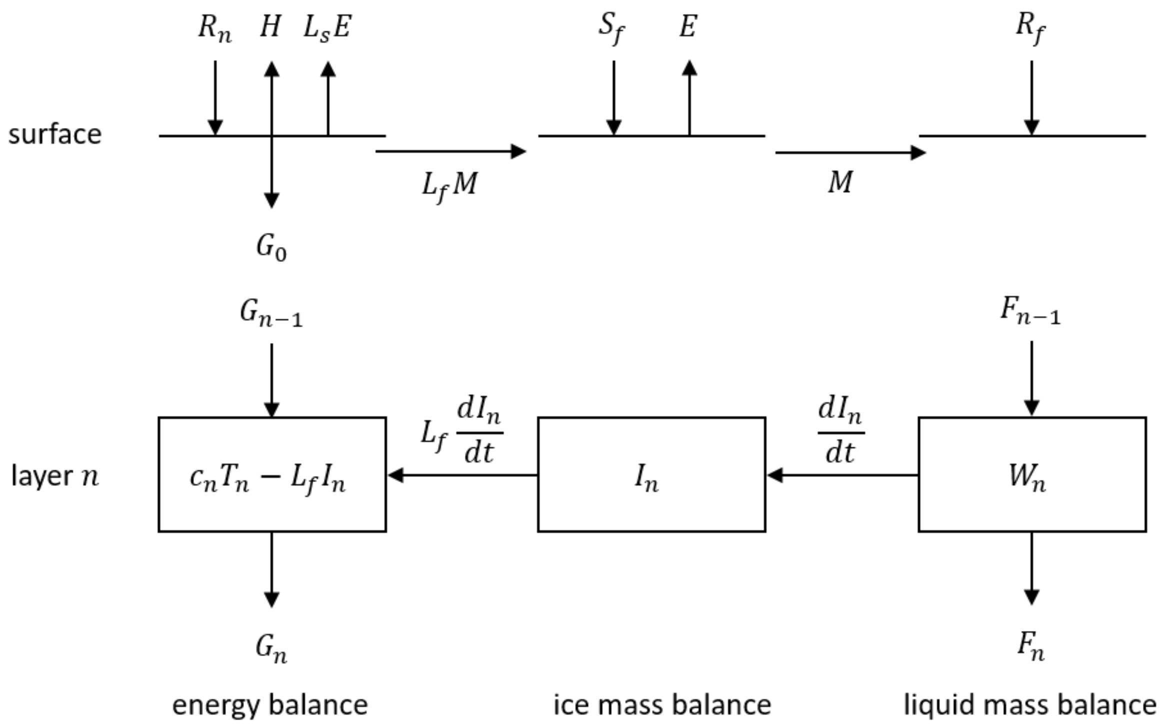

Figure 1. *Schematic of the energy, ice mass and liquid mass balances of the snow surface and an internal snow layer in FSM. The terms represent state variables, fluxes and constants described in the text.*

The mass balance equations for ice and liquid water in the top snow layer are

$$\frac{dI_1}{dt} = S_f - E - M \tag{4}$$

and

$$\frac{dW_1}{dt} = R_f + M - F_1, \tag{5}$$

where F_1 is the flux of liquid water draining out of the layer. For buried snow layers, liquid water and ice mass changes are coupled by

$$\frac{dW_n}{dt} = F_{n-1} - F_n - \frac{dI_n}{dt}. \tag{6}$$

F_{n-1} and F_n are vertical water fluxes into the top of the snow layer and out of the bottom, and dI_n/dt is the rate of freezing of liquid water within the layer. The latent heat terms for sublimation, melting and freezing couple the energy and mass balance equations.

Parametrizations are simplified methods for representing processes that are too complex or operate on too small a scale to be explicitly resolved by a model. For example, all of the conduction, advection and radiation processes by which heat can be transferred within snow are generally represented by an effective thermal conductivity in a simple linear relationship between heat flux and temperature gradient. Essery (2015) gives full details of the parametrizations used in FSM to calculate fluxes and snow properties required in the solution of the energy and mass balance equations. Two options are available for representing each of five snow properties and fluxes:

- snow albedo may be diagnosed as a function of temperature or predicted as a function of snow age;
- thermal conductivity may be a fixed parameter or a function of snow density;
- snow density may be a fixed parameter or a function of snow age;
- heat and moisture fluxes to the atmosphere may be calculated assuming neutral stratification or adjusted for atmospheric stability;
- liquid water may drain immediately from a snow layer, or an amount of water may be retained in snow at 0°C and can refreeze in cold snow.

FSM can thus be run in 32 different configurations, numbered from 0 to 31, to generate ensembles of snow simulations. Some of the configurations neglect important processes and might be expected to give poor simulations, but they are included because they match simplifications made in some existing models.

Examples

Link et al. (2019) presented examples of snowmelt rates calculated with a temperature index model using air temperature measurements from the Reynolds Creek Experimental Watershed (RCEW) in Idaho. They fitted the threshold temperature and melt factor parameters of the model to melt rates derived from snow mass measurements in 1985, and subsequently used the same parameter values to compare with measurements in 1992. Figure 2 shows that the model fits the measurements well in the winter of 1984-1985 after calibration to minimize the root mean square error, but it melts the snow too early in the warmer and less snowy evaluation winter of 1991-1992.

Reba et al. (2011) collated all of the data required for running physically based snow models on hourly timesteps at RCEW, plus evaluation data including hourly measurements of mass with a snow pillow. Here we use these data to run and evaluate FSM. For comparability with the calibrated temperature index model, FSM configuration 31 was calibrated by adjusting just two of its 14 parameters: the minimum albedo of melting snow and surface roughness length. FSM 31 and the temperature index model match snow mass measurements about equally well for the calibration winter in Figure 2a, but FSM 31 is more robust when the calibrated parameters are transferred to a simulation for the evaluation winter in Figure 2b.

Running the 32 configurations of FSM without calibration produces a wide range of different results shown in Figure 3. Many of the simulations melt the snow too early in both winters. Calibrating each configuration separately greatly reduces the model spread for 1984-1985, but **transferring the calibrated parameter values also reduces the spread for 1991-1992**. This does not mean that all of the model configurations are equally good, despite some of them neglecting processes that are expected to be important. Mass balance measurements alone do not contain enough information to tell if an energy and mass balance model is getting "the right results for the wrong reasons".

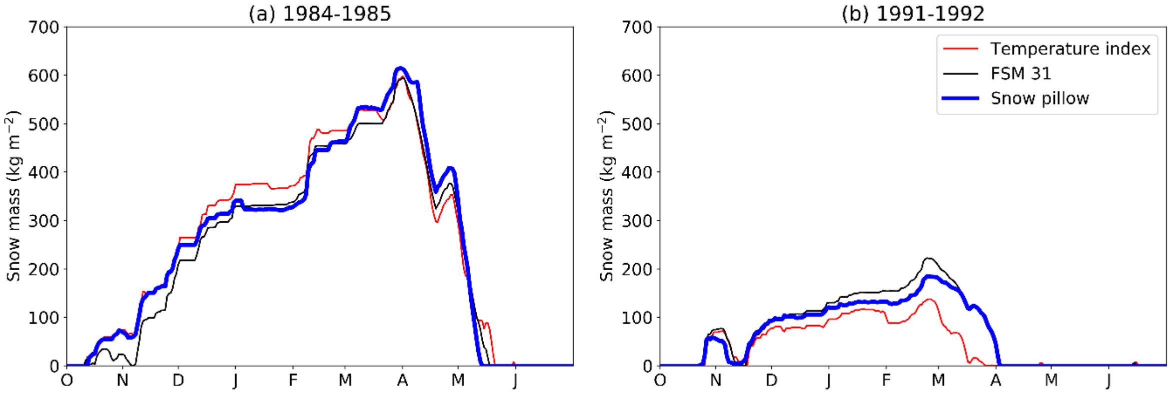

Figure 2. *Snow mass on the ground at RCEW in winters 1984-1985 and 1991-1992 measured with a snow pillow and predicted with a temperature index model and FSM configuration 31. Two parameters in each model were adjusted to minimize errors for 1984-1985; the same parameter values were used without adjustment for 1991-1992.*

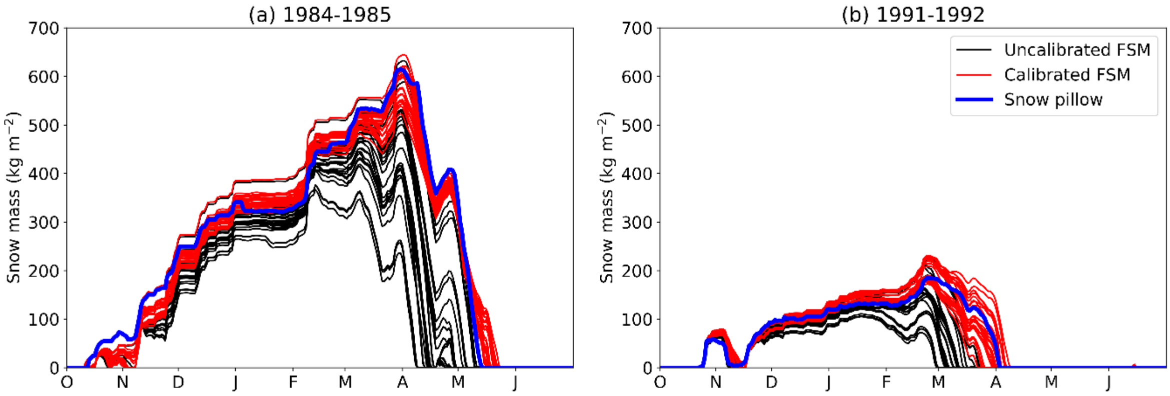

Figure 3. *Simulations of snow mass by the 32 configurations of FSM without calibration and calibrated to minimize errors in comparison with measurements for 1984-1985.*

Understanding why a model may work well in some situations but not others requires understanding the processes that contribute to snow accumulation and melt. Because shortwave and longwave radiation and sensible, latent and ground heat fluxes can all provide energy to melt snow, models can produce similar mass balance simulations while partitioning the energy balance in different ways. This is illustrated in Figure 4 by plotting the contributions of net radiation to snowmelt against contributions from sensible, latent and ground heat fluxes for the calibrated FSM ensemble. Simulations by model configurations that drain melt water from snow immediately lie close to lines that correspond with the amount of energy required to melt all of the snow that fell; model configurations that retain liquid water and allow it to freeze lie above the lines because additional energy is required to melt the frozen water. The simulations vary from having nearly equal amounts of energy provided for melt by net radiation and sensible heat, to all of the energy being provided by radiation, to radiation having to provide additional energy because the surface is cooled by sublimation. Measurements of energy balance components are not available in the RCEW dataset. Where energy balance measurements are available, they could be used to discriminate between models that appear to have similar performances in simulating snow mass balances and hence to eliminate models that give the right answer for the wrong reason.

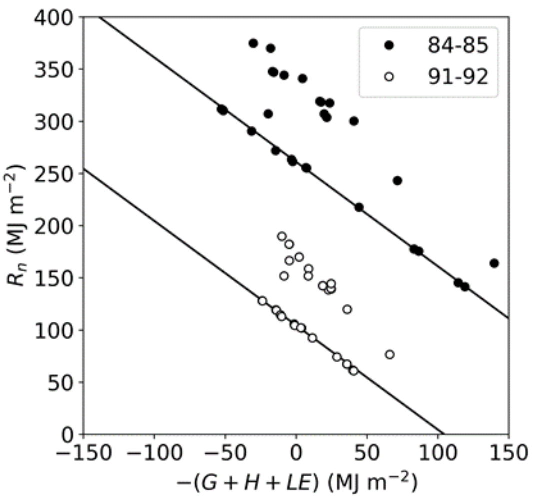

Figure 4. *Contributions of energy for snowmelt from net radiation plotted against contributions from turbulent and conducted heat fluxes in calibrated FSM simulations for 1984-1985 and 1991-1992. The lines have slope –1 and intercepts equal to the energy required to melt all of the snowfall; model configurations that lie close to the lines are ones that do not permit refreezing of surface melt water in the snow (they may lie slightly below the line because some snow is also removed by sublimation).*

Try this at home

The code for FSM can be downloaded from https://github.com/RichardEssery/FSM and compiled on a linux or Windows computer with a Fortran compiler (several free Fortran compilers are available; see https://gcc.gnu.org/wiki/GFortranBinaries). The github README provides a quickstart guide for FSM. Driving and evaluation data for one winter at the Météo-France Col de Porte snow research site (Morin et al. 2012) are provided as an example. Data for several other sites that have been used in the Earth System Model-Snow Model Intercomparison Project (ESM-SnowMIP, Ménard et al. 2019), including 20 years of data for Reynolds Creek, can be downloaded from https://www.geos.ed.ac.uk/~ressery/ESM-SnowMIP/text.zip. As a simple alternative to FSM, ESCIMO.spread (Marke et al. 2016) is an energy and mass balance snow model implemented in a spreadsheet that can be downloaded from https://www.acwr.eu/escimo.html.

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