A similarity inspired enhancement for estimating dispersion coefficients in rivers

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Abstract Estimating dispersion coefficients using the method of moments involves computing the variances of observed concentration-time profiles collected during tracer experiments. A major disadvantage is that poor quality profiles (particularly those that are incomplete) yield inaccurate estimates of the variance. Having shown that all profiles collected at a single monitoring site collapse towards a common non-dimensional profile, the paper innovatively proposes that, where necessary, the variance of the common non-dimensional profile could be used instead of the variance of an individual profile. Equations are presented that relate the properties of dimensional and non-dimensional profiles. A practical application using tracer data from the Murray Burn in Edinburgh, UK is described. The results shows that the new method (termed the enhanced method of moments) reduced the errors in dispersion coefficients, derived from observed concentration-time profiles with incompletely observed trailing edges, compared to those found using the ordinary method of moments by between 12 % and 57 %. Some features of the analysis that might have limited the success of the new method are suggested, and some ideas for refining it are proposed.

Keywords dispersion coefficient; tracer experiment; profile variance; method of moments

INTRODUCTION

A common approach for predicting the impact of pollution incidents on river systems is to simulate the movement and mixing of a pollutant as it is transported downstream by the flow of water. When the impact requires investigation over long river reaches a one-dimensional mathematical model based on the advection-dispersion equation (ADE) is usually used (Fischer *et al*, 1979; Rutherford, 1994). This simulates the spatial and temporal evolution of the cross-sectional average pollutant concentration. The successful use of this type of model requires that estimates of the dispersion coefficient are available over a wide range of flow rates. If this information is not already known modellers are faced with some difficult choices because the dispersion coefficient is very difficult to predict accurately. Wallis & Manson (2004) describe various options, but only two approaches offer a practical solution. These are (1) using one of several empirical equations for predicting the coefficient directly, typically as a function of bulk flow parameters such as velocity and channel width, and (2) undertaking tracer experiments at several different flow rates in the river of interest from which the coefficient and its flow dependence can be inferred. Although the former are generally easy to use the results are not very reliable. In contrast, the latter require investment of time and resources in executing the experimental work, but are capable of yielding a wealth of relevant information.

The usual format of a tracer experiment consists of the rapid release of a known mass of tracer at the top of a river reach followed by the collection of in-river tracer concentration-time profiles at several sites further downstream. The traditional method for estimating the dispersion coefficient in each reach between monitoring sites is the method of moments (Rutherford, 1994). The method has the advantage of simplicity, requiring only the evaluation of simple statistical properties of observed concentration-time profiles, but over the last decade or so it has become unpopular (see, for example, Seo & Cheong, 1998). The main reason for this is that computed variances of observed concentration-time profiles are very sensitive to the quality of the information contained in the trailing edges of the concentration-time profiles. Indeed, a frequent problem is that the final part of the trailing edge is missing. This can introduce very large errors into the computed dispersion coefficient.

The aim of this paper is to propose a method for overcoming the shortcoming of the method of moments when the trailing edge of a profile is not completely observed. It is based on the idea that the information from several tracer experiments can be pooled so that, for example, part of a missing trailing edge from one experiment could be replaced by information gleaned from an observed trailing edge from a different experiment. The way in which this is done exploits the previously reported phenomenon that when all observed tracer profiles at a particular location are plotted in a particular non-dimensional form they all have a similar shape (Day & Wood, 1976; Wallis, 2005). Thus a satisfactory evaluation of the variance of the common non-dimensional profile shape yields satisfactory estimates of the variances of all the pooled profiles, including those with incomplete trailing edges. This method has the potential to improve existing estimates of dispersion coefficients from tracer profiles that are almost complete and to release untapped knowledge from tracer data that has been discarded in previous studies, because trailing edges were not observed. Both of these could significantly improve the quantity and quality of published dispersion coefficients.

The paper is divided into three main sections. Firstly, background information on the similarity of observed concentration profiles is presented. Secondly, the traditional method of moments is described and an enhanced version of it is proposed. Thirdly, the potential of the enhanced method for improving the estimation of dispersion coefficients is demonstrated using observed tracer data from the Murray Burn in Edinburgh, UK.

BACKGROUND

Fig. 1 shows four observed concentration-time profiles taken from a series of tracer experiments undertaken in the Murray Burn in Edinburgh. The data were collected at the same location (Site 1) under four different (steady) flow rates following the instantaneous release of tracer at the same injection site. The following equations can be used to convert the concentration-time data into a non-dimensional form:

$$C=ccp \qquad (1)$$

$$\tau=t-tLtT-tL \qquad (2)$$

where C is the non-dimensional concentration, c is the observed concentration, c_p is the observed peak (maximum) concentration, τ is the non-dimensional time, t is the observed time, t_L is the observed time of the leading edge of the profile and t_T is the observed time of the trailing edge of the profile. The definition of the observed times of the profile edges is related to the occurrence of concentrations that are a fixed percentage of the peak concentration. Day & Wood (1976), who first suggested this way of converting such data into non-dimensional form, used 50 %: following some earlier work (Wallis, 2005), 10 % is used herein. Fig. 2 gives the result of converting the data shown in Fig. 1.

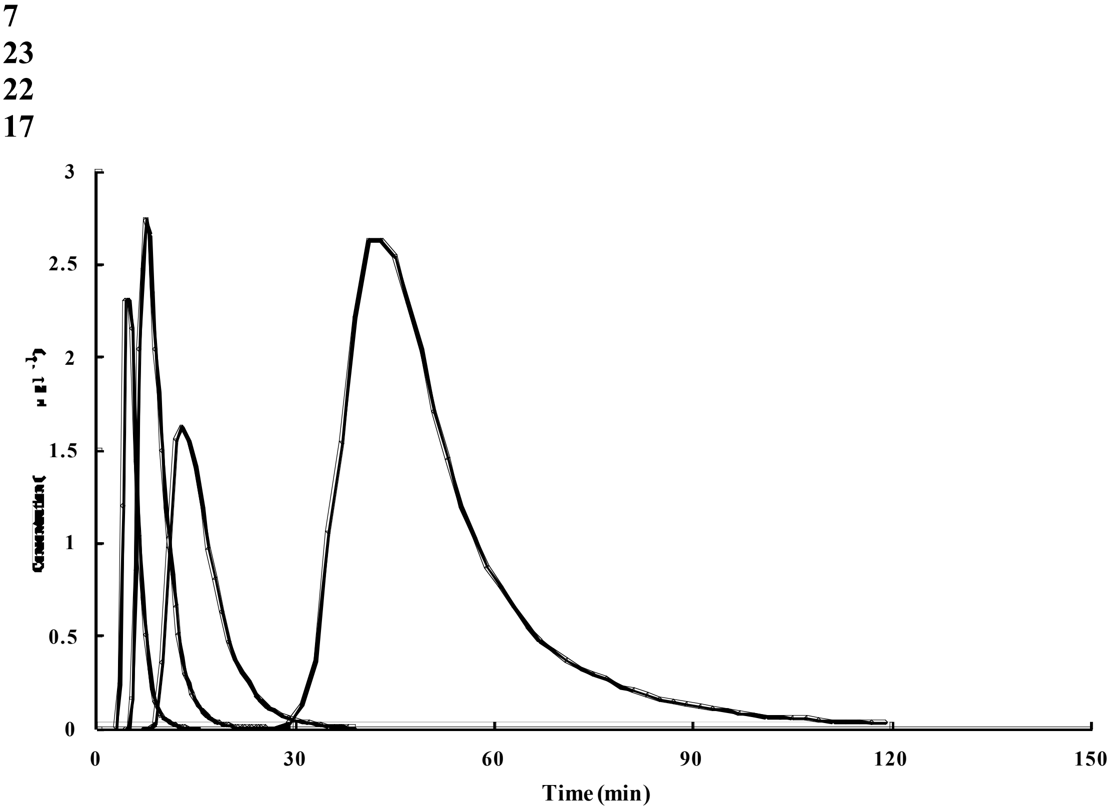

Fig. 1 Observed tracer profiles at Site 1 from four tracer experiments; labels indicate experiment number.

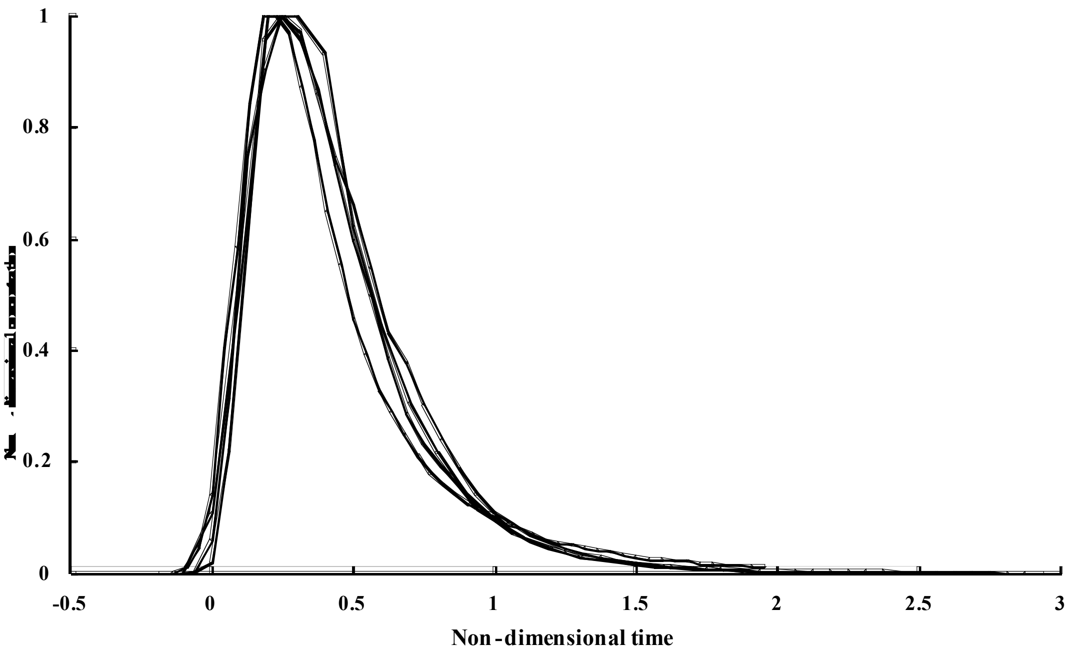

Fig. 2 Non-dimensional tracer profiles at Site 1 from four tracer experiments.

Clearly, the peak of the non-dimensional concentration is 1, and the leading and trailing edges occur at non-dimensional times of 0 and 1, respectively, corresponding to non-dimensional concentrations of 0.1. The collapse of the data towards a common non-dimensional profile is particularly striking. It is unclear, however, whether in the absence of errors the individual non-dimensional profiles would be the same, which is a tantalising prospect, or if there is some physically-based reason for some small variation between them. Apart from measurement errors in the original data, a potential source of error comes from determining the times when the concentration is 10 % of the peak, i.e. in estimating t_L and t_T. For example, because the discrete nature of the data rarely captures the required times exactly, some interpolation may be required between the closest neighbouring data points. In Fig. 2 the nearest data point was used rather than an interpolated one. Figs. 3 and 4 show the corresponding observed and non-dimensional data from the same tracer experiments at a location further downstream (Site 2).

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7
17
23

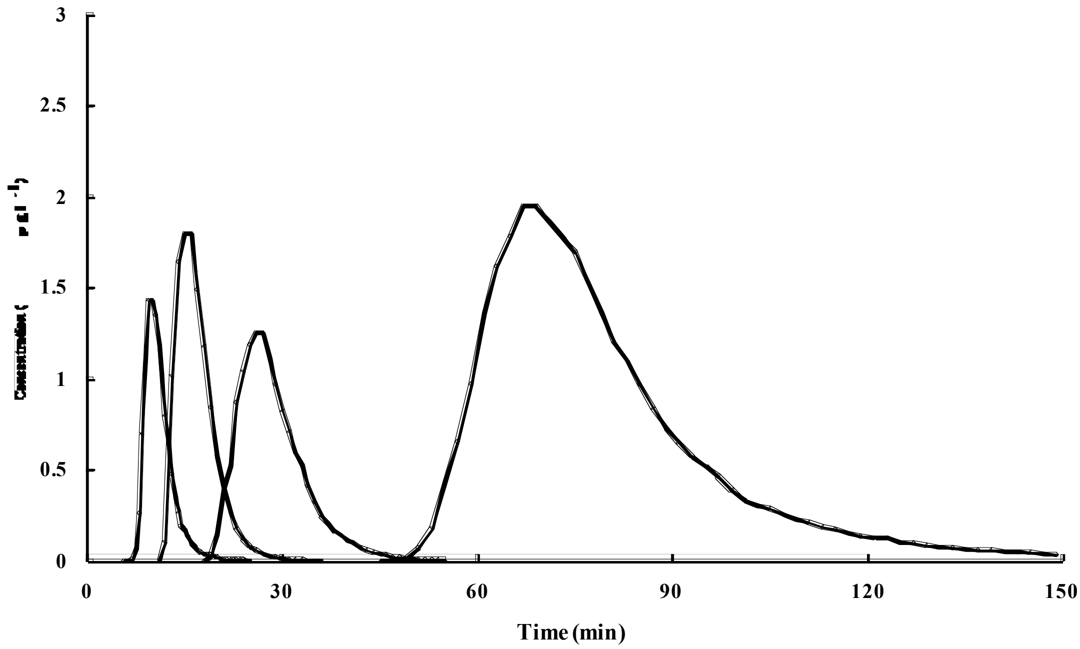

Fig. 3 Observed tracer profiles at Site 2 from four tracer experiments; labels indicate experiment number.

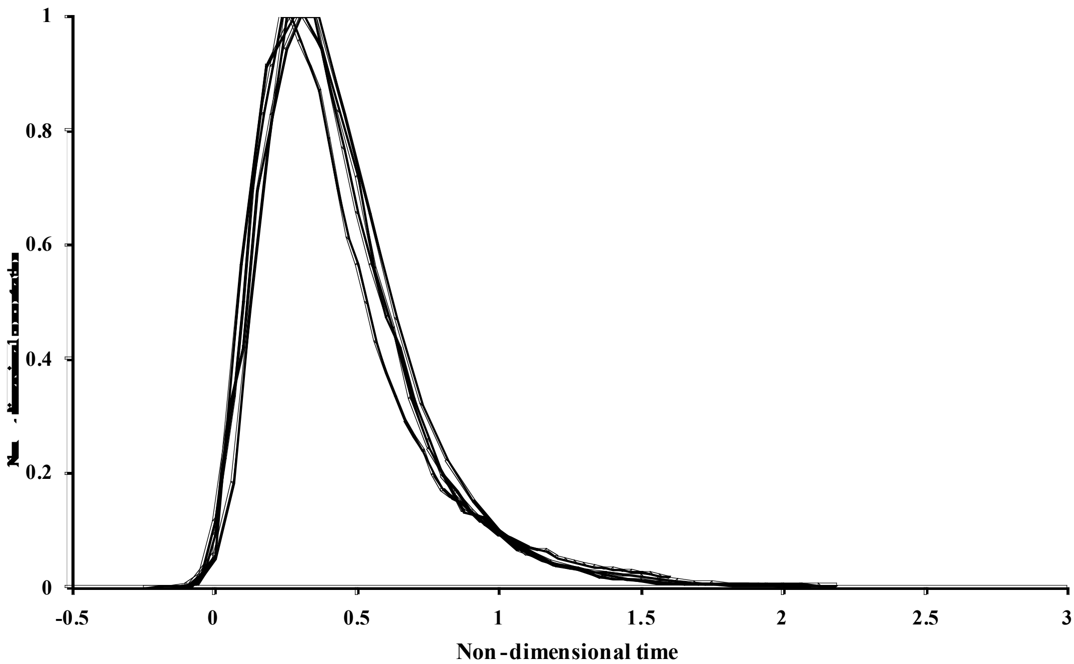

Fig. 4 Non-dimensional tracer profiles at Site 2 from four tracer experiments.

METHOD OF MOMENTS

The nth moment of a concentration-time profile is given by the following equation:

nth moment= 0∞tncx,t dt (3)

where t is time after release of the tracer, $c(x,t)$ is the concentration at location x at time t and n is an integer. By computing the first three moments (n = 0, 1, 2) the following properties of a concentration-time profile can be found:

area under the profile, α = 0∞cx,t dt (4)

centroid time, μ = 0∞tcx,tdt α (5)

variance, σ2= 0∞t2cx,t dtα- μ2 (6)

In practice the moments are evaluated numerically by replacing the integrals with summations, and the upper limit is taken as the time of the last observed concentration. The dispersion coefficient, D, for a reach of length, L, is found from:

D= 0.5L2σb2-σa2μb-μa3 (7)

where b and a refer to the downstream and upstream boundaries of the reach.

ENHANCED METHOD OF MOMENTS

By analogy with the above, the properties of a non-dimensional concentration-time profile are found using:

area under the profile, ατ = 0∞Cx,τ dτ (8) centroid time, μτ = 0∞τCx,τdτ ατ (9)

variance, στ2= 0∞τ2Cx,τ dτατ- μτ2 (10)

The relationship between the properties of the dimensional concentration-time profiles and the properties of the non-dimensional profiles is straightforward, being determined by the nature of equations (1) and (2). Thus:

area, α=ατcptT-tL (11)

centroid time, μ=μτtT-tL+tL (12)

variance, σ2=στ2tT-tL2 (13)

which reflect the facts that the area is related to the height and width of a profile, the centroid time is related to the timing and width of a profile, and the variance is related only to the width of a profile. Hence α, μ and σ^2 can be estimated from α_τ, μ_τ and σ_τ^2 provided c_p, t_L and t_T are observed, and then the dispersion coefficient is calculated from equation (7).

APPLICATION TO THE MURRAY BURN

The Murray Burn is a small river that flows through the Heriot-Watt University campus at Riccarton in Edinburgh. The tracer data used in the paper were collected at both ends of a 137 m long study reach, and are taken from four individual tracer experiments conducted at flow rates between 14 and 260 l s^{-1}. Data at the top of the study reach are shown in Fig. 1 and data at the bottom of the study reach are shown in Fig. 3. The study reach has a longitudinal slope of 0.025 and a mean width of 3.7 m. The bed material consists of a wide range of grain sizes, being composed predominantly of cobbles of nominal sizes between 1 cm and 15 cm. Flow depths vary both along the reach, and with flow rate, being typically < 0.3 m. The tracer was injected 120 m upstream of the top of the study reach.

The data shown in Figs. 1 and 3 constitute four pairs of upstream and downstream concentration-time profiles from which dispersion coefficients can be estimated. Since the experiments were undertaken at different flow rates, the variation of the dispersion coefficient with flow rate can also be examined. This is precisely the sort of information that would be required when setting up a pollution alarm model for predicting the impact of pollution incidents on, for example, a river used for supplying water for domestic consumption.

Table 1 (column 3) shows dispersion coefficients obtained using the method of moments with the dimensional profiles (this is termed the ordinary method of moments (Omom)). Since all eight profiles are complete (or very nearly complete) and are generally well-resolved it can be expected that the computed dispersion coefficients are reliable. This is supported both by the smooth increase of the dispersion coefficients with flow shown in Fig. 5 (Cases 1 – 4 are introduced below) and by comparison with dispersion coefficients for experiments 7 and 23 given in Wallis and Manson (2005), which are 0.985 m^2 s^{-1} and 0.587 m^2 s^{-1}, respectively. These values were obtained by optimising a numerical model of the ADE and are, therefore, independent from the ordinary method of moments.

Table 1 Dispersion coefficients (D) for Murray Burn evaluated in five different ways.

Experiment	Flow rate ($l\ s^{-1}$)	D, Omom ($m^2\ s^{-1}$)	D, Case 1 ($m^2\ s^{-1}$)	D, Case 2 ($m^2\ s^{-1}$)	D, Case 3 ($m^2\ s^{-1}$)	D, Case 4 ($m^2\ s^{-1}$)
7	134	0.979	1.819	-ve	-	-
17	14	0.650	1.164	-ve	0.961	0.442
22	260	3.084	3.360	0.671	2.827	2.953
23	60	0.650	1.153	-ve	0.176	0.825

The power of the enhanced method of moments described earlier is illustrated in Fig. 5, which compares the ordinary method of moments dispersion coefficient results from Table 1 (column 3) with those obtained from a series of numerical experiments that are described below. The aim of these numerical experiments was to illustrate how better estimates of dispersion coefficients (compared to those obtained from the ordinary method of moments) can be obtained when concentration-time profiles have incomplete trailing edges. The dispersion coefficient values are given in Table 1 (columns 4 – 7).

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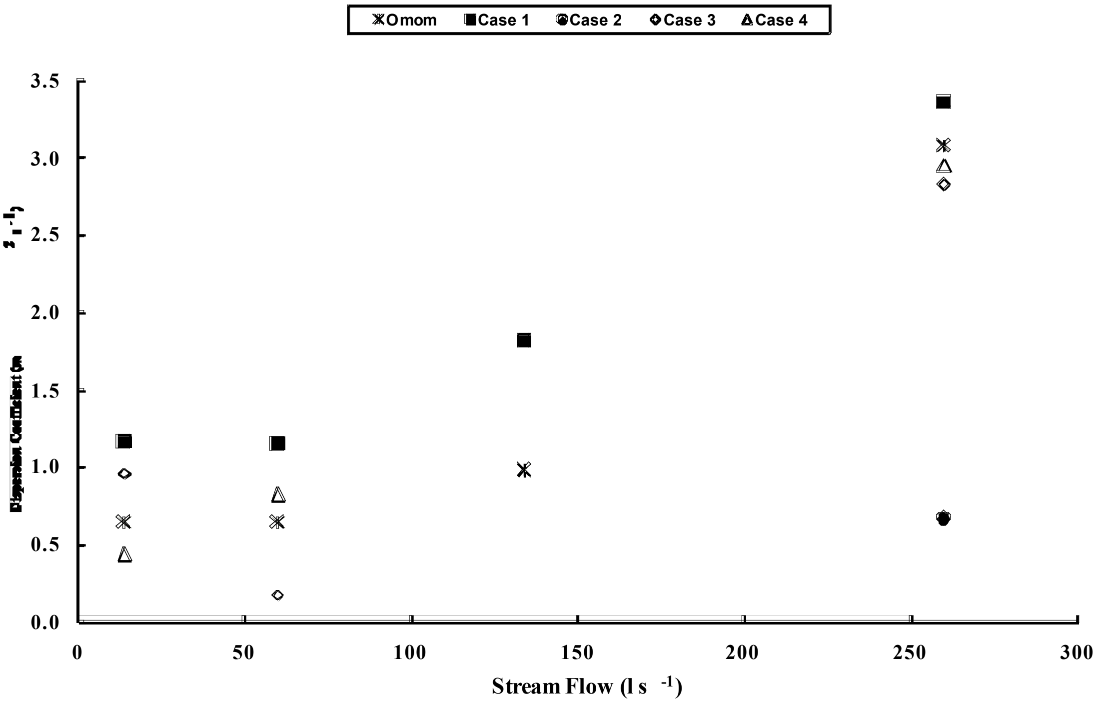

Fig. 5 Computed dispersion coefficients for four tracer experiments (negative values omitted); labels indicate experiment number.

A scenario was assumed in which only the concentration-time profiles from experiment number 7 were in fact complete, and that the final part of the trailing edge of some of the other profiles had not been observed. This was achieved by truncating them at the first data point that had a concentration less than 10 % of the peak concentration. The following analyses were undertaken for experiment numbers 17, 22 and 23: Case 1 – ordinary method of moments using complete downstream profiles and truncated upstream profiles; Case 2 – ordinary method of moments using complete upstream profiles and truncated downstream profiles; Case 3 – enhanced method of moments using complete downstream profiles and truncated upstream profiles, and using experiment number 7 to provide the properties of the required upstream non-dimensional concentration-time profile; Case 4 – enhanced method of moments using complete upstream profiles and truncated downstream profiles, and using experiment number 7 to provide the properties of the required downstream non-dimensional concentration-time profile. Note that in Cases 3 and 4, the truncated profiles were only used to estimate c_p, t_L and t_T, because of the available surrogate information from experiment 7. For completeness, Cases 1 and 2 were also undertaken for experiment 7, but Cases 3 and 4 were superfluous because experiment 7 was used to provide the profile properties in the enhanced method of moments – the results would have been the same as those shown in Table 1 (column 3).

Figure 5 illustrates that the results from Cases 3 and 4 (enhanced method of moments with incomplete profiles) are closer to the original values (ordinary method of moments with complete profiles) than the results from Cases 1 and 2 (ordinary method of moments with incomplete profiles). Also, a common failing of the ordinary method of moments, namely that it can produce negative values, has been eliminated. A negative value arises when the dimensional downstream profile has a smaller variance than the corresponding upstream profile. The usual cause of this is a missing final part of the trailing edge on the downstream profile, as indeed was the situation for Case 2. Unsurprisingly, therefore, experiments 7, 17 and 23 returned negative dispersion coefficients for this case. Clearly, negative dispersion coefficients have no physical meaning and are therefore not shown in Fig. 5: they are denoted by "-ve" in Table 1. With the enhanced method of moments the negative dispersion coefficients have been eliminated because the surrogate variance of the non-dimensional downstream profile provided by experiment 7 enables a good (although not necessarily an optimum) estimate of the variance of the complete dimensional downstream profile to be made.

The degree of improvement in the dispersion coefficient estimates for experiments 17, 22 and 23 obtained by using the enhanced method of moments in comparison to the ordinary method of moments can be quantified as follows. Comparing Cases 1 and 3, the average magnitude of the percentage error reduced from 55 % to 43 %, respectively: comparing Cases 2 and 4, the average magnitude of the percentage error reduced from 78 % to 21 %, respectively. These percentage errors were evaluated by comparing dispersion coefficients from Cases 1 – 4 with the original method of moments dispersion coefficients in Table 1 (Column 3), and ignoring any negative values. It would be unwise to associate too much significance with these errors magnitudes because they are based on a very small number of experiments. Nevertheless, they confirm that the enhanced method has real potential for improving the estimation of dispersion coefficients from incompletely observed tracer data in comparison to the ordinary method of moments.

DISCUSSION

The degree of success of the enhanced method is clearly dependent on how similar the non-dimensional profiles are to one another at a particular site. Fig. 2 shows that there are small differences between the four profiles at Site 1, and Fig. 4 shows that there are small differences between the four profiles at Site 2 also. The effect of these differences on the properties of the non-dimensional profiles is shown in Table 2, which shows that whilst there is a similar variability in the area and centroid at the two sites, there is greater variability in the variance at Site 1 than at Site 2. This is probably the reason why the reduction in average percentage error of the dispersion coefficient between Cases 1 and 3 is rather modest compared to that between Cases 2 and 4 (although it should also be recognised that the latter is based on only one experiment, because of the negative dispersion coefficients found for Case 2 with experiments 17 and 23).

Table 2 Properties of non-dimensional concentration profiles from the Murray Burn.

	Site 1			Site 2		
Experiment	Area	Centroid	Variance	Area	Centroid	Variance
7	0.567	0.455	0.103	0.592	0.436	0.078
17	0.527	0.425	0.114	0.514	0.426	0.084
22	0.560	0.429	0.084	0.532	0.453	0.083
23	0.516	0.436	0.079	0.539	0.429	0.071
Mean	0.540	0.436	0.095	0.544	0.436	0.079
s.d.	0.027	0.013	0.017	0.033	0.012	0.006

s.d. = standard deviation

Although the potential of the enhanced method for improving the evaluation of dispersion coefficients from the moments of concentration-time profiles has been demonstrated, more work on two issues, in particular, is required to refine things further. Firstly, on minimising the variability between site-specific non-dimensional profiles derived from separate tracer experiments and, secondly, on improving the estimate of the surrogate profile properties derived from the site-specific non-dimensional profiles.

In regard to the former, a potentially significant source of error in constructing a non-dimensional profile is the identification of the times that characterise the leading and trailing edges (t_L and t_T). In the analysis of the Murray Burn data described above the data point with the closest concentration to 10 % of the peak was used to identify these times. Further work should investigate the merits of interpolating between the data points that span the required concentration in order to improve the estimation of these times, particularly if the profiles are coarsely sampled. This may reduce the variability between non-dimensional profiles from different experiments. There may be advantages also in choosing a different concentration to characterise the times of the leading and trailing edges because the sensitivity of the identification of these times may vary with the slope of the profile.

In regard to the latter issue introduced above, using a consistent range of non-dimensional time over which to evaluate the surrogate profile properties may be beneficial, as may the use of mean surrogate properties derived from several profiles. In the example described above, profile properties were based on the entire available profile (with the end of the trailing edge occurring at non-dimensional times which varied between 1.6 and 2.9), and only one profile was used to provide the surrogate properties (which simulated a worst case scenario of only one experiment being available that contained a completely observed concentration-time profile).

CONCLUSIONS

The paper describes a method for improving the accuracy of computed variances of incompletely observed concentration-time profiles from tracer experiments in rivers. The enhanced method of moments is similar to the ordinary method of moments, but improves it by pooling information from several observed profiles collected at the same site during separate experiments. The pooling is possible because there is evidence that profiles collected at the same site exhibit similarity and can be converted to a common non-dimensional profile. This implies that all site-specific non-dimensional profiles have very similar properties, such as centroid time and variance. Hence only a single completely observed concentration-time profile might be sufficient to provide good estimates of these properties for use with incompletely observed profiles.

The potential of the enhanced method to improve the estimation of dispersion coefficients in rivers was illustrated through a practical example using tracer data from the Murray Burn in Edinburgh. Not only were errors in estimated dispersion coefficients reduced compared to using the ordinary method of moments, but cases where incompletely observed trailing edges cause the ordinary method of moments to yield (non-physical) negative values were effectively handled by the enhanced method. Some ideas for improving the implementation of the enhanced method to observed tracer data were proposed.

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