Multilinear flood routing model for alluvial rivers

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Abstract Limited time schedules and project costs usually do not allow for detailed field investigations, which would supply sufficient and up-to-date appropriate channel geometry and roughness data describing the morphologic and hydraulic channel characteristics for hydrodynamic modeling. This hinders the application of hydraulic models for flood routing especially in flood forecasting on rivers with unstable channels. In such cases the application of complete distributed hydraulic flow routing models is neither justifiable nor advantageous. Therefore, in the HRON conceptual rainfall- runoff model, which is in preparation in several catchments in the Flood Forecasting System of Slovakia, the application of a nonlinear hydrologic flood routing method was chosen as a rational alternative. In the contribution this nonlinear hydrologic routing model is described and verified on an alluvial river with strongly discharge dependent flood wave celerity. This model, which model belongs to the family of multilinear models, is built based on the state-space formulation of the linear reservoir cascade model. It incorporates empirical wave celerity vs. discharge relationship, which accounts for nonlinearity of the flood routing process. The shape and the parameters of that were fitted by several methods, including constrained optimisation with the help of a genetic algorithm and neutral networks. The modelling results showed that the proposed inclusion of empirical information on the variability of the travel-time parameter of the model enabled satisfactory accuracy of the prediction of flood propagation.
Keywords routing model; travel time; artificial neural network; enetic algorithm

INTRODUCTION

The simulation of the flood wave transformation between two river profiles using hydraulic methods is in many cases problematic due to lack of hydraulic and morphological data. Under such conditions, as a rational alternative to hydraulic routing seem to be hydrological routing models.

In this study are presented simulation results of a hydrological routing model cascade of linear reservoirs (KLN). KLN has two parameters: number of reservoirs (*n*) and time parameter (*k*). In Szolgay (2003) the multi-linear concept of KLN was developed, where the model parameter *k* varied according to a pre-defined flood peak travel time vs. discharge relationship. The flood peak travel time vs. discharge relationship was then a parameter of the KLN. Here different options of the parameterization and calibration of this model are reviewed and compared.

MULTILINEAR ROUTING MODEL KLN

The discrete cascade of linear reservoirs (KLN) was described by Szöllösi - Nagy (1981).The basic scheme is the cascade of linear reservoirs, where the output from one reservoir is the input for the following one. Furthermore the lateral inputs into modelled river reach can also be represented by external inputs which are fed into reservoirs in the cascade.

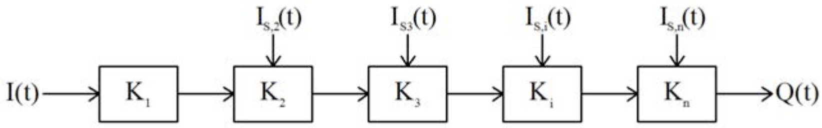

Fig. 1 The scheme of the cascade of linear reservoirs with lateral inputs included, where $I_{(t)}$ is the input to the cascade, $Q_{(t)}$ the output from cascade, K_i k_i the parameter of the *i*-th reservoir in the cascade and $I_{s,i}$ is the lateral inflow to *i*-th reservoir in the cascade.

The model consists of a series of n linear reservoirs, each with the time constant k. The storage-discharge relationship of the i-th reservoir in the cascade has the following form:

$$S_i = kQ_i \tag{1}$$

The (n*1) state vector S_i of the model represents the volumes of water stored in each reservoir at given time; Q_i represents the outflow from the i-th reservoir at given instance for the corresponding storage.

As output from one reservoir in cascade is the input for the following one, moreover each reservoir is allowed to have an external input $I_{S,i}$, which stands for the lateral inflow into the corresponding reservoir along the reach. The continuity equation for the i-th reservoir in the series is therefore written as:

$$\frac{dS_i}{dt} = Q_{i-1} + I_{S,i} - Q_i \tag{2}$$

If all the inputs to the cascade $I_{S,i}$ are considered to be constant during the sampling interval (a,a+1) of the length T, then the governing state-space equations of the model (the state equation and the outflow equation) can be written in the following form (Szolgay, 1982):

$$\underline{S}(a+1) = \underline{F}(a+1,a)\underline{S}(a) + \underline{G}(a+1,a)\underline{I}(a+1,a) \tag{3}$$

$$\underline{Q}(a+1) = \underline{H}(a+1)\underline{S}(a+1) \tag{4}$$

where $\underline{S}$ and $\underline{Q}$ are the (n*1) vectors of the reservoir's volumes and outflows respectively, and $\underline{H}$ is the (n*n) matrix, which equals $\underline{Id}$*(1/k), where $\underline{Id}$ is the identity matrix. The elements of the (n*n) state and input transition matrices $\underline{F}$ and $\underline{G}$ are defined as:

For $i \geq j$

$$F(i,j) = \frac{T^{i-j} e^{-T/k}}{(i-j)!\,k^{(i-j)}} \tag{5}$$

$$G(i,j) = k - \sum_{f=0}^{i-j} \frac{T^f e^{-T/k}}{f!\,k^{f-1}} \tag{6}$$

Otherwise

$$F(i,j) = 0 \tag{7}$$

$$G(i,j) = 0 \tag{8}$$

The transformation of the flood wave is a non-linear process; however the routing model uses constant parameters and assumes linearity, what may result in crude approximations.

As an alternative to the use of such a nonlinear storage outflow relationship, the whole model can be assumed to respond linearly to the input at any point in time, but with the model parameters recalculated as a function of the flow values at selected points. Such techniques, commonly referred to as multilinear modelling, usually distinguish different components in the input hydrograph that correspond to characteristic flow regimes, each which is subsequently routed through a linear sub-model. The overall output of the system is non-linear and consists of the outputs from the linear sub-models. Such various inflow components can be obtained by dividing the input hydrograph into horizontal or vertical segments. The former method is known as the amplitude distribution scheme; the latter the time distribution scheme. Kundzewicz (1984) gave an extensive description of the principles of these methods.

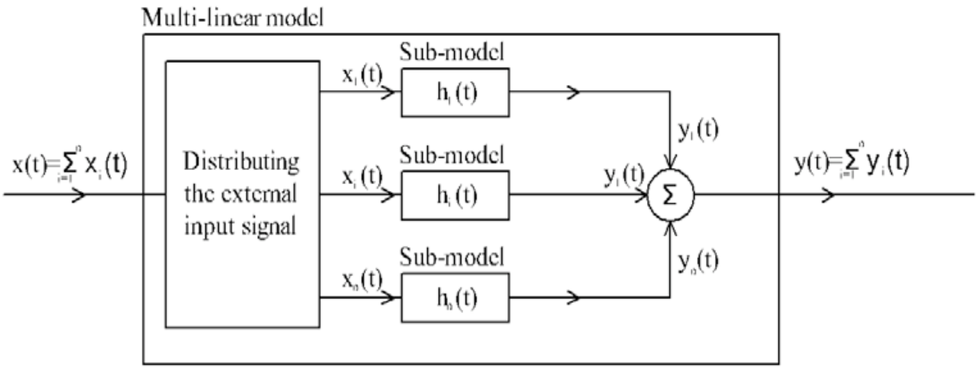

Fig. 2 The general structure of the multi-linear model. (Kundzewicz, 1984).

Multilinear concept of KLN may be described as scheme of two or more KLN models with the same number of the reservoirs (*n*) but with a time varying values of the parameter *k*. In the model implementation presented here the model parameter *k* changes in each computational time step. As shown in Szolgay (2003) the value of *k* can be calculate from flood peak travel time – discharge relationship, because the product *n* (number of reservoirs) and *k* (time parameter) is proportional to the travel time of the flood wave (Kalinin and Miljukov,1957).

Price (1973) suggested the general shape of the wave speed - discharge relationship (Fig. 3). This shape can be interpreted as consisting of two power functions, one each for the main channel and the overbank flow respectively, joined by an S-shaped transition curve.

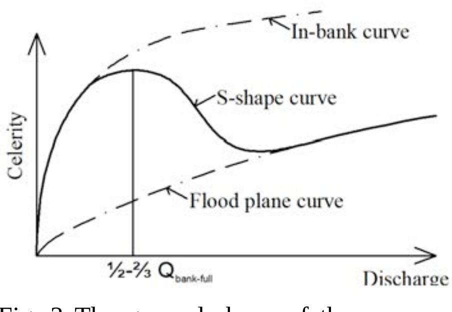

Fig. 3 The general shape of the wave speed (celerity) – discharge relationship described by Price (1973).

Substituting wave speeds by the travel-time of flood peaks, this relationship can be transformed to the flood peak travel time – discharge relationship (Fig. 4), which can be used as the indicator of the time parameter of KLN.

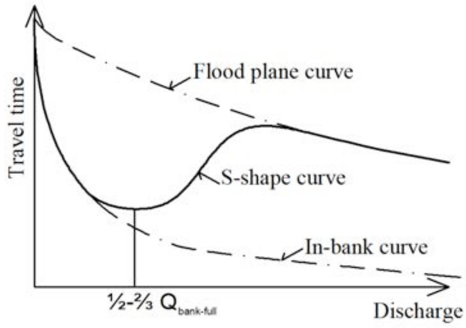

Fig. 4 The travel time – discharge relationship which can be used as indicator of parameter for KLN.

CASE STUDY

The models presented in this study were calibrated using data from the lower part of the Morava River, which is located between the gauges at Moravský Svätý Ján and Záhorská Ves in western Slovakia. The reach is 34.76 km with a slope of around 0.2%. The full bank flow-capacity is between 200 and 250 m3/s. Lateral inflow was estimated by hydrological analogy from the two measured tributaries Zaya and Rudava. The flood peak travel time data collected in Danáčová (2005) from this river reach are shown in Figure 5.

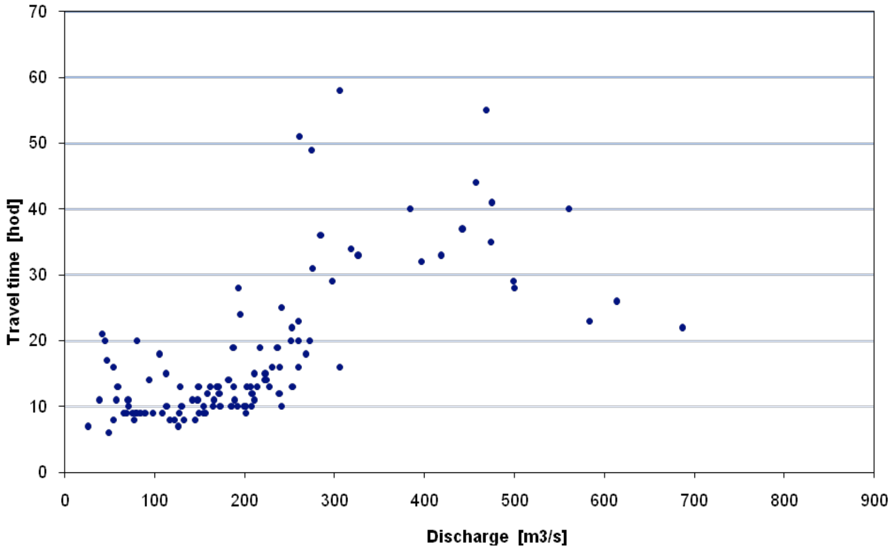

Fig. 5The flood peak travel time data plotted against the peak discharge values on the Morava River. These points were obtained by Danáčová (2005) from several hydrographs in the period 1992 – 2002. The pattern described in Price (1973) is almost fully visible in the data.

THE MODEL PARAMETRISATION

The model parameter *k* changes in each computational time step when the multilinearity concept is applied. The shape of travel time – discharge relationship needs to be estimated in order to calibrate the KLN model.

There are several ways how the flood peak travel time – discharge relationship can be estimated. The first is by manual calibration, when the travel time – discharge relationship is fitted into the flood peak travel time – discharge data collected from historical hydrographs. This way was calibrated the multilinear concept of KLN in Danáčová (2005).

In Šúrek (2008) KLN model was calibrated against historical data using a Genetic Algorithm (GA), to consider the optimal shape of the time-discharge relationship. In the calibrating process, the goodness of fit between the measured hydrograph and simulated one is valued using the Nash-Sutcliffe coefficient (7).

Here the travel time-discharge relationship was estimated using artificial neural networks (ANN). In order to include all several possible effects, several setups of the ANN, which could possibly affect the model parameter, were tested. The ANN was trained to estimate the optimal values of model parameter and is used on-line during the routing procedure. An ANN was applied as a model parameter simulator.

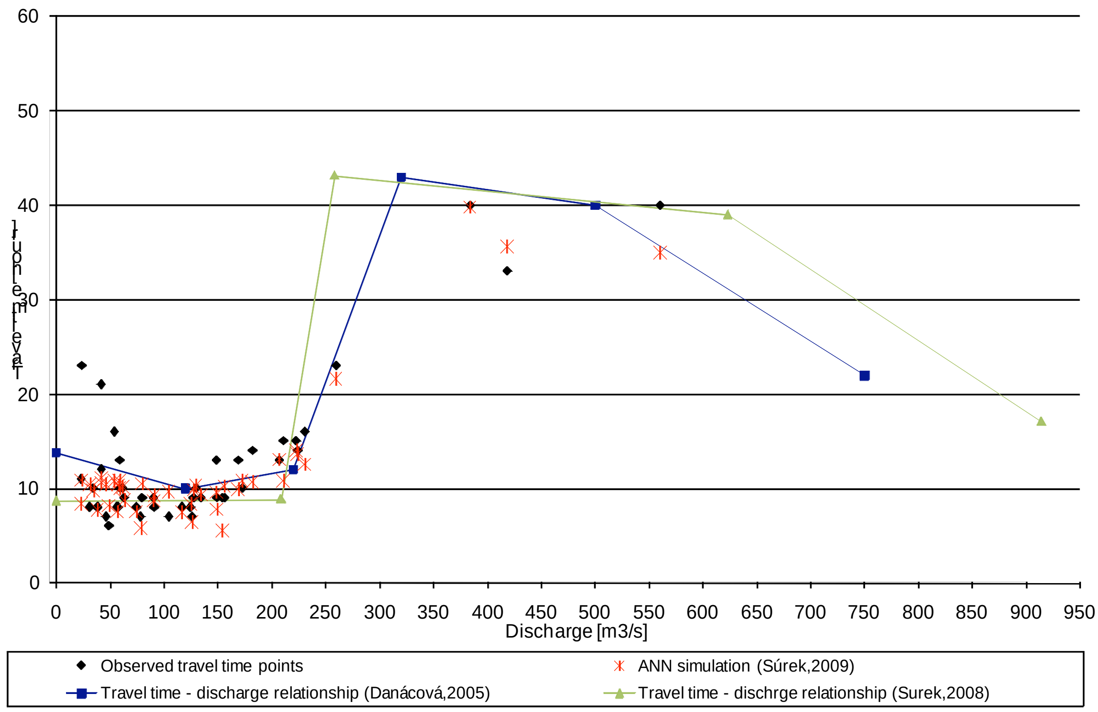

Fig. 6 Observed flood peak travel time compared against the flood peak travel time - discharge relationships estimated by ANN and found in Šúrek (2008) and Danáčová (2005).

On the following figures (7, 8, 9) the results of the input hydrographs transformation are presented. During visual inspection of the model performance the focus was on the ability of the model to reproduce the observed output hydrograph and the timing of the peak.

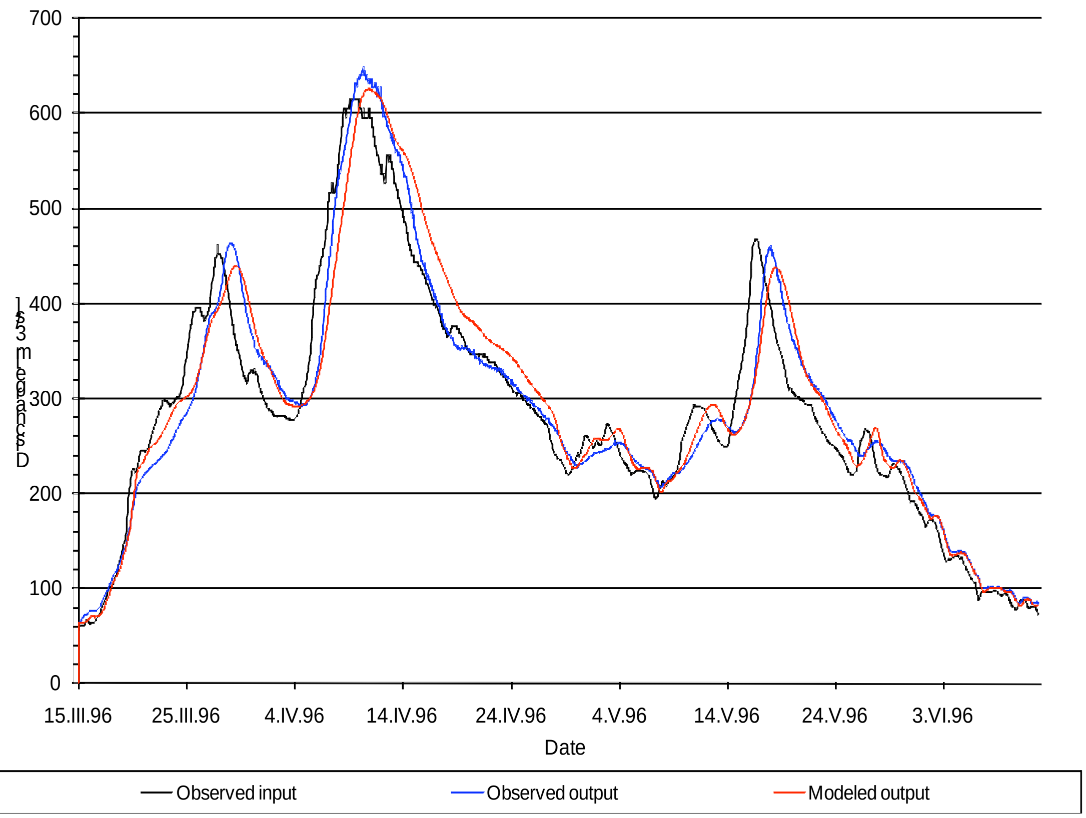

Fig. 7 The verification of KLN model parameterized by flood peak travel time – discharge relationship estimated empirically. The modelled output hydrograph fits to observed one with Nash Sutcliffe value of 0, 974 (Danáčová, 2005).

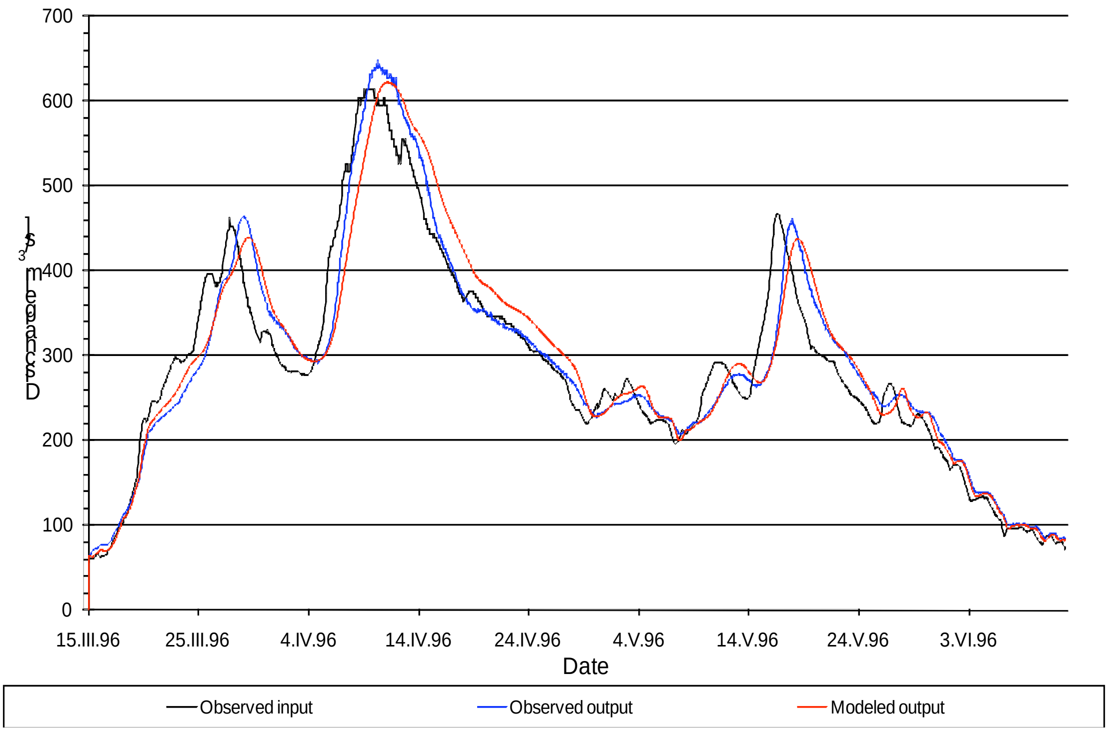

Fig. 8 The verification of KLN model parameterized by flood peak travel time – discharge relationship optimized by GA. The modelled output hydrograph fits to observed one with Nash Sutcliffe value of 0.973 (Šúrek,2008).

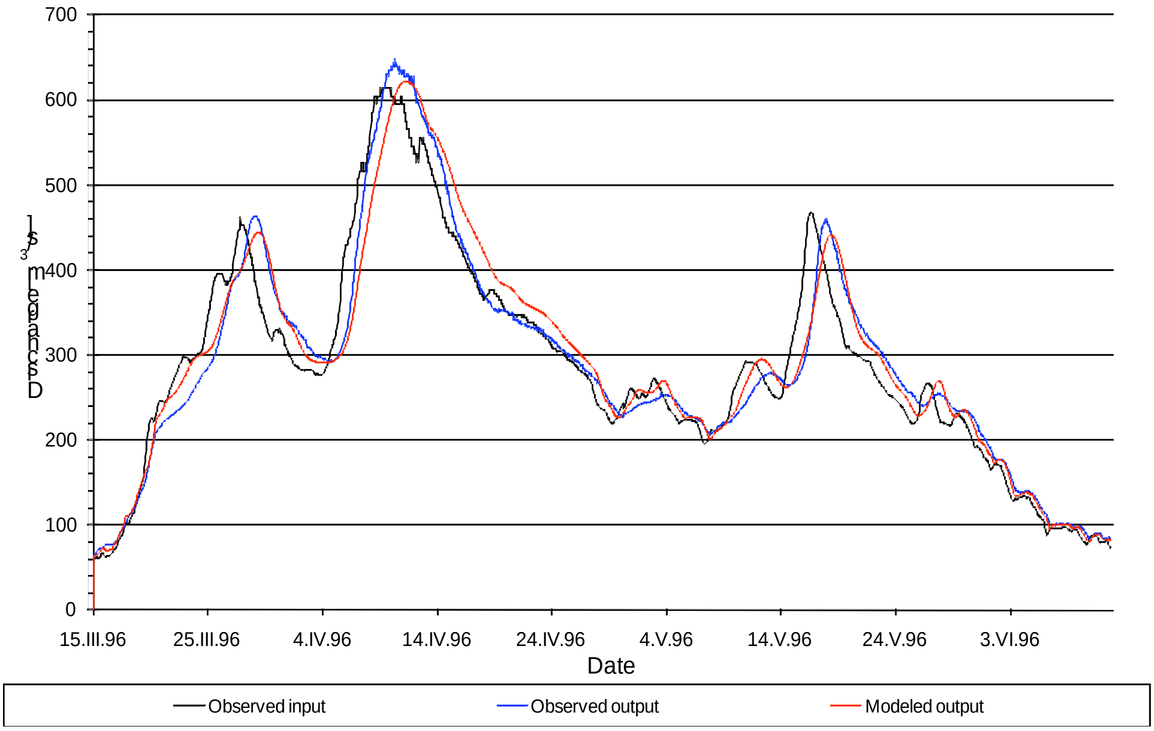

Fig. 9 The verification of the multi linear model with the build-in ANN. The modelled output hydrograph fits to observed one with Nash Sutcliffe value of 0.975 (Šúrek, 2009).

Table 1. Comparing Nash Sutcliffe values from different studies.

Flood wave	Discharge value of the flood peak [m3/s]	Manual calibration (Dánačová, 2005)	Calibration using GA (Šúrek, 2008)	KLN parameterized by ANN (Šúrek, 2009)
06.06.1992 - 19.06.1992	120.9	0.942	0.952	0.921
13.01.1993 - 14.02.1993	154.7	0.921	0.934	0.935
08.11.1996 - 09.12.1996	207.4	0.976	0.974	0.978
20.12.1995 - 06.01.1996	212.7	0.995	0.929	0.948
29.08.1995 - 13.09.1995	222.8	0.982	0.982	0.982
07.12.1993 - 01.03.1994	251.2	0.987	0.987	0.986
17.11.1991 - 19.11.1991	259.5	0.989	0.957	0.959
24.05.1994 - 06.06.1994	272.6	0.963	0.983	0.944
04.09.1996 - 09.11.1996	305.8	0.983	0.976	0.960
15.10.1998 - 05.12.1998	438.4	0.983	0.985	0.969
09.02.1997 - 18.03.1997	457	0.985	0.989	0.976
11.03.1993 - 06.05.1993	470.6	0.958	0.964	0.946
20.01.1992 - 27.04.1992	583.5	0.984	0.980	0.989
15.03.1996 - 12.06.1996	622.4	0.974	0.973	0.975
Average:		0.973	0.969	0.962

CONCLUSIONS

This study compared the simulation results of the hydrological routing model consisting from linear reservoirs in series (KLN) published in Danáčová (2006), Šúrek (2008) and with a model parametrisation scheme using ANN.

The best results of the model verification were obtained in Danáčová (2005) by an empirical approach. The overall average value of the NS criterion of 0.973 was better than 0.962 for Šúrek (2008) or that achieved in this study. However when considering is the visual inspection of the flood wave hydrographs as an indicator of model performance, the approach presented in this study performed best.

Acknowledgements

This work was supported by the Slovak Research and Development Agency under the contract No. APVV-0443-07.

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