COMPUTATION OF DIRECT RUNOFF AMOUNTS FROM STORM RAINFALL (1)

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ABSTRACT

Direct runoff amounts (Q) are computed for small watersheds from storm rainfall (P), antecedent soil moisture index (ASM), and the amount of rainfall retained (P_1) before direct runoff begins. The prediction equation $Q = (P - P_1)^2/[(P - P_1) + (c + kP_1)]$ requires an estimate of P_1 which is related to ASM in a linear fashion, $P_1 = a - b$ (ASM). The parameters a, b, c, and k are amenable to physical interpretations.

The ASM, using two levels of moisture storage, is determined by using an unsaturated flow-modulated soil moisture accounting procedure where actual evapotranspiration is dependent on day length, function of daily mean temperature, season, and the availability of soil moisture.

Application of the prediction equation to three small watersheds, representing diverse runoff potentials, gives a satisfactory specification of direct storm runoff. The correlation coefficients are highly significant, exceeding the 1 percent level.

RÉSUMÉ

Le débit d'eau (Q), qui ruisselle à la surface d'un petit bassin, peut être calculé en connaissant la précipitation (P), l'humidité initiale de la terre (ASM) et la quantité de pluie (P_1) absorbée au commencement de l'orage avant le début de l'écoulement d'eau à la surface de la terre. L'équation

$$Q = \frac{(P - P_1)^2}{(P - P_1) + (c + kP_1)}$$

qui relie ces variables nécessite que l'on estime (P_1) qui est une fonction linéaire et ASM, $P_1 = a - b$ (ASM). Les paramètres a, b, c, et k ont une interprétation physique.

En utilisant deux degrés d'humidité différents, on détermine (ASM) au moyen d'un procédé de mesure d'humidité dans un sol où l'écoulement ne sature pas ce sol. Ces mesures sont corrigées selon la saison. L'évapotranspiration réelle est fonction de la longueur de la journée, de la température journalière moyenne, de la saison et de l'humidité du sol.

L'application, de l'équation citée ci-dessus, à trois petits bassins, représentant des potentiels d'écoulement différentes, détermine d'une manière satisfaisante la quantité d'eau, produite par un orage, qui s'écoule à la surface de la terre. Les coefficients de corrélation sont très significatifs, plus petit que 1 pourcent.

1. INTRODUCTION

Direct runoff estimates from storm rainfall for agricultural or upstream watersheds are required in the economic appraisal of flood prevention programs, in the design of the hydraulic features of structures for watershed and downstream protection, and

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to show the effects of existing and proposed watershed projects. Given the direct runoff, design hydrographs may be developed by methods of hydrograph synthesis [*USDA*, *Anonymous*, 1957]. The use of available long-term rainfall data, in conjunction with a rainfall-runoff prediction model, permits the determination of various runoff probabilities and long-term sediment yields when given a relationship between direct runoff and sediment.

This study was made to develop a rainfall-runoff prediction model by establishing rainfall-runoff-retention-relationships from the hydrologic characteristics of soil and readily available climatological data.

2. DATA UTILIZED

Approximately two years of runoff and soil moisture data from two watersheds (*) under Forest Service management located near Oxford, Mississippi, and a watershed under Agricultural Research Service control at the North Mississippi Branch Experiment Station, Holly Springs, Mississippi, were utilized together with available precipitation and air temperature data. The soil moisture data were obtained at weekly intervals with the neutron probe at three or four sites in each watershed. Readings were taken at the 7-inch depth and at each foot depth to 9 feet. The watersheds range in size from 2 to 3 acres.

All the watersheds have similar topograpny, a yearly average precipitation of about 52 inches, and a temperature range from a January mean of 43°F to 80°F in July. Three types of cover are represented :

1. Poor pasture of overgrazed native grasses (Watershed WP-4, Holly Springs, Mississippi).
2. Abandoned field largely covered with broomsedge (*Andropogon spp.*) and not grazed (Abandoned Field I).
3. Depleted hardwood forest protected against grazing (Hardwood III).

The soils on the abandoned field watershed and the depleted hardwood watershed are of loessial origin, principally of the Providence series, with a silt loam texture and a fragipan at 14-24 inches. The fragipan restricts but does not prevent percolation of water. Soils on the poor pasture watershed are similar except that about one-third of the area is composed of Upper Coastal Plains soils (Ruston series) with a sandy loam texture and moderate internal drainage. The fragipan on a small ridge area in this watershed is near the 48-inch depth.

3. DEVELOPMENT OF RAINFALL-RUNOFF PREDICTION MODEL

The complete water balance of a watershed must be ascertained in establishing the rainfall-runoff-retention relationships. The water balance for the watersheds studied is represented by the equation :

$$P = ET + G + Q + \Delta S \qquad (1)$$

where P = precipitation, ET = evapotranspiration, G = percolation loss to ground water, Q = direct runoff, and ΔS = change in soil moisture storage. Determination of the water balance from climatological data requires the prediction of evapotranspiration and, in addition, percolation losses to ground water from predicted rainfall retention. A flow diagram of the water balance components appears as Figure 1.

(*) Permission to use data obtained by the Southern Forest Experiment Station, USDA, Oxford, Mississippi, is gratefully acknowledged.

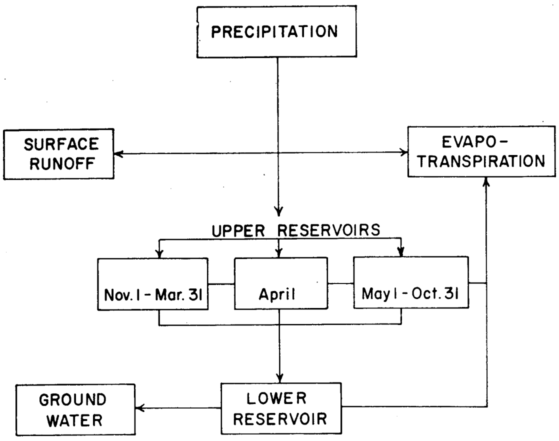

Fig. 1 — Flow diagram of water balance components.

3.1. *Antecedent Soil Moisture Index*

The antecedent soil moisture index (*ASM*) for all watersheds cited is the predicted amount of moisture in the selected upper 18 inches of the soil profile in excess of the amount existing under extreme drought conditions. The soil moisture content at this dry condition approximated that remaining under a tension of 15 atmospheres and ranged from 1.5 inches on the depleted hardwood watershed to 2.3 inches on the poor pasture watershed while the total soil moisture at saturation in the 18-inch profile ranged from 6.5 to 6.0 inches. The choice of the 18-inch depth for *ASM* determination was predicated upon the existence of a fragipan just below this depth over much of the area of the watersheds and the major concentration of roots above this level.

Soil moisture, as represented by *ASM*, is available for evapotranspiration losses to the atmosphere and percolation to subsurface soils. Assuming that sufficient moisture is readily available to satisfy the evaporation opportunity as afforded by the existing meteorological condition — whether the soil is bare or covered by vegetation — the actual evaporation or evapotranspiration would then be essentially equal to "potential" evapotranspiration as proposed by *Thornthwaite* (1948). For such moist conditions a number of techniques have been developed for estimating potential evapotranspiration from meteorological and/or pan evaporation data (*Penman*, 1948; *Thornthwaite*, 1948; *Blaney*, 1952). For this study an estimate of potential evapotranspiration (*PET*) based on readily available, long-term, temperature records is desirable. Such a procedure has been presented in an earlier paper (*Hamon*, 1961) and is used here in a modified form as a result of additional testing. The equation

$$PET = kD\varrho_t \tag{2}$$

estimates daily evapotranspiration in inches, where $k = 0.0065$; D = possible duration of sunshine in units of 12 hours; ϱ_t = saturated water vapor density in grams per cubic meter at the daily mean temperature.

Under initial nonlimiting moisture conditions evapotranspiration proceeds at the potential rate until a soil moisture deficit is reached where moisture availability is insufficient to support this rate. Various methods of accounting for the decrease in evapotranspiration with decreasing soil moisture have been proposed (*Thornthwaite and Mather*, 1954; *Veihmeyer and Hendrickson*, 1955; *Lemon*, 1957; *Kohler*, 1958;

Holmes and Robertson, 1959). An intermediate procedure of those proposed to account for this decrease, based on considerable field data, considers the reduction in the potential rate to be proportional to the reduction in available moisture.

Precipitation retained in the top 18-inch soil profile for loss as evapotranspiration and percolation is proportioned between an upper and a lower moisture retention reservoir for soil moisture accounting, Figure 1. The maximum amounts of moisture that may be retained in the upper reservoirs for loss at the potential evapotranspiration rate and the adjustments applied to potential evapotranspiration to estimate the evapotranspiration loss of moisture stored in the lower reservoir, modulated as to season, are shown in Figure 2. Curve (*n*) was used during the dormant season with recession in *PET* obtained by squaring the percent of moisture remaining of the possible 0.8-inch deficiency in the lower reservoir. Similarly, curve (*i*) was applied during April as a cubed percentage, curve (*m*) during May as a squared percentage, and curve (*j*) for June-October as a linear percentage of the available moisture remaining in the lower reservoir to obtain the corresponding recession in *PET*.

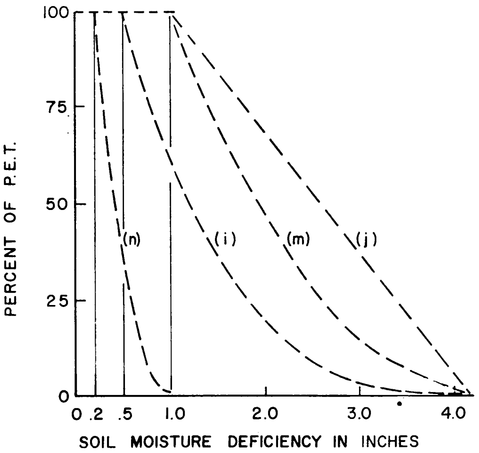

Fig. 2 — Adjustments of potential evapotranspiration as soil moisture deficiency increases. (Watershed *WP*-4. Curve (*n*) for November-March; curve (*i*) for April; curve (*m*) for May; curve (*j*) for June-October).

Percolation losses or unsaturated flow from the 30- to 42-inch layer just above the fragipan on the ridge location in Watershed *WP*-4 during April, 1959, when storm-rainfall amounts were such that little if any rainfall percolated below the one-foot level, were satisfactorily represented as :

$$g = [(2M - W)/W]^2 X\, 0.05 W \qquad (3)$$

where $g = 0$ when $M \leqslant 0.5$ W. All quantities are in inches and g = daily percolation; M = actual initial moisture in excess of that at 15 atmospheres tention; W = total moisture at saturation minus the amount of water at 15 atmospheres tension. This equation was used to compute percolation from both the upper and lower reservoirs on the assumption that the percolation from the 30- to 42-inch layer was representative of percolation from the upper soil layers.

The *ASM* was computed by a daily accounting of precipitation, direct runoff, evapotranspiration, and percolation. When the quantity of rainfall retained exceeded the holding capacity of the upper reservoir, the excess was added to the lower reservoir. In case the lower reservoir capacity was exceeded, the excess was then lost to ground water.

To adjust for a more rapid initial percolation loss from the lower reservoir when rainfall was added under dry antecedent conditions, as indicated by soil moisture data, an auxiliary reservoir was established. The minimum storage in this auxiliary reservoir for computational purposes was considered as half the maximum storage (0.5 W) of the lower reservoir. The total quantity of excess rainfall and percolated water from the upper reservoir were added to both the lower and auxiliary reservoirs. But the amount of percolation to ground water was determined by applying Equation 3 to the auxiliary reservoir when the artificial storage therein exceeded the storage in the lower reservoir.

The computed percolation loss to ground water was subtracted from the lower reservoir and 1.5 times this quantity was subtracted from the auxiliary reservoir. This procedure was continued until the water stored in the auxiliary reservoir was exhausted or reduced to the quantity of water stored in the lower reservoir with normal procedures then followed. In case the holding capacity of the auxiliary reservoir was exceeded, the excess overflow was allocated to ground water with an equal amount subtracted from the lower reservoir.

The *ASM* quantity on any day consists of the retained moisture in both the lower and upper reservoirs. This quantity plus the unavailable water equals the total moisture. Computed daily values of the total moisture in the top 18 inches of soil for Watershed *WP*-4 are compared with observed values in Figure 3. The appropriate recessional rates were selected, as shown in Figure 2, to give the best specification of observed data with acceptance of the computed percolation losses.

3.2 *Initial Runoff and ASM*

The *ASM* index for Watershed *WP*-4, computed for the previous day, and observed runoff were used as coordinates for a scatter diagram plotting of storm rainfall as shown for selected rainfall amounts in Figure 4. A total of 78 rainfall events out of 172 events over the 2-year period gave no runoff. To determine the apparent relationship between *ASM* and the rainfall retained (P_1) before runoff begins, the higher storm rainfalls that gave no runoff for *ASM* ranges and all rainfalls that produced 0.04 inch or less of runoff were plotted in Figure 5. A straight line representing the boundary between rainfall events that produced runoff and no runoff was arbitrarily selected as:

$$P_1 = 0.50 - 0.10\,(ASM) \tag{4}$$

(All rainfall events that produced runoff exceeding 0.04 inch plotted above the line). The same relationship for the abandoned field is :

$$P_1 = 2.50 - 0.52\,(ASM) \tag{5}$$

and for the depleted hardwood watershed is :

$$P_1 = 3.00 - 0.68\,(ASM) \tag{6}$$

The relationship between P_1 and *ASM* is identical to that obtained by *Hartman et al* (1960) for the Blacklands Experimental Watershed near Riesel, Texas, and is of the simple linear form :

$$P_1 = a - b(ASM) \tag{7}$$

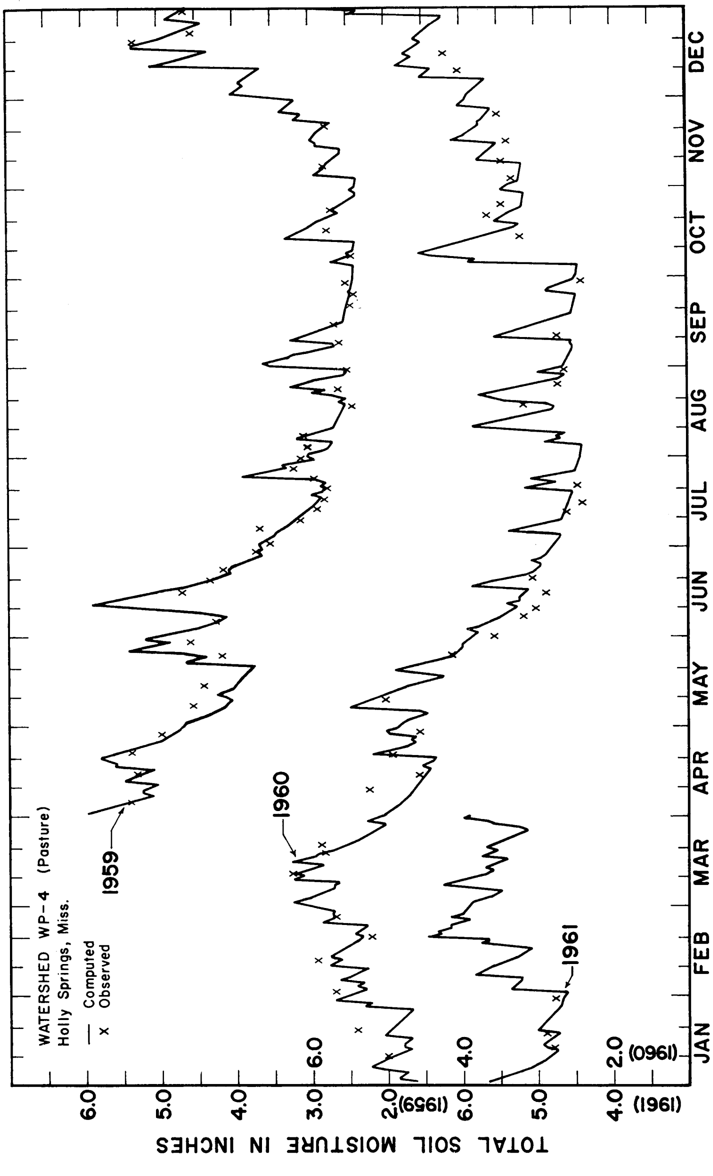

Fig. 3 — Comparison of computed and observed soil moisture in the top 18-inch soil profile (Watershed *WP-4*).

3.3 *Rainfall-Runoff Prediction Model*

The rainfall-runoff prediction model was developed by starting with the proportion as noted in the Engineering Handbook of the Soil Conservation Service (*USDA, Anonymous*, 1957) : (8)

$$Q/P = (P - Q)/S$$

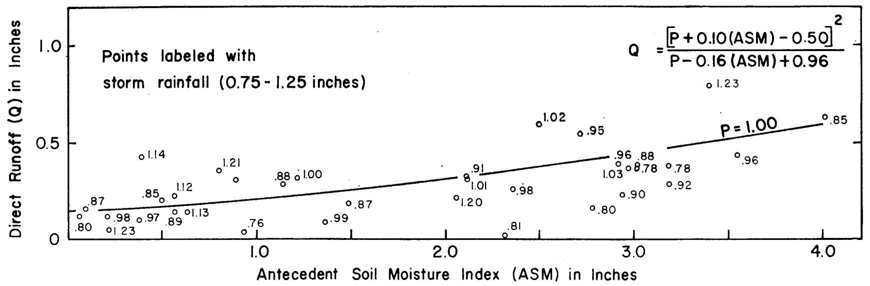

Fig. 4 — Storm runoff as related to antecedent soil moisture index for selected storm rainfalls (Watershed *WP*-4).

where Q = direct storm runoff, P = rainfall, and S = potential retention of rainfall. A certain quantity of rainfall (P_1) is retained before runoff begins and if ($P - P_1$) is then substituted for (P), Equation 8 may be written as :

$$Q = (P - P_1)^2/[(P - P_1) + S] \tag{9}$$

with the restraint that $Q = 0$ when $P_1 \geqslant P$.

This equation is used by the Soil Conservation Service for predicting storm runoff in the form $Q = (P - 0.2S)^2/(P + 0.8S)$ by assigning $P_1 = 0.2S$. Values of S have been obtained by plotted P versus Q for gaged watershed in various parts of the country. Data from the three Mississippi watersheds studied and the Blacklands Experimental Watershed (*Hartman et al*, 1960) indicate that the term S represents a storage factor that may be represented by a two-parameter function of P_1 as :

$$S = c + kP_1 \tag{10}$$

Inserting this form of S into Equation 9 the rainfall-runoff prediction model results

$$Q = (P - P_1)^2/[(P - P_1) + (c + kP_1)] \tag{11}$$

The parameters c and k were determined for each watershed by curve fitting. This was accomplished by drawing a smooth curve to represent data as shown in Figure 4

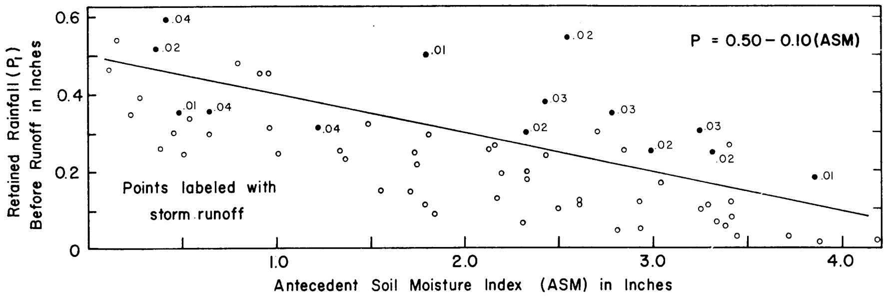

Fig. 5 — Rainfall retained before runoff begins in relation to antecedent soil moisture index (Watershed *WP*-4).

for each watershed for the first estimates of the parameters c and k, then adjusting these parameters to give the highest correlation coefficients when all the data were considered. The value of c for the three watersheds under different cover (poor pasture, abandoned field, and depleted hardwood) but with similar soils was essentially constant and was assigned the value of 0.16. Values of k selected for the highly compacted poor pasture, abandoned field, and depleted hardwood were 2.6, 0.62, and 0.20, respectively. Graphical representation of the prediction equation fitted to the poor pasture watershed data is shown in Figure 6 and for the depleted hardwood watershed in Figure 7.

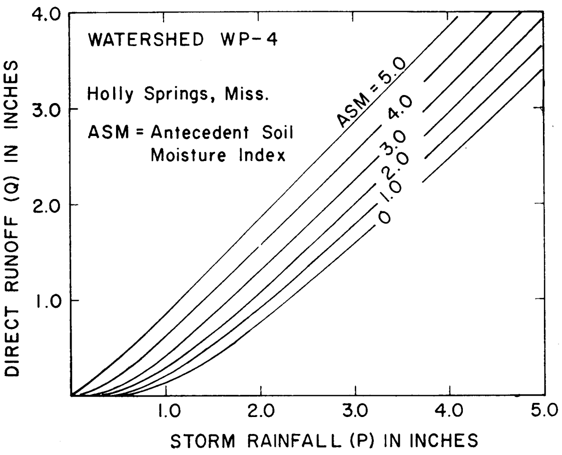

Fig. 6 — Direct runoff as related to storm rainfall in terms of the antecedent soil moisture index for a poor pasture watershed.

Observed versus computed runoff for Watershed *WP*-4 is plotted in Figure 8. For the 94 events (including all rainfall events with either observed or computed runoff) the average observed storm runoff was 0.34 inch and the coefficient of determination (r^2) by least squares was 0.91. Similarly, $r^2 = 0.82$ and 0.75 for the abandoned field and depleted hardwood watersheds, respectively. The correlation coefficients are highly significant, exceeding the 1 percent level.

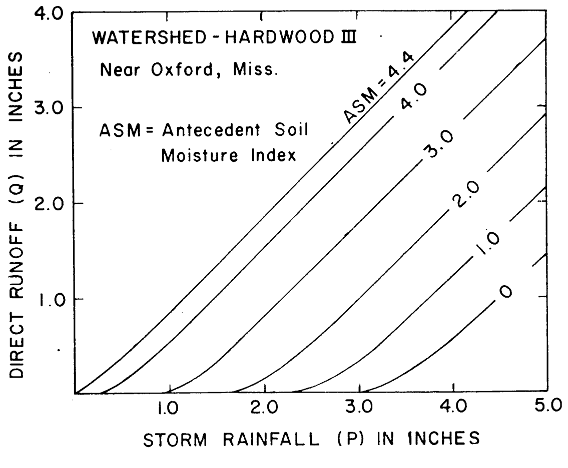

Fig. 7 — Direct runoff as related to storm rainfall in terms of the antecedent soil moisture index for a depleted hardwood watershed.

4. DISCUSSION

The storm rainfall-runoff prediction model is developed by equating the ratio, $Q/(P - P_1)$, to $(P - P_1 - Q)/S$ where P_1 is the rainfall required to initiate runoff and S represents a storage factor. The term P_1 includes the rainfall initially infiltrated and retained as interception and surface storage. The storage factor, S, is dependent upon P_1, available storage in the upper soil profile, and percolation into the lower soil profile. Since S is related to ASM indirectly through P_1 by the two-parameter equations $S = c + kP_1$, adjustments can be made for different combinations of the principle influencing factors of infiltration, percolation, and available storage.

For the three watersheds studied, the existence of a fragipan at the 16- to 24-inch depth over much of the area of the watersheds restricted percolation; thus, the parameter k was successively smaller as the available storage in the top 18 inches was reduced by higher retentions of rainfall before the initiation of runoff. This is shown by the inverse relationship of k to the a term in Equation 7 [$P_1 = a - b\,(ASM)$]. Ordinarily this parameter is highly conservative since the majority of soil-cover complexes exhibit an increasing depth of plant root activity to increase potential storage with higher percolation rates for higher initial infiltration losses as represented by P_1. Observations of rainfall required to initiate runoff on small experimental plots with corresponding soil moisture data obtained by the neutron probe at appropriate times for a wide variety of hydrologic soil-cover complexes should furnish sufficient data to evaluate the parameters of the rainfall-runoff prediction equation.

The ASM values used in establishing the relationship of P_1 to ASM were for the day prior to the storm event; therefore, the rainfall-runoff prediction model should be equally applicable to daily precipitation and runoff amounts.

Accurate computations of runoff from storm or daily rainfall amounts depend upon a strong correlation between rainfall amounts and intensities. The scatter of points in Figure 8 is partially due to variable rainfall intensities. This is exemplified

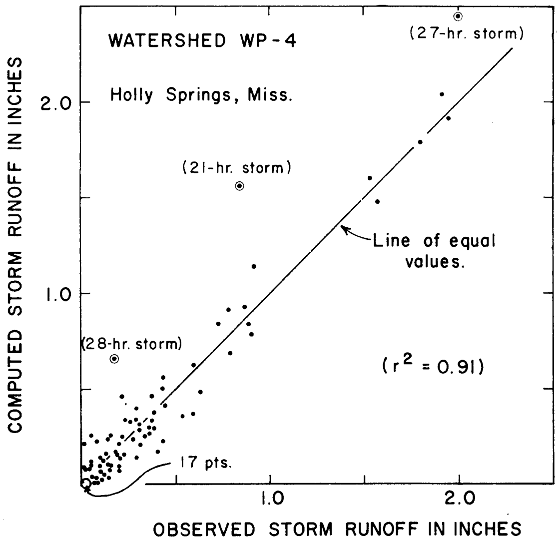

Fig. 8 — Comparison of computed and observed runoff for a poor pasture watershed (Watershed *WP*-4).

TABLE 1

Components of the water balance for Watershed WP-4, Holly Springs, Mississippi, (all values in inches)

Period of time	P	Q	PET	ET	G	ΔS
1959-60						
Apr.	3.46	0.56	3.04	2.52	1.15	− 0.77
May	6.05	2.44	4.81	3.09	0.23	+ 0.29
June	5.92	3.02	5.10	3.82	0.45	− 1.37
July	3.13	0.19	5.75	3.54	0	− 0.60
Aug.	4.94	0.92	5.75	3.53	0	+ 0.49
Sept.	2.08	0.47	4.32	2.64	0.07	− 1.10
Oct.	2.51	0.44	3.03	2.10	0	− 0.03
Nov.	3.13	0.52	1.62	1.26	0.06	+ 1.29
Dec.	6.54	2.04	1.37	1.33	2.24	+ 0.93
Jan.	4.77	1.70	1.29	1.26	0.93	+ 0.88
Feb.	4.11	1.66	1.17	1.14	1.10	+ 0.21
Mar.	5.43	2.83	1.50	1.30	1.94	− 0.64
Annual	52.07	16.79	38.75	27.53	8.17	− 0.42
1960-61						
Apr.	2.30	0.26	3.31	1.81	0.41	− 0.18
May	3.78	1.12	4.05	3.28	0.40	− 1.03
June	2.38	0.22	5.30	3.19	0	− 1.03
July	2.79	0.33	6.13	2.86	0	− 0.40
Aug.	6.04	0.92	5.71	4.72	0.18	+ 0.22
Sept.	2.22	0.05	4.40	2.31	0	− 0.14
Oct.	5.83	2.28	2.94	2.11	0.55	+ 0.89
Nov.	3.08	0.83	1.74	1.49	0.22	+ 0.54
Dec.	4.27	0.66	1.10	0.92	0.84	+ 1.86
Jan.	0.78	0.03	1.07	0.87	1.03	− 1.15
Feb.	7.80	3.45	1.61	1.19	1.79	+ 1.37
Mar.	8.61	4.87	2.37	2.08	1.77	− 0.11
Annual	49.88	15.02	39.73	26.83	7.19	+ 0.84
2-yr. Av.	51.0	15.9	39.2	27.2	7.7	
Ratio %		Q/P 31		ET/P 53	G/P 15	

Note : P = Precipitation; Q = Direct runoff; PET = Potential evapotranspiration; ET = Actual evapotranspiration; G = Accretion to ground water; ΔS = Change in soil moisture storage.

by the three long duration storms, labeled in the figure, which fall outside the normal distribution of points. By neglecting these three points, the value of r^2 is increased from 0.91 to 0.97.

The determination *ASM* values using the unsaturated flow model considers all elements of the water balance for the watersheds studied. Where runoff is not measured it must be predicted to carry forward the soil moisture accounting procedure. Using measured storm runoff and storm precipitation, the complete water balance data resulting from daily computations of evapotranspiration and percolation for the poor pasture watershed are tabulated in Table 1. The average precipitation was near normal and proportioned as follows : 31 percent to direct runoff, 53 percent to actual evapotranspiration, 15 percent to ground water, and about 1 percent to change in soil moisture storage in the 18-inch profile.

5. CONCLUSIONS

The rainfall-runoff prediction model developed is amenable to physical interpretation and enables the computation of runoff estimates beyond the range of available data and for extended periods from rainfall and temperature data. An antecedent soil moisture index, constituting the primary variable in the relationship, may be adequately determined by using an unsaturated flow-modulated soil moisture accounting procedure with potential evapotranspiration determined from climatological data.

Computations of direct storm runoff from storm rainfall for three watersheds on similar soils but with different cover conditions and widely varying runoff potential gave satisfactory results as verified by correlation coefficients that exceeded the 1 percent level. The significant departures were due to long-duration storm rainfall.

The rainfall-runoff prediction model is especially adapted for computers and the use of long-term rainfall and temperature data in such a model will enable the determination of direct runoff probabilities and water balances estimates.

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