Developing a sheet erosion equation for a semiarid region

CELSO AUGUSTO GUIMARÃES SANTOS, KOICHI SUZUKI, MASAHIRO WATANABE
Department of Civil and Environmental Engineering, Ehime University, 3-Bunkyo-cho, Matsuyama, Ehime 790-77, Japan

VAJAPEYAM SRIRANGACHAR SRINIVASAN
Department of Civil Engineering, Federal University of Paraíba, Av. Aprígio Veloso, 882, Campina Grande, Paraíba 58109-970, Brazil

Abstract An empirical equation for soil loss rate is presented in the form of $E = \alpha L^{\beta_1} I^{\beta_2} S_0^{\beta_3}$, where L is the length of slope, S_0 is the degree of slope, I is the rainfall intensity, α is a coefficient depending mainly on the vegetation cover of the soil surface and on soil characteristics, and β_1, β_2 and β_3 are constant exponents. The exponents β_1, β_2 and β_3 are determined using data from a simulation using a runoff-erosion model based on kinematic wave approximations; the model is applied to hypothetical plots with characteristics representative of a semiarid area. The proposed equation is tested using data obtained from nine field experimental plots located in a typical semiarid region of northeastern Brazil and with different conditions of surface cover.

INTRODUCTION

As the degree of slope and the length of slope increase, discharge and the velocity of water flow on the slope increase and, equally, so does the rate of detachment and removal of soil particles. The development of general relationships for predicting the rate of sheet erosion has continued since Zingg (1940) related soil loss to the degree and length of slope. Musgrave (1947) then reported the results of analyses of soil-loss measurements which also provided a relationship like the Zingg equation. However, Musgrave assumed that the rate of sheet erosion also depends on the rainfall. This relationship can be expressed in the form

$$E = \alpha L^{\beta_1} I^{\beta_2} S_0^{\beta_3}$$

where L is the length of slope, S_0 is the degree of slope, I is the rainfall intensity, α is a coefficient depending mainly on the vegetation cover of the soil surface and on soil characteristics, and β_1, β_2 and β_3 are constant exponents. Nevertheless, no empirical equation for predicting sheet erosion precisely and practically has been developed, because the erosion process inherently contains many uncertainties that are difficult to solve analytically.

In order to develop a useful and practical model, the authors have studied erosion process mechanisms on bare slope surfaces (Santos *et al.*, 1994). In this paper, a relationship is proposed for the sediment yield of a semiarid region using data simulated by a runoff-erosion model and the equation is then tested using data from a typical semiarid region of northeastern Brazil.

Bare land without vegetation is widely distributed throughout semiarid areas in northeastern Brazil. Reduction of the agricultural productivity of the land in these areas due to the loss of fertile topsoil by water erosion has become one of the most urgent problems. Once the agricultural productivity and vegetation of the land are lost, it will take an extremely long time for the productivity and vegetation to recover.

SUMMARY OF THE RUNOFF-EROSION MODEL

In this paper the runoff-erosion model developed by Lopes (1987) called Watershed Erosion Simulation Program (WESP) was used to determine the exponents β_1, β_2 and β_3, because this model was specially developed for small basins. WESP represents a physically-based, distributed parameter, event-oriented, nonlinear, numerical model that is capable of accommodating change in topography, surface roughness, and soil properties.

The infiltration process is modelled using the Green & Ampt equation and Darcy's law during a steady rain, which can be written in the form:

$$f(t) = K_s\left(1 + \frac{N_s}{F(t)}\right) \tag{1}$$

where $f(t)$ is the infiltration rate (m s^{-1}), K_s is the effective soil hydraulic conductivity (m s^{-1}), N_s is the soil moisture-tension parameter (m), $F(t)$ is the cumulative depth of infiltrated water (m) and t is the time variable (s).

The overland flow caused by rainfall excess is considered one dimensional. Manning's turbulent flow equation is given by:

$$u = \frac{1}{n} R_H^{2/3} S_f^{1/2} \tag{2}$$

where $R_H(x,t)$ is the hydraulic radius (m), u is the local mean flow velocity (m s^{-1}), S_f is the friction slope, and n is the Manning friction factor of flow resistance. Here the assumption of the kinematic approximation that the friction slope is equal to the plane slope ($S_0 = S_f$) is used; i.e. the gravity and friction components are the dominant factors of the momentum equation. This approximation results in the local velocity equation for planes ($R_H = h$):

$$u = \alpha' h^{m-1} \tag{3}$$

where h is the depth of the flow (m), and α' (equal to $(1/n)S_0^{1/2}$) and m (equal to 5/3) are parameters related to surface roughness and geometry, respectively.

The sediment transport is considered as the erosion rate in the plane reduced by the deposition rate within the reach. The erosion occurs due to raindrop impact as well as surface shear. The sediment continuity equation is used to express the sediment transport rate in the reach as a function of the concentration, the discharge and the depth. The equation is solved numerically with a four-point implicit finitedifference scheme to calculate the sediment flow as a function of time and distance. The sediment flux Φ to the flow is written as:

$$\Phi = e_I + e_R - d \quad (4)$$

where e_I is the rate of sediment detachment by rainfall impact, e_R is the rate of sediment detachment by shear stress, and d is the rate of sediment deposition. The rate e_I (kg m^{-2} s^{-1}) is obtained from the relationship:

$$e_I = K_I I r_e \quad (5)$$

in which K_I is the soil detachability parameter (kg s m^{-4}), I is the rainfall intensity (m s^{-1}), and r_e is the effective rainfall rate (m s^{-1}). The rate e_R (kg m^{-2} s^{-1}) is expressed by the relationship:

$$e_R = K_R \tau^{1.5} \quad (6)$$

where K_R is a soil detachability factor for shear stress (kg m N$^{-1.5}$ s^{-1}), and τ is the effective shear stress (N m^{-2}). And d (kg m^{-2} s^{-1}) is expressed as:

$$d = \varepsilon V_s C \quad (7)$$

where ε is a coefficient that depends on the soil and fluid properties (set to 0.5 in this study), V_s is the particle fall velocity (m s^{-1}), and $C(x,t)$ is the sediment concentration in transport (kg m^{-3}).

SIMULATION FOR HYPOTHETICAL PLOTS

In the present study, the hypothetical plots are simulated using the above model in order to determine the exponents of the regression equation, β_1, β_2 and β_3. The soil moisture-tension parameter N_s is equal to zero and the effective soil hydraulic conductivity K_s is equal to 5 mm h^{-1} in equation (1), because a saturated case is considered here. The main erosion parameters in the model were proposed by Santos *et al.* (1994) for a micro-basin located in a Brazilian semiarid region. These parameters are the coefficient to measure soil detachability by rainfall impact K_I

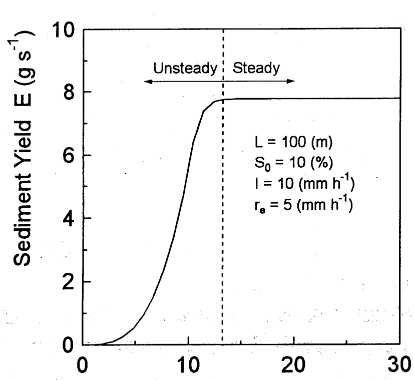

Fig. 1 Example of a simulated sediment hydrograph.

(equal to 5.1×10^8 kg s m^{-4}) in equation (5), and the soil detachability factor by shear stress K_R (equal to 2.1 kg m N$^{-1.5}$ s^{-1}) in equation (6).

The hypothetical plots are 30, 100 and 300 m in length, 4, 10 and 20% in slope with unit width. These nine hypothetical plots under three constant rainfall intensities of 10, 20 and 50 mm h^{-1} give a set of 27 values that are used to determine the exponents of the sheet erosion equation. Figure 1 shows an example of the sediment hydrograph simulated for a hypothetical plot with 10% slope. This figure shows how the sediment yield calculated by the model reaches a steady state. Only the constant sediment yield associated with the steady state is discussed in this paper.

Figure 2 shows how the sediment yield E changes linearly with the effective rainfall intensity r_e, length of slope L and degree of slope S_0 on a logarithmic scale.

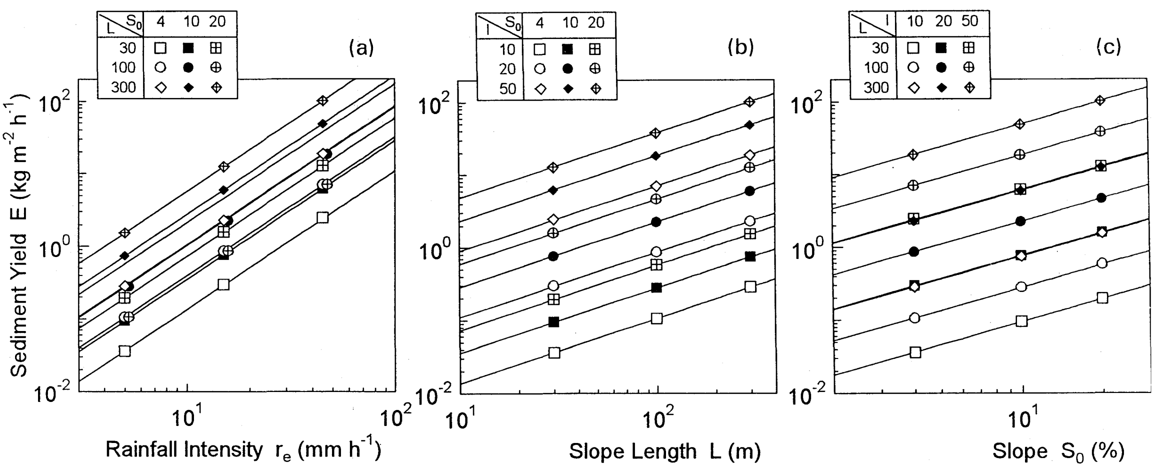

Fig. 2 Simulated relationship between sediment yield E and (a) effective rainfall intensity r_e, (b) length of slope L, and (c) degree of slope S_0.

APPLICATION TO EXPERIMENTAL FIELD PLOTS

The experimental field plots are located in the Experimental Basin of Sumé, which has been operated since 1972 (Cardier & Freitas, 1982) by UFPB (Federal University of Paraíba, Brazil), SUDENE (Superintendency of Northeast Development, Brazil) and ORSTOM (French Office of Scientific Research and Technology for Overseas Development) to obtain data concerning runoff and sediment yield produced by heavy rainfall in a natural environment. The experimental basin incorporates four micro-basins, nine experimental plots, one sub-basin, and several micro-plots operated by simulated rainfall. The surface conditions as well as the slope for each micro-basin or experimental plot are different.

The nine experimental plots are 4.55 m in width by 22.5 m in length (100 m^2), and their mean slope, vegetation cover condition and year of installation are given in Table 1. The same conditions of land management and vegetation were maintained for plots 1 and 4, and plots 2 and 3 to permit comparison of results for different slopes.

On the bare plots (plots 1 and 4), the vegetation was removed when it reached

Table 1 Characteristics of the 100 m^2 plots.

Plot no.	Mean slope (%)	Vegetation cover	Year of installation
1	3.8	Bare soil	1982
2	3.9	Dead vegetation cover	1982
3	7.2	Dead vegetation cover	1982
4	7.0	Bare soil	1982
5	9.5	Native vegetation (*caatinga*)	1982
6	4.0	Cactus (downslope tillage)	1983
		Corn	1989
7	4.0	Cactus (contoured tillage)	1983
		Bean	1989
8	4.0	Bare and revolved soil	1986
9	4.0	New *caatinga* (1981)	1986

5 cm high. The plots with dead vegetation cover (plots 2 and 3) had their vegetation cut when it reached 20 to 25 cm high, but the dead vegetation was not removed from the plots. Plots 6 and 7, with cacti, were cleaned when the vegetation reached 5 cm high. Plot 8 was kept constantly without vegetation and the soil was revolved. The plots with native vegetation cover (plots 5 and 9) received no human intervention. In order to study the effect of soil erosion by other cultivation practices, the cacti in plots 6 and 7 were removed in 1989 before the rainy season, and then corn and beans were planted instead at the beginning of the rainy season using tillage methods characteristic of the region. Corn and beans were chosen because they are mainly cultivated in that region. Typical surface conditions for plots 6 and 1 are shown in Fig. 3.

Several rainfall events were chosen between 1987 and 1988 based on the work of Santos *et al.* (1994). The relationship between observed runoff depth L_o and observed rainfall depth R_o is shown in Fig. 4(a) and (b) for the plots with and without vegetation cover, respectively. It can be seen that when the rainfall depth increases the runoff depth also increases, but for the plots with vegetation cover there are more events with a small runoff depth, i.e. less than 0.1 mm. Figure 5(a) and (b) shows the relationship between the observed sediment yield E_o and the rainfall intensity I, also for the plots with and without vegetion cover, respectively. It is clearly observed

(a) (b)

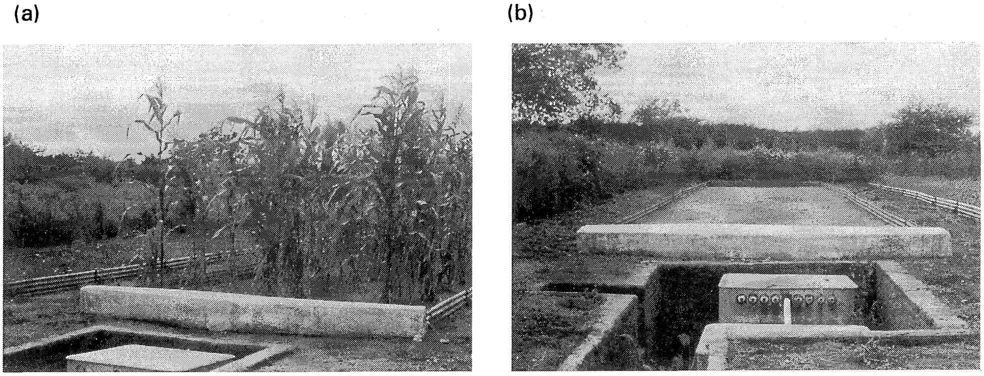

Fig. 3(a) Plot 6 with corn with downslope tillage. (b) Plot 1 without vegetation cover.

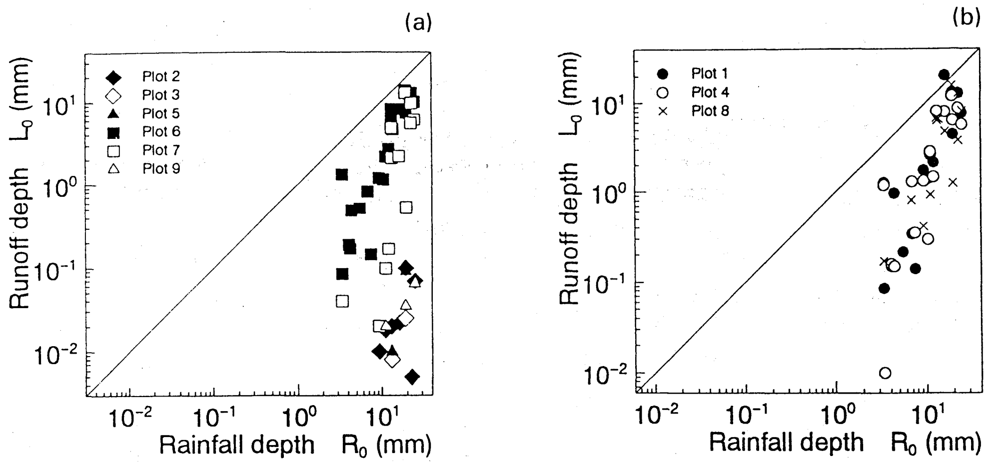

Fig. 4 The relationship between observed runoff depth L_o and observed rainfall depth R_o for the plots with (a) vegetation cover, (b) bare soil.

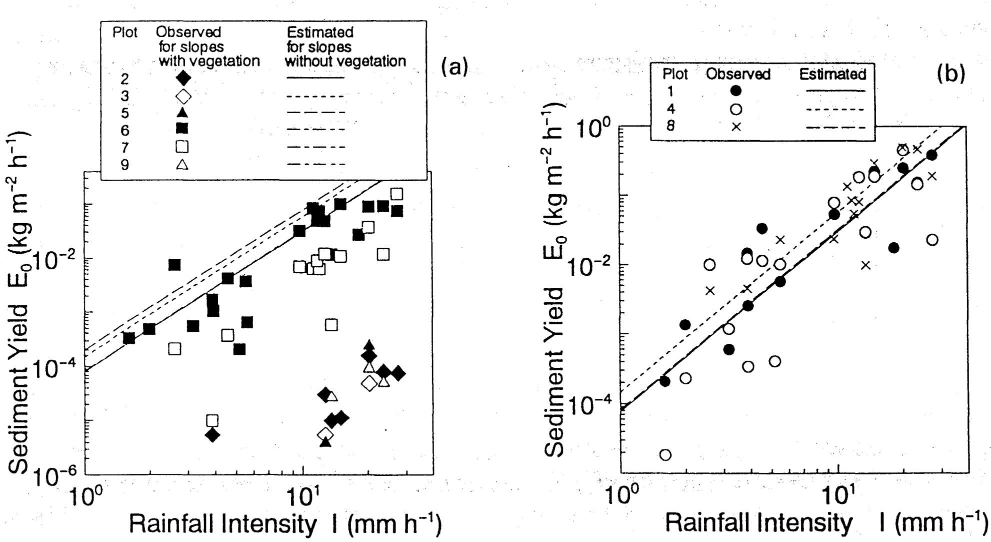

Fig. 5 The relationship between observed sediment yield E_o and rainfall intensity I for the plots (a) with vegetation cover, (b) bare soil.

that when the soil protection is minimum, the erosion increases, and the best land protection seems to be either natural vegetation (*caatinga*) or dead cover.

REGRESSION EQUATION

In order to clarify the influence of rainfall intensity, degree of slope, and length of slope on soil loss, the following equation was adopted here. Musgrave (1947) synthesized the results of analyses of soil-loss measurements for some 40 000 storms occurring on fractional-acre plots in the United States, and expressed the rate of soil loss E by the following relationship:

$$E = \alpha L^{\beta_1} I^{\beta_2} S_0^{\beta_3} \tag{8}$$

where α is the inherent erodibility of the soil and a cover parameter, β_1, β_2 and β_3 are exponential constant numbers, L is the length of slope, S_0 is the degree of slope, and I is the rainfall intensity.

In this paper, the values of the exponents β_1, β_2 and β_3 are obtained using the least squares approximation for the data from the hypothetical plots. The approximation is shown by solid straight lines in Fig. 2(a), (b) and (c). Thus, the following equation is proposed:

$$E = 1.91 \times 10^{-5} L^{0.9} r_e^{1.91} S_0^{1.04} \tag{9}$$

where E is in kg m^{-2} h^{-1}, L in m, S_0 is in percent, and r_e is the effective rainfall intensity given by $r_e = I - K_s$, wherein K_s is nearly 5 mm h^{-1}, which is the final infiltration rate or the effective soil hydraulic conductivity as written is equation (1).

The above equation can now be applied to the nine experimental plots in order to test its applicability. The simulation results produced by equation (9) using the data for the nine experimental plots are shown by the lines in Fig. 5(a) and (b). The equation was determined using data from hypothetical plots without vegetation cover. It can be seen from these results that when the soil is protected by some form of vegetation, the order of magnitude of sediment yield for this region decreases around 10^{-2}. Among the plots with vegetation, the best vegetation cover is the native vegetation and the worst is cactus with downslope tillage. This is also why the data for experimental plots 6 and 7 with cactus cover follow the line for the estimates produced using equation (9).

Figure 6 shows the relationship between the inherent erodibility of the soil α and the vegetation cover conditions. It can be seen that the value of the parameter α for a bare plot is between 10^{-6} and 10^{-8}, but for a plot with *caatinga* (native vegetation) the value becomes 10^{-8} to 10^{-10}.

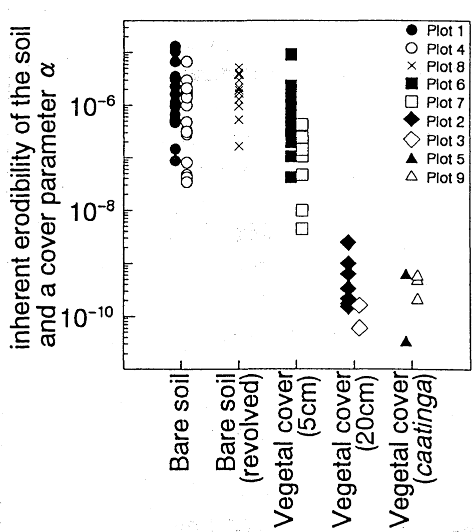

Fig. 6 The relationship between the inherent erodibility of soil α, and the vegetation cover condition.

CONCLUSIONS

In order to clarify the mechanism of sheet erosion in semiarid regions and the influence of rainfall intensity, degree and length of slope on soil loss, simulations of hypothetical plots were conducted using a runoff-erosion model. The hypothetical plots had semiarid characteristics, and the proposed equation was tested with data from nine experimental plots located in a Brazilian semiarid area with different vegetation cover conditions. The following conclusions were obtained:

- When the soil is protected by some form of vegetation, the sediment yield value for this region decreases around ca. 10^{-2}. Among the plots with vegetation, the best vegetation cover is the native vegetation and the worst is cactus with downslope tillage.
- The value of the erodibility parameter α for a bare plot is between 10^{-6} and 10^{-8}, but for a plot with native vegetation the value becomes 10^{-8} to 10^{-10}.
- The equation for predicting the rate of sheet erosion E (kg m^{-2} h^{-1}) for a semiarid area in northeastern Brazil can be written as:

  $$E = 1.91 \times 10^{-5} L^{0.9} r_e^{1.91} S_0^{1.04}$$

  where L (m) is the length of slope, r_e is the effective rainfall intensity (mm h^{-1}), and S_0 is the degree of slope (%).

Acknowledgement The field work reported in this paper was carried out by UFPB (Brazil), SUDENE (Brazil) and ORSTOM (France).

REFERENCES

Cardier, E. & Freitas, B. J. (1982) *Bacia Representativa de Sumé, Primeira Estimativa dos Recursos de Água* (Representative Basin of Sumé, first estimation of the water resources) (in Portuguese). SUDENE Recife, Pernambuco, Brazil.

Lopes, V. L. (1987) A numerical model of watershed erosion and sediment yield. PhD Thesis, University of Arizona, Tucson, Arizona, USA.

Musgrave, G. W. (1947) The quantitative evaluation of factors in water erosion: a first approximation. *J. Soil Wat. Conserv.* **2**(3), 133-138.

Santos, C. A. G., Suzuki, K., Watanabe, M. & Srinivasan, V. S. (1994) Optimization of coefficients in runoff-erosion modeling by Standardized Powell method. *J. Hydroscience and Hydraulic Engineering JSCE* **12**(1), 67-78.

Zingg, A. W. (1940) Degree and length of land slope as it affects soil loss in runoff. *Agric. Engr* **21**, 59-64.