Comparison between two mathematical models for the computation of sediment yield from a basin

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Abstract Two mathematical models were used for the estimate of sediment yield resulting from rainfall and runoff at the outlet of the Kossynthos basin (250 km^2; Thrace, Greece). Both models consist of three sub-models: a simplified rainfall–runoff sub-model, a surface erosion sub-model and a sediment transport sub-model for streams. The two models differ only in the surface erosion sub-model. In the rainfall–runoff sub-model the soil moisture variation in the root zone is considered. The surface erosion sub-model of the first model is based on the "Universal Soil Loss Equation", while the corresponding sub-model of the second model is based on the relationships of Poesen (1985) for splash detachment and splash transport. The sediment transport sub-model for streams is based on the concept of sediment transport capacity (Yang & Stall, 1976). The degree of conformity between the annual values of sediment yield at the basin outlet according to both models is satisfactory.

NOTATION

a	slope gradient (°)
c_t	total sediment concentration by weight (ppm)
C	cover factor
D_{50}	median particle diameter (m)
DR	sediment delivery ratio (%)
E_p	potential evapotranspiration (mm)
f	friction factor
g	gravity acceleration (m s^{-2})
h_o	surface runoff (mm)
i	energy slope
IN	deep percolation (mm)
k, k'	proportionality coefficients
KE	rainfall kinetic energy (J m^{-2})
n	index for the time step of the variables
N	rainfall amount (mm)
q	surface runoff (m^3 s^{-1} m^{-1})
q_f	sediment transport by runoff (m^3 s^{-1} m^{-1})
q_r	downslope splash transport per unit width (kg m^{-1})
q_{rs}	mass of detached particles per unit area (kg m^{-2})
q_t	sediment transport capacity by overland flow (m^3 s^{-1} m^{-1})
r	entrainment ratio
r_s	soil resistance to drop detachment (J kg^{-1})

S	available soil moisture (mm)
S_{max}	maximum available soil moisture (mm)
u	average water velocity (m s^{-1})
u_{cr}	critical average water velocity (m s^{-1})
u_*	shear velocity (m s^{-1})
w	terminal fall velocity of sediment particles (m s^{-1})
YA	annual value of sediment yield at the basin outlet (t)
YD	annual value of surface erosion amount for the whole basin (t)
ν	kinematic viscosity (m^2 s)
ρ	water density (kg m^{-3})
ρ_s	sediment density (kg m^{-3})

INTRODUCTION

The classical "Universal Soil Loss Equation" (USLE) or its modifications have been applied in the past by the author of this paper for estimating sediment yield due to rainfall and runoff at the outlets of the sub-basins of a large basin. The investigated basin is located in central Europe (Hrissanthou, 1988). In spite of the empirical nature of the USLE and the fact that this equation was initially developed for small agricultural fields, the computed annual values of sediment yield at the basin outlet were satisfactory compared with the corresponding measured values.

After the establishment of the USLE, several models for estimating surface erosion due to rainfall and runoff were also developed for small experimental fields. One of these newer models is the model of Poesen (1985). In the present paper both the USLE and the relationships of Poesen are used as surface erosion sub-models of two different mathematical models for estimating sediment yield at the outlet of the Kossynthos basin in Greece. Both mathematical models include two identical sub-models: a rainfall–runoff sub-model and a stream sediment transport sub-model. The individual sub-models are briefly described in the following sections.

RAINFALL–RUNOFF SUB-MODEL

A simple water balance model was used for the computation of surface runoff (Giakoumakis & Tsakiris, 1992). The water is stored in the root zone which may be simulated with a shallow container. The difference $S_{max} - S_n$ is the soil moisture deficit for the time increment considered. It is obvious that the depth of the available soil moisture increases through the precipitation N_n and decreases through the potential evapotranspiration E_{pn} and the deep percolation IN_n. The balancing equation is written below:

$$S_n' = S_{n-1} + N_n + E_{pn} \tag{1}$$

The surface runoff h_{on} and the deep percolation IN_n for the time step n can be evaluated as follows:

if $S_n' < 0$ then $S_n = 0$, $h_{on} = 0$ and $IN_n = 0$

if $0 \leq S_n' \leq S_{max}$ then $S_n' = S_n$, $h_{on} = 0$ and $IN_n = 0$

if $S_n' > S_{max}$ then $S_n = S_{max}$, $h_{on} = k(S_n' - S_{max})$ and $IN_n = k'(S_n' - S_{max})$ where $k' = 1 - k$

FIRST SURFACE EROSION SUB-MODEL

This sub-model is based on the USLE with an improved erosivity factor (Foster *et al.*, 1977). The sediment transport capacity by overland flow is calculated by the relationships of Beasley *et al.* (1980). The sub-model is described in detail in Hrissanthou (1988).

The sediment supply from surface erosion to a stream may be estimated by means of the following controls: If the available sediment in the stream basin exceeds overland flow sediment transport capacity, deposition occurs on the basin and the sediment transported to the stream equals sediment transport capacity. If the available sediment in the basin is less than overland flow sediment transport capacity and if the flow's erosive forces exceed the resistance of the soil to detachment by flow, detachment occurs; in this case sediment transported to the stream equals the available sediment.

SECOND SURFACE EROSION SUB-MODEL

The following relationships of Poesen (1985) were used for estimating surface erosion:

$$q_{rs} = C(KE)r_s^{-1}\cos a \tag{2}$$

$$q_r = q_{rs}[0.301\sin a + 0.019D_{50}^{-0.22}(1 - e^{-2.42\sin a})] \tag{3}$$

The original relationship of Poesen for splash detachment is valid for bare soils. Therefore, an additional factor is necessary to express the decrease of splash detachment because of the vegetation. It is believed that the dimensionless vegetation factor C of the USLE is appropriate to express the vegetation influence.

The sediment transport by runoff q_f can be expressed as follows (Nielsen *et al.*, 1986):

$$q_f = rq_t \tag{4}$$

The entrainment ratio r equals 1 for noncohesive soils while for cohesive soils it is less than 1.

The well known formula of Engelund & Hansen (1967) for sediment transport capacity by streamflow was modified especially for overland flow:

$$q_t = \frac{(2g/f)^{1/6}}{(\rho_s/\rho - 1)^2 g^{1/2} D_{50}} q^{5/3} i^{5/3} \tag{5}$$

The available sediment on the soil surface equals the sum "downslope splash transport + sediment transport by runoff". The sediment quantity reaching a stream from the corresponding basin area results by comparing this sum with the sediment transport capacity by overland flow as described in the previous section.

A fine difference between the first and the second surface erosion sub-model is that sediment transport due to rainfall and runoff is computed by the second sub-model, while soil detachment due to rainfall and runoff is calculated by the first sub-model based on the USLE.

STREAM SEDIMENT TRANSPORT SUB-MODEL

The sediment yield at the outlet of the stream considered may be computed by the concept of sediment transport capacity by streamflow. The following relationships were used to compute sediment transport capacity by streamflow (Yang & Stall, 1976):

$$\log c_t = 5.435 - 0.286 \log \frac{wD_{50}}{\nu} - 0.457 \log \frac{u_*}{w} + \left(1.799 - 0.409 \log \frac{wD_{50}}{\nu} - 0.314 \log \frac{u_*}{w}\right) \log\left(\frac{ui}{w} - \frac{u_{cr} i}{w}\right) \qquad (6)$$

$$\frac{u_{cr}}{w} = \frac{2.5}{\log(u_* D_{50}/\nu) - 0.06} + 0.66 \qquad \text{for } 1.2 < u_* D_{50}/\nu < 70 \qquad (7)$$

$$\frac{u_{cr}}{w} = 2.05 \qquad \text{for } u_* D_{50}/\nu \geq 70 \qquad (8)$$

The sediment yield at the outlet of the stream considered reflects the same basic controls as the sediment supply to the stream from surface erosion: If the available sediment in the stream exceeds sediment transport capacity by streamflow, deposition occurs and the sediment outflow equals sediment transport capacity. If the available sediment is less than streamflow sediment transport capacity, bed detachment may occur and the sediment outflow equals the available sediment.

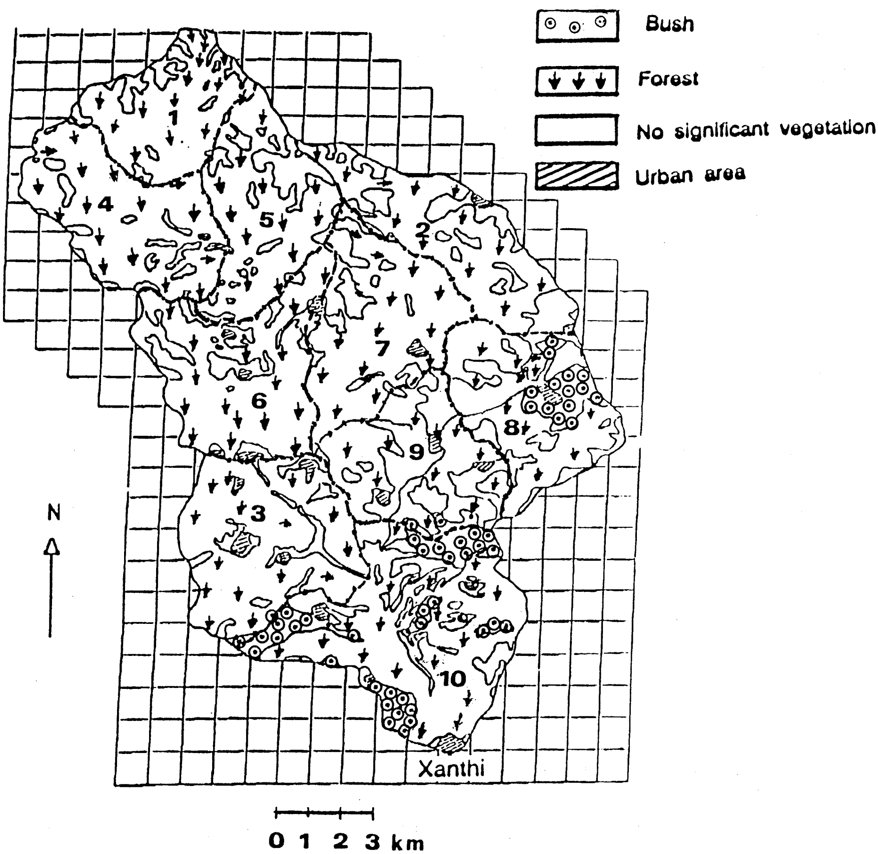

Fig. 1 Vegetation map of Kossynthos basin divided into 10 sub-basins.

APPLICATION TO KOSSYNTHOS BASIN

The mathematical models described above were applied to the Kossynthos basin. The basin of the Kossynthos stream lies north of Xanthi (Thrace, Greece) and has an area of about 250 km^2. It consists of forest (74%), bush (4.5%), urban area (1.5%) and an area with no significant vegetation (20%) (Fig. 1). The highest part of the basin has an altitude of about 1700 m. The largest stream length from the basin outlet (Xanthi) up to the basin boundary is about 35 km.

The whole basin was divided into 10 natural sub-basins (Fig. 1) for more precise calculations, the sub-models were applied to each sub-basin separately. Only the main stream of each sub-basin was considered in the sediment transport sub-model for streams, because numerous unavailable data for the geometry and hydraulics of the entire stream system would otherwise be required. A sediment routing plan is necessary in order to specify the sediment motion from sub-basin to sub-basin.

Monthly rainfall and other meteorological data for 12 years (1980–1991) from six rainfall stations were available. Therefore, the calculations were performed on a monthly basis. This way of working renders necessary the following assumptions: uniform conditions exist over a sub-basin and steady-state conditions exist throughout each month for the runoff and erosion processes.

The monthly values of sediment yield at the basin outlet resulting from the models for a certain year were added to produce the annual value of sediment yield YA due to surface and stream erosion. The ratio of YA to the corresponding annual value of surface erosion amount YD for the whole basin is called the sediment delivery ratio (DR). The computer results from both mathematical models for YA, YD and DR for the years 1980–1991 are contained in Table 1. It is observed that the second model supplies disproportionately extreme values of surface erosion compared with the first model for too high or too low rainfall values (years 1990 and 1991). This fact is due to the extreme values of sediment transport by runoff (second model), compared with the values of soil detachment by rainfall and runoff (first model), for extreme events.

Table 1 Computational results for *YA*, *YD* and *DR* for different years.

Year	Model 1 *YA* (t)	Model 1 *YD* (t)	Model 1 *DR* (%)	Model 2 *YA* (t)	Model 2 *YD* (t)	Model 2 *DR* (%)
1980	340 500	578 000	59	311 000	739 000	42
1981	199 000	412 000	48	198 000	528 500	37
1982	223 000	440 000	51	216 000	690 500	31
1983	109 000	348 000	31	91 500	169 500	54
1984	182 000	306 500	59	157 500	291 500	54
1985	212 500	415 000	51	194 500	340 500	57
1986	129 500	270 000	48	116 000	244 000	48
1987	313 000	502 500	62	301 500	657 500	46
1988	197 000	376 500	52	194 000	414 000	47
1989	122 000	317 500	38	116 000	194 000	60
1990	256 000	529 500	48	252 500	1 526 000	16
1991	60 000	234 000	26	42 000	42 000	100

REMARKS AND CONCLUSIONS

The lack of sediment yield data was the main reason for applying two different mathematical models to the basin considered. The small deviation between the results of both models is an encouraging indication for the size order of the computed sediment yield.

The most important drawbacks of the modelling chain are reported below:

(a) The temporal development of the physical processes over the considered time period is not followed. The models compute only total values of runoff, surface erosion and sediment transport.

(b) The equations used for surface erosion and sediment transport were not adapted to local conditions; especially, the equations for surface erosion were developed for small experimental fields.

(c) Snowmelt runoff, gully and bank erosion were neglected.

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