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Developments in physically-based overland flow modelling

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Abstract The most powerful developments in rainfall–runoff modelling refer to a physically-based approach to allow for spatial variations as well as impacts, nowadays caused by man-induced land-use changes. With increased computer power, two-dimensional overland flow modelling became the state of the art. But due to highly varying flow characteristics over complex surfaces, the use of point equations for the detailed description of spatially occurring processes rapidly leads to a data deficit in the requirements. In addition, the assumptions of gradually varied flow in the point-scale technology used do not hold, unless one smoothes the important microtopographic surface structure. To overcome these deficiencies spatially-averaged conservation equations for interacting rill and interrill area overland flows are introduced. On the basis of the kinematic wave approximation the combined rill (channel) and interrill (sheet) flow is treated as one-dimensional flow with an additional interaction term. The overland flow part with the quasi-two dimensional conservation equations of the presented catchment modelling system is then linked to the flow in the channel network. It is routed by the fully dynamic St Venant equations.

INTRODUCTION

The hydrological component represents a major part within the integrated watershed management approach, considering, assessing and combining the complex relationships between environmental features as well as the cumulative effects of disturbances on environmental quality across complex landscapes. Therefore, the requirements for hydrological models simulating overland flow and related transport processes on a physical basis, by using mathematical algorithms, increased significantly in the same way as the demand for higher modelling precision has risen. Although, new tools such as geographic information systems have been developed to enable the assessment of spatial heterogeneity, present physical modelling of surface transport processes still requires assumptions of spatial homogeneity. The justified degree of simplification is determined by model scale, data requirements, availability and processing of data as well as by expenses for man-power and instrumentation involved in the overall watershed management task.

Due to the difficulties of quantitative estimates on surface transport processes by physically-based modelling approaches, simulations were done on the basis of conceptual models. As they combine the simplicity of an empirical concept with the wider applicability of the more rigorous physically-based approach, their advantage lies in the representation of the significant features of a physical process in mathematical terms. Although several decades have elapsed since these methods were introduced, they still play a major role in computer-based complex hydrological models where, in addition, the movement of nutrients, chemicals and pesticides is estimated and predicted. Their predictive capabilities are limited however to the simulation of various simple scenarios, as well as the evaluation of the resulting consequences as only a limited understanding of the underlying physical process is reflected by the conceptual approach.

Over the last few decades hydrologists have constantly decreased the lack of physical basis of mathematical models. But simulating overland flow by a fully dynamic, two-dimensional model accounting for microtopographic flow characteristics (Zhang & Cundy, 1989) will hardly be feasible if a catchment exceeds a certain size. A first practical technique for solving the conservation equations was based on the approximate kinematic wave theory (Lighthill & Whitham, 1955; Iwagaki, 1955). It was then first introduced to the modelling of watershed flow problems by Henderson & Wooding (1964) who considered in theory the runoff from a V-shaped catchment (Wooding, 1965a,b, 1966) (Fig. 1(a)). A detailed analysis on the kinematic wave criteria was then published by Woolhiser & Liggett (1967). The practical validity of the kinematic model has to be judged by direct reference to the physics of the processes involved e.g. by considering more accurate differential equations of higher order (Morris & Woolhiser, 1980; Viera, 1983).

The concept of kinematic cascades (Fig. 1(b)) was introduced by Brakensiek (1967). Based on the kinematic wave theory it helped to overcome geometric

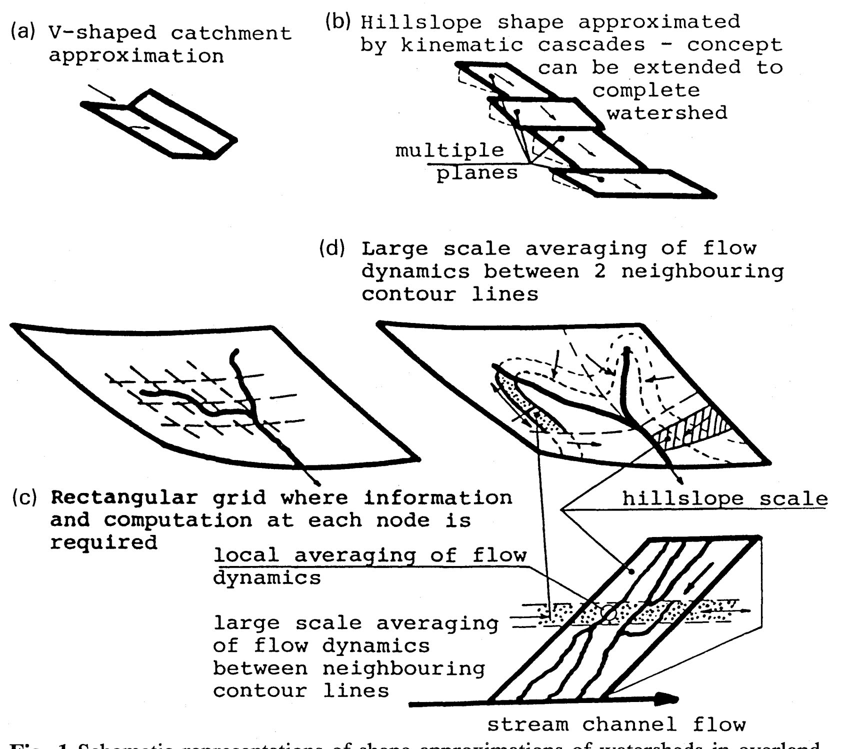

Fig. 1 Schematic representations of shape approximations of watersheds in overland flow modelling: (a) V-shaped catchment, (b) kinematic cascades—KINEROS, (c) rectangular grids—SHE model, etc., (d) spatially averaging technique of flow dynamics.

landscape restrictions in modelling complete watersheds. This type of model represents the first of its kind to combine a physically-based approach with an operational method as well as offering a certain flexibility to varying hillslope shapes.

Due to increased computer power over the years a big effort was also put into mathematical models coping with the spatially highly variable overland flow phenomena, as well as surface transport processes by using a fully dynamic two-dimensional approach (Abbott *et al.*, 1986a,b; Bathurst, 1986; Lane & Nearing, 1989; Nearing *et al.*, 1989). Aiming at considering microtopographic characteristics, a grid-based approach as well as the use of the point-scale technology poses the problem of estimating local friction and local x- and y-direction slope values at multiple single/nodal points of a computational network over a watershed due to the two-dimensional solution of the point-scale overland flow equations (Fig. 1(c)). In practice, this results in a very substantial parameter estimation problem. To overcome grid-based routing disadvantages as well as the difficulties of flow division and convergence in one-dimensional kinematic routing on a grid basis, an improved two-dimensional kinematic routing concept on a triangular irregular network was developed by Goodrich *et al.* (1991).

These and other physically-based models are essential in highlighting the physical processes involved. But in general, they are confronted with difficulties in estimating overland flow parameters due to the heterogeneity of the land surface microtopography. In addition they also have difficulties in computing overland flows at small grid spacing in order to gain realistic results. All recent models are based on a dynamic approach in one or another way, considering point-scale overland flow equations. But their correct solutions require gradually varied flow. Therefore in the case of a rapidly changing microtopography on a hillslope, it is not possible to use the point-scale equations, unless one smoothes the natural surface microtopography (Tayfur *et al.*, 1993). To account for a realistic overland flow situation, the importance of the natural hillslope system, broken into rill and interrill processes (Emmett, 1978; Meyer *et al.*, 1975), has to be understood and then considered.

A MATHEMATICAL OVERLAND FLOW MODEL

A physically-based modelling technique represents a major advance in understanding and predicting overland flow on complex hillslope profiles. Based on the derivation of spatially averaged conservation equations for interacting rill and interrill area overland flows (Tayfur & Kavvas, 1994), this section will show ways to overcome the common difficulties in:

- estimating overland flow parameters of heterogeneous surfaces;
- computing overland flows at small grid spacing for realistic results by
- considering the microtopography of the surface at the same time,
- representing the flow over complex landscape surfaces.

In real life situations, overland flow, over land surfaces, is characterized by flow on interrill areas as well as flow in rills discharging into the stream channel network of a catchment (Fig. 1(d)). Hence, a good hydrological model not only has to assume the occurrence of surface flow as sheet flow, it also has to consider the influence of rills on the flow dynamics to avoid serious misinterpretation of results and finally has to have the capability to express the combined flow dynamics over the landscape forming hillslopes.

Kavvas & Govindaraju (1992) combined in a simple and strictly one-dimensional way—no interaction between flows in parallel rills and neighbouring interrill areas—the rill flow dynamics. As interrill areas always contribute a certain amount of flow towards rills, which in a realistic approach therefore can't be assumed as straight and parallel, a two-dimensional expansion of the combined flow dynamics was then developed by Tayfur & Kavvas (1994), where flow interaction between interrill areas and rills exists.

The interrill area flow is conceptualized as a two-dimensional sheet flow and a one-dimensional channel flow is assigned to the rills. The overland flow dynamic is simulated by a kinematic wave approximation of the St Venant equations as this is a very practicable approach (Govindaraju *et al.*, 1992; Woolhiser, 1975). The kinematic wave arises out of a continuity equation,

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = R \tag{1}$$

e.g. with A as the cross-sectional area of flow, Q as the discharge and R as a source/sink term as well as a unique relationship of the form:

$$Q = Q(A; x, t) \tag{2}$$

For the case of two-dimensional overland flow over a plane of constant width this results in:

$$\frac{\partial h_o}{\partial t} = i_e(x,y,t) - \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} \tag{3}$$

and

$$q_x = q_x(h_o; x, t) \tag{4a}$$

$$q_y = q_y(h_o; y, t) \tag{4b}$$

where q_x and q_y denote the discharge per unit width in the x-direction and in the y-direction respectively, $h_o(x,y,z)$ the interrill area sheet flow depth and i_e the rate of rainfall excess. In most situations of hydrological relevance an explicit independence of q on t can be assumed leading to $q_x = q_x(h_o;x)$ and $q_y = q_y(h_o;y)$ respectively. It can be expressed in power law form $q = ch^m$ with c and m as parameters giving q_x and q_y as

$$q_x = C_x h_o^{5/3} = \frac{\sqrt{S_{ox}}}{n\sqrt[4]{1+\sqrt{\frac{S_{oy}}{S_{ox}}}}} h_o^{5/3} \tag{5a}$$

and

$$q_y = C_y h_o^{5/3} = \frac{\sqrt{S_{oy}}}{n\sqrt[4]{1+\sqrt{\frac{S_{ox}}{S_{oy}}}}} h_o^{5/3} \tag{5b}$$

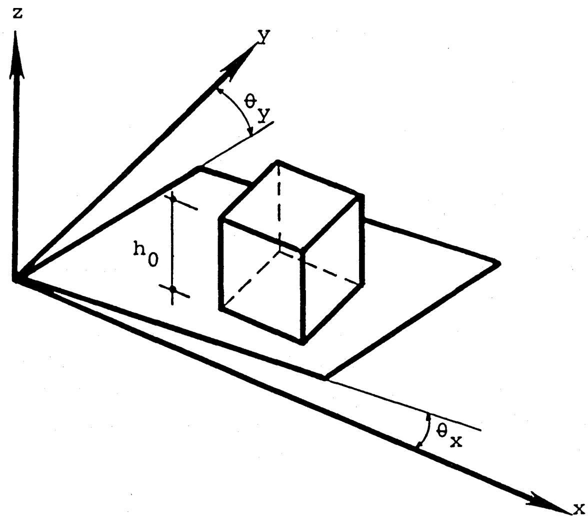

Fig. 2 Schematic flow element representing the sheet flow dynamics on an interrill area.

where n denotes Manning's roughness coefficient, S_{ox} the bed slope in the x-direction (S_{ox} = $\tan\theta_x$) and S_{oy} the bed slope in the y-direction (S_{oy} $=\tan\theta_y$) (Fig. 2).

As equation (3) represents an equation conserving mass at a single point, its practical application will be impracticable as each model parameter has to be monitored at every single node/point location on the hillslopes. Therefore equation (1) is locally averaged over an individual interrill area having a width l (Fig. 1(d) and Fig. 3) to obtain the locally averaged sheet flow equation:

$$\frac{\partial \bar{h}_o}{\partial t} = \bar{i}_e - \frac{\partial}{\partial x}(C_x \bar{h}_o^{5/3}) - \frac{1}{2}\pi^{5/3}\frac{C_y}{l}\bar{h}_o^{5/3} = f_o(\bar{h}_o; x, y, t) \qquad (6)$$

where $\bar{h}_o$ and $\bar{i}_e$ denote the averaged parameter values. Although equation (6) is one-dimensional, it contains two-dimensional properties as the second term on the right

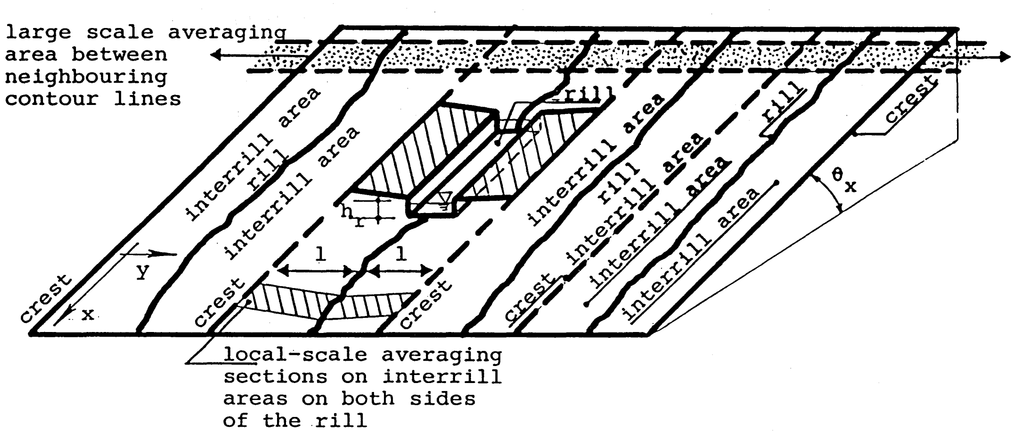

Fig. 3 Schematic landscape representation of a part of a watershed with a local-scale averaging rill and interrill section as well as with a large-scale averaging area between two neighbouring contour lines.

hand side of the equation represents the water flux discharging from the interrill area section into the rill.

The locally averaged one-dimensional equation for rill flow is similar to the overland flow equation. For a rectangular rill cross-section as well as under the kinematic wave assumption, the flow equation can be stated as:

$$\frac{\partial h_r}{\partial t} = i_e + q_l - \frac{\sqrt{S_{ox}}}{n}\frac{1}{b_r}\frac{\partial}{\partial x}\frac{(bh_r)^{3/2}}{\sqrt{b+2h_r}} = f_r(h_r, b; x, t) \tag{7}$$

where b denotes the rill width, h_r the rill flow depth, S_{ox} the bed slope of the rill and q_l the net lateral inflow from the interrill area section into the rill—q_l can then be separated into a rill inflow component from the right-hand side of the rill as well as the left-hand side and can be estimated according to equations (5a) and (5b).

But as natural landscapes consist of hillslopes with microtopographic surface structures characterized by a large number of rills and interrill areas, it is unrealistic to evaluate the physical and hydraulic properties of each rill and interrill section in order to avoid gross errors in the flow predictions. Therefore the locally averaged flow equations (6) and (7) have to be averaged along the transect and/or contour lines of the watershed representing hillslopes (Fig. 1(d) and 3). This large-scale averaging is based on the stochastic averaging theory. Its application to the overall overland flow dynamics (combined averaged interrill and rill flow) over a certain length L results in an equation for an overall mean flow depth $\bar{h}$ (Govindaraju *et al.*, 1992):

$$\frac{\partial \bar{h}}{\partial t} = \langle f_r(h_r, b; x, t)\rangle . \lambda(x,y,t) + \langle f_o(h_o; x, y, t)\rangle . [1 - \lambda(x,y,t)] \tag{8}$$

Equation (8) reflects the distribution of the rills over a hillslope through the parameter λ (x,y,t) as well as the fact that any stochastic parameter such as the rill widths and depths etc. have to be averaged which is expressed by $\langle\rangle$. The multiple solution of equation (8) for various hillslope transects respectively watershed contour lines leads then to averaged flow velocities and discharges. Overland flow towards the draining watershed channels can then be computed.

THE OVERLAND FLOW MODEL WITHIN A WATERSHED FRAMEWORK

The spatially-averaged conservation equations for overland flow with interacting rill and interrill dynamics represent the innovative component of the new watershed modelling system for the transport over complex surfaces. Other presently available process components incorporated into the expending system are rainfall, infiltration and channel flow. The system, expendable for soil erosion, nutrient transport etc., allows for spatial and temporal variability of the relevant parameters. The watershed is subdivided by contour lines where the controlling parameters are averaged over the area between neighbouring contours. The input file is structured in such a manner that the relevant simulation parameters for the estimation of the flow dynamics along chosen streamlines are provided for each area between neighbouring contour lines. Consequently the resulting overland flow discharges from the hillslopes, which finally drain into the channel network of the watershed, are computed. However, due to the splitting of the basin by contour lines as well as stream lines, spatial variability is considered. For every time increment during the simulation of a rainstorm a relevant input file has to be prepared. Thus, the system not only allows for areal variability but also considers the temporal variations within a storm event.

Monitored rainfall data are gathered from various raingauges representing the rainfall characteristics of the catchment. It is planned to provide a pattern of rainfall intensities as simulation input for a situation of a moving thunderstorm over a research area.

The infiltration is simulated on the basis of the Green & Ampt (1911) parameters. Due to its simplicity as well as widely available parameters this model is very popular in large-scale watershed modelling. Using an approximation procedure, certain assumptions regarding the hydraulic properties of the soil have to be considered:

- homogeneous soil profile,
- infiltration only occurs in the vertical direction,
- infiltration through macropores is neglected,
- shrinking and expansion of the soil is negligible,
- no encrustation on the soil surface occurs during rainfall.

As the Green & Ampt model was used for the ponded infiltration into a homogeneous soil with initial uniform water content (Fig. 4), the infiltration rate is given by:

$$f = K + K\frac{n\psi_f}{F} \tag{9}$$

and its integration leads to:

$$Kt = F - n\psi_f \ln(1 + \frac{F}{n\psi_f}) \tag{10}$$

with $n = \phi_c - \theta_i$ denoting the available porosity, $\phi_c = \phi - \phi_r$ the effective porosity, ϕ_r the residual saturation, θ_i the initial soil water content, ψ_f the wetting front capillary

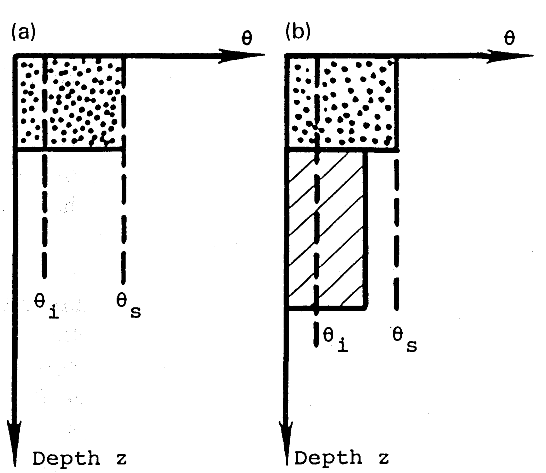

Fig. 4 Schematic representation of a soil water infiltration front for a precipitation event with a rainfall intensity (a) constant in time as well as (b) varying in time.

pressure head [cm], t the time [h], F the total amount of infiltrated water [cm], f the infiltration rate [cm h^{-1}] and K the hydraulic conductivity [cm h^{-1}]. These parameters can be estimated on the basis of soil texture classes (Rawls *et al.*, 1983).

The runoff produced on the hillslopes forms small stream channels where it is finally transported through the stream channel network towards the outlet of the watershed. Considering a complex fourth-order dendritic channel structure on the basin scale (Fig. 5), Rajbhandari (1989) applied an efficient method for the solution of the two partial differential equations representing the conservation of mass and momentum to simulate the network's flow dynamics. For the computational improvement the overlapping Y-segment (Sevuk, 1973) and/or sequential routing technique (Yen & Akan, 1976) is used as well as a sparse matrix solution technique based on a special Gauss elimination technique (Gupta & Tanji, 1977) to gain a fast solvable matrix of banded structure of the dynamic equations. An implicit weighted four-point finite-difference scheme (Fread, 1978) is used for the solution of the unsteady flow equations, besides the Newton–Raphson iterative procedure (Amein & Fang, 1970) for their sequential solving. The same solution technique is applied to the overland flow part. For the channel flow, the dynamic wave model, the diffusion wave model as well as the kinematic wave model can be applied to solve the St Venant equations.

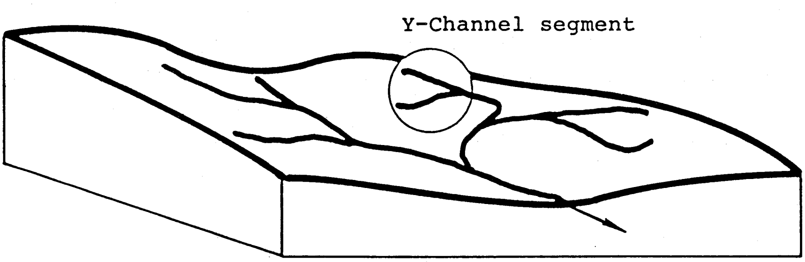

Fig. 5 Schematic representation of a fourth-order stream channel network of a watershed which can efficiently be solved either by the overlapping of Y-segments which stepwise substitute for the complete network or by sequential and separate routing in each channel from the most upstream branch downwards, satisfying the continuity requirements at each junctions.

CONCLUSIONS

Current research aims at representing complex landscape surfaces and the flow over those surfaces. The concept of the stochastically averaged flow dynamics, leading to extended overland flow equations based on the kinematic wave approximation for sheet flow and rill flow, characterizes the controlling surface geometry in terms of a limited number of parameters, besides the approach to combine rill flow dynamics with sheet flow dynamics. In point scale technology, by avoiding the intensive data requirements as well as time-consuming data processing, the spatially averaged conservation equation technology uses significantly less information about the landsurface microtopography, while the computation is simpler, and hence, faster. The plausibility of this technology is also shown by the fact that in practice one never has all the necessary information for each point of a detailed numerical scheme laid over a watershed. Estimations of slope gradients for hillslopes by the use of GIS storing DTM information with a resolution of 30 × 30m represents the common data availability in practical hydrological engineering. Based on such a database, a comparison of the new spatially averaged conservation equations with two other popular watershed models (EPIC and AGNPS), which are based on simple but very practical and powerful approaches (SCS method in the EPIC model and mass balance between neighbouring cells—continuity equation considering steady overland flow dynamics—in the AGNPS model), will be validated in an Alpine catchment.

The new averaged equations representing the overland flow dynamics are the major component within a newly developed hydrological system for the modelling of transport processes over complex landscape surfaces. Just considering the flow processes within a watershed, the overland flow part was linked to the Green & Ampt based infiltration component as well as to a component simulating the unsteady flow dynamics in a dendritic stream channel system draining the basin.

Critically analysing the equation for the overland flow dynamics as well as the approximate procedure for the infiltration simulation, the physically-based concept will more efficiently produce results of sufficient precision in hydrological engineering applications in comparison to other physically-based approaches. Merely the simulation of rainstorms characterized by several peaks with significant dry periods in between the peaks (also considering large variation in intensities), limits the application of this concept. The assumption of a rigid wetting front in the case of a Hortonian overland flow situation (storm intensity > vertical infiltration rate at saturated soil conditions) is valid. But the concept shows deficiencies during single storm situations where the rainfall intensity of a later peak is smaller than that of a previous peak and/or the vertical infiltration rate of the saturated soil. In such situations a significant hysteresis effect regarding the pF-curve can occur, strongly depending on the soils. However, in dealing with storm events of strongly varying intensities an adapted infiltration approach has to be considered.

The flow dynamics in the channel network are substituted by Y-segments. The diffusion wave approximation of the St Venant equations can result in less precision near the junction of the Y-system, as the effect of local and convective acceleration due to interacting channels can't be considered by this type of model. The kinematic wave approximation shows the inability to account for backwater effects from downstream. Only the full dynamic wave model accounts for the complete flow dynamics. However, reasonable results combined with efficient computation needs the correct evaluation of the expected flow situation in the stream channel network.

In general it can be stated that models based on the spatially averaging technology of conservation equations offer a powerful and versatile hydrological tool. Although an erosion as well as a nutrient transport component for the presented watershed system based on the same averaging principles has recently been developed, the potential of the new technology has not yet been fully exploited. In particular climatic and/or groundwater modelling may prove a promising field for further developments and applications.

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REFERENCES

Abbott, M. B., Bathurst, J. C., Cunge, J. A., O'Connell, P. E. & Rasmussen, J. R. (1986a) An introduction to the European Hydrological System—Système Hydrologique Européen, "SHE" 1: History and philosophy of a physically-based, distributed modelling system. *J. Hydrol.* **87**, 45–59.

Abbott, M. B., Bathurst, J. C., Cunge, J. A., O'Connell, P. E. & Rasmussen, J. R. (1986b) An introduction to the European Hydrological System—Système Hydrologique Européen, "SHE", 2: Structure of a physically-based modelling system. *J. Hydrol.* **87**, 61–77.

Amein, M. & Fang, C. S. (1970) Implicit flood routing in natural channels. *J. Hydraul. Engng, ASCE* **96**(12), 2481–2500.

Bathurst, J. C. (1986) Physically-based distributed modelling of an upland catchment using the Système Hydrologique Européen. *J. Hydrol.* **87**, 79–102.

Brakensiek, D. L. (1967) Simulated watershed flow system for hydrograph prediction: a kinematic application. *Proc. Int. Hydrol. Symp., Pap. no. 3.* Fort Collins, Colorado.

Emmett, W. W. (1978) Overland flow. In: *Hillslope Hydrology* (ed. by M. J. Kirkby),. 145–176. John Wiley, New York.

Fread, D. L. (1978) NWS operational dynamic wave model. In: *Proc. Verification of Math. and Phys. Models* (26th Annual Hydraul. Div. Spec. Conf., ASCE, College Park, Maryland), 455–464.

Goodrich, D. C., Woolhiser, D. A. & Keefer, T. O. (1991) Kinematic routing using finite elements on a triangular irregular network. *Wat. Resour. Res.* **27**(6), 995–1003.

Govindaraju, R. S., Kavvas, M. L. & Tayfur, G. (1992) A simplified model for two-dimensional overland flows. *Adv. Wat. Res.* **15**, 133–141.

Green, W. H. & Ampt, G. A. (1911) Studies on soil physics. I: The flow of air and water through soils. *J. Agric. Sci.* **4**, 1–24.

Gupta, S. K. Tanji, K. K. (1977) Computer program for solution of large, sparse, unsymmetric systems of linear equations. *J. Numerical Meth. in Engng* **11**, 1251–1259.

Henderson, F. M. & Wooding, R. A. (1964) Overland flow and groundwater flow from a steady rainfall of finite duration. *J. Geophys. Res.* **69**(8), 1531–1540.

Iwagaki, Y. (1955) Fundamental studies of the runoff analysis by characteristics. *Disaster Prevention Research Institute, Bulletin no. 10, Kyoto University, Kyoto.*

Kavvas, M. L. & Govindaraju, R. S. (1992) Hydrodynamic averaging of overland flow and soil erosion over rilled hillslopes. In: *Erosion, Debris Flows and Environment in Mountain Regions* (ed. by D. E. Walling, T. R. Davies & B. Hasholt) (Proc. Chengdu Symp., July 1992), 101–111. IAHS Publ. no. 209.

Lane, L. J. & Nearing, M. A. (1989) Water erosion prediction project landscape profile model documentation. *NSERL Report no. 2, ARS, USDA, Purdue Univ., West Lafayette.*

Lighthill, M. L. & Whitham, G. B. (1955) On kinematic waves, I. Flood movement in long rivers. *Proc. Roy. Soc.* **A229**, 281–316.

Meyer, L. D., Foster, G. R. & Romhens, M. J. M. (1975) Sources of soil eroded from upland slopes. In: *Proc. 1972 Sediment Yield Workshop*, 177–189. USDA Sediment Lab., ARS, Oxford, Mississippi.

Morris, E. M. & Woolhiser, D. A. (1980) Unsteady one-dimensional flow over a plane: partial equilibrium and recession hydrographs. *Wat. Resour. Res.* **16**(2), 355–360.

Nearing, M. A., Foster, G. R., Lane, L. J. & Finkner, S. C. (1989) A process-based soil erosion model for USDA-Water Erosion Prediction Project technology. *Trans. Am. Soc. Agric. Engrs.*

Rajbhandari, H. (1989) Dynamics of flow in channel networks. MSc Thesis, Dept Civil and Environ. Engng, Univ. California, Davis.

Rawls, W. J., Brakensiek, D. L. & Miller, N. (1983) Green–Ampt infiltration parameters from soils data. *J. Hydraul. Engng, ASCE* **109**(1).

Sevuk, A. S. (1973) Unsteady flow in sewer networks. PhD Thesis, Dept Civil. Engng, Univ. Illinois, Urbana—Champaign.

Tayfur, G. & Kavvas, M. L. (1994) Spatially averaged conservation equations for interacting rill-interrill area overland flows. *J. Hydraul. Engng, ASCE* **120**(12), 1426–1448.

Tayfur, G., Kavvas, M. L., Govindaraju, R. S. & Storm, D. E. (1993) Applicability of St Venant equations for two-dimensional overland flows over rough infiltrating surfaces. *J. Hydraul. Engng, ASCE* **119**(1), 51–63.

Viera, J. H. D. (1983) Conditions governing the use of approximations for the St Venant equations for shallow water surface flow. *J. Hydrol.* **60**, 43–58.

Wooding, R. A. (1965a) A hydraulic model for the catchment-stream problem. I, Kinematic wave theory. *J. Hydrol.* **3**, 254–267.

Wooding, R. A. (1965b) A hydraulic model for the catchment-stream problem. II, Numerical solutions. *J. Hydrol.* **3**, 268–282.

Wooding, R. A. (1966) A hydraulic model for the catchment-stream problem. III, Comparison with runoff observations.

J. Hydrol. **4**, 21–37.

Woolhiser, D. A. (1975) Simulation of unsteady overland flow. In: *Unsteady Flow in Open Channels* (ed. by K. Mahmood & V. Yevjevich), vol. 2, 485–508. Water Resources Publications, Fort Collins, Colorado.

Woolhiser, D. A. & Liggett, J. A. (1967) Unsteady one-dimensional flow over a plane—the rising hydrograph. *Wat. Resour. Res.* **3**(3), 753–771.

Woolhiser, D. A., Smith, R. E. & Goodrich, D. E. (1990) *KINEROS, a Kinematic Runoff and Erosion Model: Documentation and User Manual.* USDA, ARS-77.

Yen, B. C. & Akan, A. O. (1976) Flood routing through river junctions. In: *Proc. Symp. of Waterways and Rivers '76*, vol. 1, 212–231; vol. 2, 485–508. Am. Soc. Civ. Engrs.

Zhang, W. & Cundy, T. W. (1989) Modelling of two-dimensional overland flow. *Wat. Resour. Res.* **25**(9), 2019–2035.