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Numerical sediment transport models—theoretical and practical aspects

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Abstract This paper deals with theoretical and practical aspects of the application of numerical sediment transport models. The main emphasis is given to models dealing with bed-load transport. Nine sediment transport models are analysed and compared with respect to dimensions being used (1-D, 2-D, 3-D), steady or unsteady calculation, coupled or uncoupled models, armouring, grain sorting, load calculation, fractionwise transport, quantification of sediment input via tributaries, side erosion, automatic variation of river bed width. Based on the comparison one model is applied to a theoretical river section in order to perform a sensitivity analysis. The sensitivity analysis showed that the roughness estimation is particularly important for the simulation, followed by the grain-size distributions of the subsurface and surface layer and input material. For the fact that natural variations of the input parameters are even larger than those used in the analysis the accurate measurement of all relevant data is essential for the quality of the simulation.

INTRODUCTION

Due to intensive modifications in land use, the erection of water power plants, river engineering works and climatic changes, many rivers all over the world show modified sediment transport conditions. The major results are either a surplus of sediments in impounded sections and reservoirs or a lack of material in remaining free flowing sections, leading to river bed degradation problems (Habersack & Nachtnebel, 1997). Although physical models are often used to simulate defined reaches long river sections of many kilometres and study periods of several decades limit their applicability. Numerical sediment transport models are then used to simulate sedimentation and erosion in rivers over long time periods and distances.

The aims of this paper are to discuss first the theoretical aspects of numerical sediment transport models with special emphasis on the comparison of nine selected models. Then the model HEC2SR is used to perform a sensitivity analysis concerning often used calibration parameters. The possible effect of parameter changes is compared to natural data variations in order to derive some information concerning practical necessities.

THEORETICAL ASPECTS OF NUMERICAL SEDIMENT TRANSPORT MODELS

The numerical sediment transport models analysed in this paper provide a description of the physical conditions and changes of alluvial rivers. Generally the models

consist of two major parts:
- water routing,
- sediment routing.

Concerning water flow the continuity and momentum equations in the longitudinal direction are:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial z}{\partial x} + gAI_R = 0 \tag{2}$$

where Q is the discharge, A is the cross-sectional area of flow, t is the time, x is the longitudinal coordinate, g is the gravitational acceleration, I_R is the energy gradient.

For the fact that there exists no analytical solution for the differential equations numerical solutions like the finite difference method, either using an implicit or explicit scheme (Cunge *et al.*, 1980), or the method of characteristics have been developed. The sediment routing mostly consists of:
- computation of sediment transport capacity using a suitable formula,
- determination of the actual sediment discharge by making corrections for grain sorting, hiding, exposure effects or the availability of sediments,
- upstream boundary condition for sediment input,
- numerical solution for the sediment continuity equation, which is:

$$(1-p)\frac{\partial A_b}{\partial t} + \frac{\partial Q_s}{\partial x} - q_s = 0 \tag{3}$$

where p is the porosity of the bed material, A_b is the cross-sectional area of the channel, Q_s is the bed-material discharge and q_s is the lateral inflow rate of sediment per unit length.

COMPARISON OF EXISTING MODELS

At the moment a variety of numerical sediment transport models exists. Within this paper mainly bed-load transport models are discussed. Based on a comparison of 12 selected computer stream sedimentation models (Shou-Shan, 1988) and additional software used by the author Table 1 compares nine frequently used models. The models were developed by the following institutions:

HEC-6:	Hydraulic Engineering Center, Davis,	Thomas (1982), Gee (1988)
IALLUVIAL:	Iowa Institute of Hydraulic Research,	Karim & Kennedy (1982a,b)
CHARIMA:	Iowa Institute of Hydraulic Research,	(1984), Holly Jr (1988)
STARS:	Bureau of Reclamation, Denver,	Strand *et al.* (1988)
GSTARS:	Bureau of Reclamation, Denver,	Yang *et al.* (1988)
SEDICOUP:	Iowa Institute of Hydraulic Research,	Holly Jr (1988)
MORMO:	VAW Zürich,	Hunziker (1995)
FLUVIAL-12	San Diego State University,	Chang (1988, 1993)
HEC2SR	Simons, Li and Ass., Inc., Fort Collins,	Li *et al.* (1988)

Table 1 shows that all models are capable of calculating the 1-D sediment transport, armouring, and sorting processes. Major differences exist concerning steady or unsteady, coupled or uncoupled modelling procedures, side erosion, automatic width adjustment and especially transport formulae.

Based on the comparison of the models in Austria mainly the models HEC2SR, FLUVIAL-12 (Habersack, 1997) and the model MORMO are used.

Table 1 Characteristics of transport models (based on Shou-Shan, 1988; Sereinig, 1995).

Characteristics and transport formulae	HEC-6	IALLUVIAL	CHARIMA	STARS	GSTARS	SEDICOUP	MORMO	FLUVIAL-12	HEC2SR
1-D	•	•	•	•	•	•	•	•	•
Semi 2-D				•	•			•	•
Semi 3-D					•				•
Quasi-dynamic	•	•	•	•	•		•	•	•
Unsteady			•			•		•	
Coupled						•			
Iteratively coupled			•						
Uncoupled	•	•		•	•		•	•	•
Armouring	•	•	•	2)	•	•	•	•	•
Sorting	•	•	•	•	•	•	•	•	•
Total load	•	•	•	•	•		•	•	
Separate calculation						•			•
Non equilibrium calculation						•		•	•
Fractionwise transport	•	•	•	2)	•	•	•	•	•
Variable roughness	•	•	1)						
Tributaries	•	•	•	3)	3)	•	•	3)	•
Side erosion	•				•	•		•	4)
Automatic width adjustment					•			•	
TLTM		•	•						
Ackers–White (1973)			•	•	•			•	
Engelund–Hansen (1967)			•	•	•		•	•	
Power law									
Phillips & Sutherland (1979)						•			
Toffaleti (1968)	•			•					
Madden	•								
Meyer-Peter (modif.)				•					
Einstein (1959, modif.)				•					
Yang (1973)					•			•	
Yang (1984)				•	•				
Vaw (Smart & Jäggi, 1983)							•		
Einstein (suspension)									•
Meyer-Peter & Müller (1948)							•	•	•
Graf (1970)								•	
Parker (1990)								•	

1) Roughness value varies with discharge.

2) Just when using formulae from Einstein and Toffaleti.

3) Not defined.

4) Lateral erosion through limitation of river bed erosion.

APPLICATION OF A MODEL AND SENSITIVITY ANALYSIS

On the basis of the calibration procedure of the model HEC2SR for the observed bed degradation processes at the River Drau in Austria (Sereinig, 1995) a schematic geometry data set was used for a sensitivity analysis (Fig. 1, Spanzel, 1997). The aims of the sensitivity analysis were:

- description of most important input variables for the simulation,
- analysis of changed parameters in relation to the results,
- definition of a priority list concerning calibration parameters,
- comparison of parameter deviations with natural variations.

The sensitivity analysis was performed in a one-parametric way, meaning that the effect on the river bed development is shown by varying one parameter and fixing the others. A sensitivity factor is used for the graphical and numerical representation of the results. Kabala & Milly (1990) calculate the sensitivity for discrete input and output as follows:

$$S_i = \frac{\mathrm{d}O_i / O_0}{\mathrm{d}\alpha_j / \alpha_0} = \frac{\mathrm{d}O_i}{\mathrm{d}\alpha_j}\frac{\alpha_0}{O_0} \tag{4}$$

with S_i as relative sensitivity, α_0 as basis value of the input parameter and O_0 as basis value of the output parameter. The indices i and j are relevant for the output and input values.

The length of the simulated reach was 4800 m, the river bed width was 28 m and the slope was 1.5‰ (Fig. 1). The discharges varied between 100 and 400 $m^3\ s^{-1}$. Within this paper the major calibration parameters were the grain-size distribution of the subsurface, surface, input material, thickness of the surface layer, the porosity, the amount of sediment input into the modelling reach and roughness values. Although roughness is no real calibration parameter (Söhngen & Kellermann, 1997) in the calibration itself roughness values are changed in order to get realistic bed level changes.

Within the sensitivity analysis the grain-size distribution of the subsurface was decreased and increased by 5, 10 and 20% and additionally the grain sizes <1 or

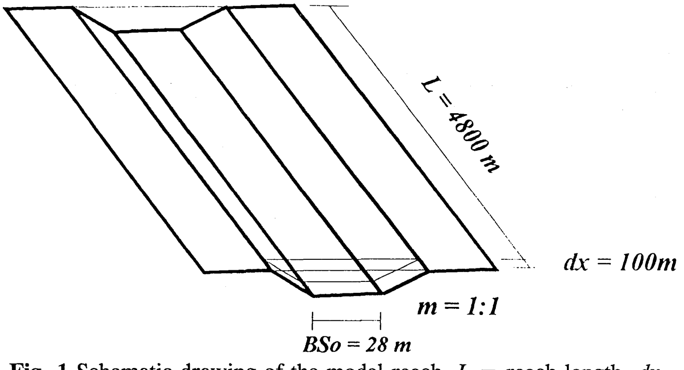
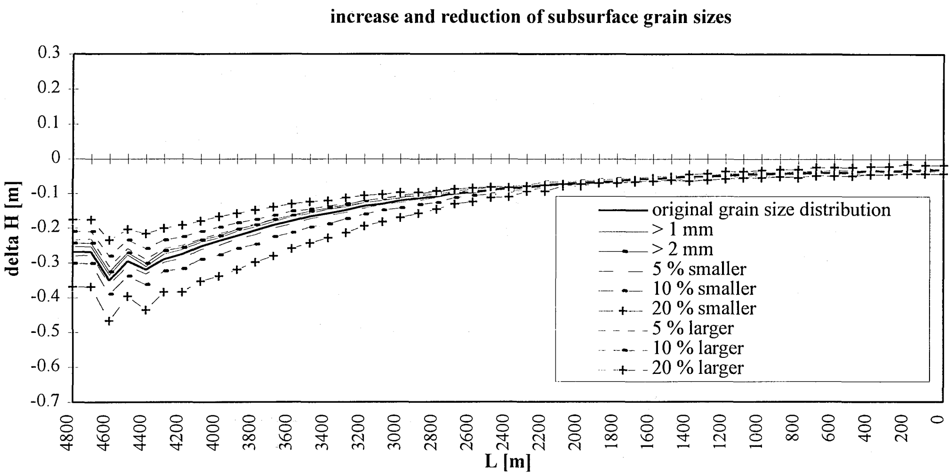
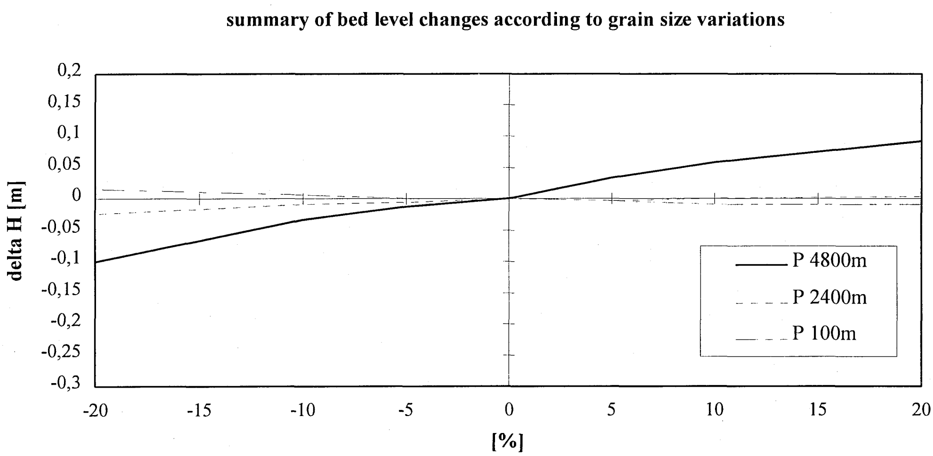


Fig. 1 Schematic drawing of the model reach, L = reach length, dx = cross-section distance, BSo = river bed width.





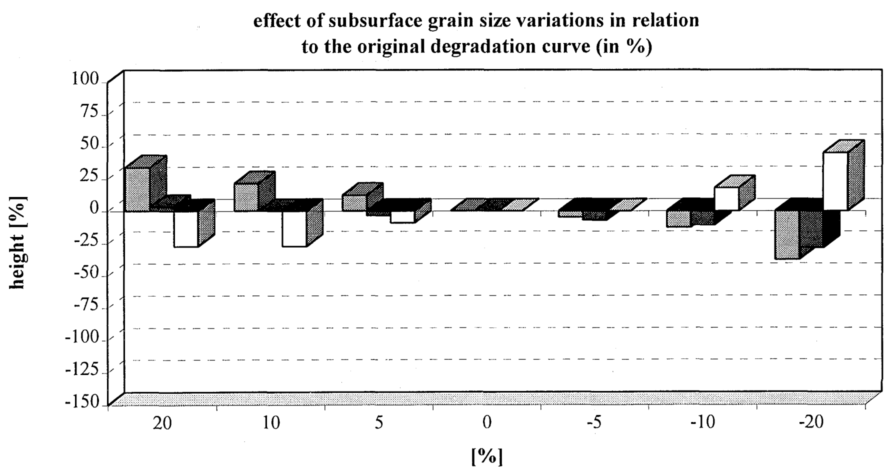


Fig. 2 Sensitivity in relation to changes of the grain-size distribution of the subsurface material.

<2 mm were neglected. Figure 2 shows the consequences for the river bed development. An increase of the grain sizes of the subsurface leads to a significant reduction of the degradation up to 25%. After about 2.5 km the degradation is again larger, which is caused by the lower input from the upstream part. The sensitivity factor S_i varied between 0.91 and 2.5.

Finer surface grain sizes cause significant degradation rates with a nearly linear relation between river bed and grain-size changes. In all cases the whole simulation reach is affected. A decrease of the grain sizes of the input material causes larger degradation rates (up to 25%) with S_i values between 2.05 and 2.73. The variation of both the subsurface and the surface material results in S_i values between 2.5 and 6.14, meaning that these higher values indicate a significantly increased sensitivity. The sensitivity of the grain-size distribution of the input material varies between 0.91 and 2.5.

The variation of the thickness of the surface layer larger than 15% leads to differences in the bed elevations. At the end of the model reach aggradation could be observed. S_i values between −1.14 and −0.45 occurred.

With the variations of porosity values up to 50% no significant increase of the degradation exists. S_i values from −0.52 up to 0 were calculated.

Differences in the amount of input material lead to bed level changes in the whole model reach, where the S_i values were varying between −1.24 and −0.98.

The increase of the roughness (Strickler) value from 30.5 (original value during the calibration) by 5% caused already a 100% larger degradation rate (Fig. 3). On the other hand a reduction leads to reduced degradation and at about 20% lower values of aggradation occur. In all cases the whole model reach was affected by the changes. Because of the nonlinear consequences of roughness changes in the numerical simulation the resulting bed level changes are nonlinear in relation to the roughness variation. The S_i values varied between −19.43 and −8.06, which were the highest sensitivities calculated.

Priority of calibration parameters

The use of a schematic geometry allows to derive a priority list of the individual calibration parameters concerning their importance for the degradation process. Figure 4 shows that changes of the roughness value cause the greatest influence on the simulation. A 20% change of the roughness leads to 400% differences in the river bed development.

Variations of the grain-size distributions of both the surface and subsurface of around 20% lead to changes up to 100%. Single variations of either surface or subsurface grain sizes of 20% result in about 50% bed level differences. 20% changes of the input material grain-size distribution lead to about 40% changes. In contrast to these parameters the thickness of the surface layer has a minor influence. The porosity variations show no significant effects in the model HEC2SR for the used schematic geometry and hydrology.

Therefore the following priority of the calibration parameters for the used model and boundary conditions can be derived:

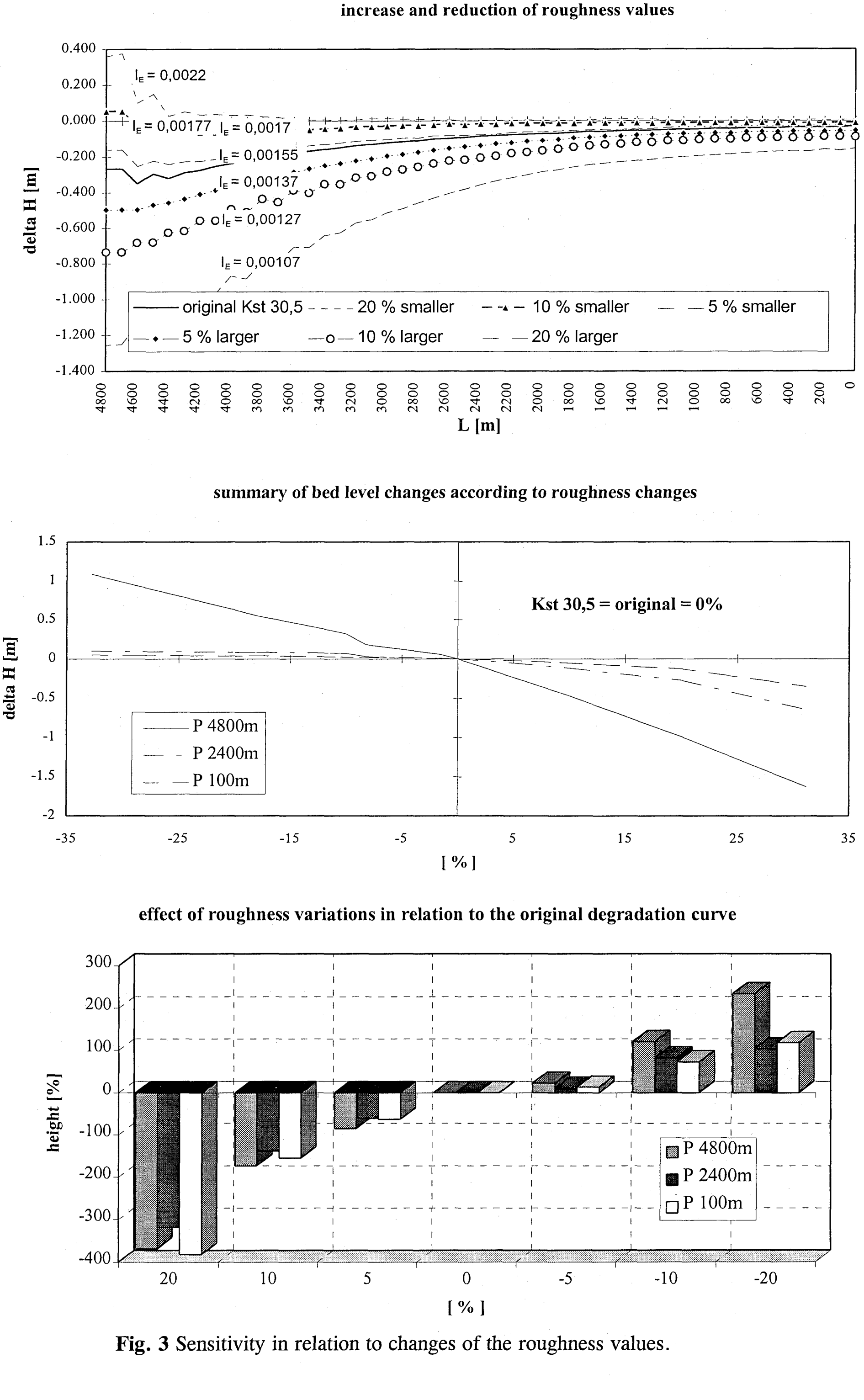


Fig. 3 Sensitivity in relation to changes of the roughness values.

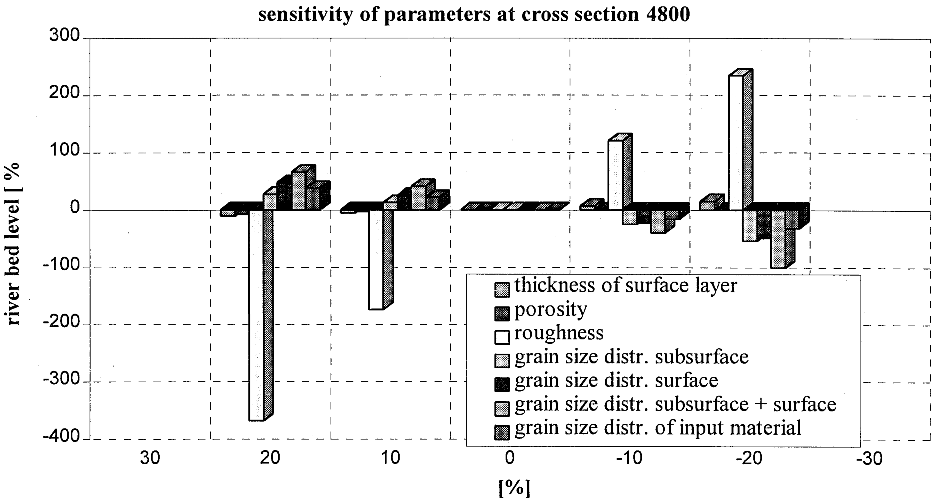


Fig. 4 Results of parameter variations.

– roughness	}extremely sensitive
– grain-size distribution of subsurface *and* surface layer	}extremely sensitive
– grain-size distribution of subsurface layer	}very sensitive
– grain-size distribution of surface layer	}very sensitive
– grain-size distribution of input material	}very sensitive
– amount of input material	}very sensitive
– thickness of the surface layer	}sensitive
– porosity	}negligible

Comparison of the results with natural variabilities

The sensitivity analysis leads to the conclusion that changes of the original calibration values have a significant influence on the simulated river bed development. Major investigations of natural variations of grain-size distributions showed larger deviations than the differences used in the sensitivity analysis (Habersack, 1997). This means that numerical simulations need a detailed investigation of input parameters, especially grain-size distributions and roughness values (which are related to each other). Furthermore the input of material into the model reach is important. Consequently sediment transport measurements are essential.

CONCLUSIONS

Within the paper it is shown that there exists a variety of different numerical sediment transport models which differ especially concerning steady or unsteady, coupled or uncoupled modelling procedures, side erosion, automatic width adjustment and especially transport formulae.

Regarding practical aspects the application of the model HEC2SR for a sensitivity analysis is discussed. Basically the calibration parameters used during simulations have been investigated with respect to their influence on the river bed level development. Generally the roughness values and the combined variations of the grain-size distributions of subsurface and surface material showed an extreme sensitivity. Single changes of the grain-size distributions of the subsurface and surface material as well as that of the input material and its amount caused significant differences in the model results. The thickness of the surface layer is responsible for minor changes and the porosity is negligible.

From a practical point of view one further major conclusion of this paper is that the investigation of the input parameters in nature, which vary to an even larger extent than the values in the sensitivity analysis, is a crucial point for the modelling procedure and proper results of simulating river bed development.

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REFERENCES

Ackers, P. & White, W. W. (1973) Sediment transport: new approach and analysis. *J. Hydraul. Div. ASCE* **99**(HYII), Proc. Paper 10167, 2041–2060.

Chang, H. H. (1988) Introduction to Fluvial-12—mathematical model for erodible channels. In: *Twelve Selected Computer Stream Sedimentation Models Developed in the United States* (ed. by F. Shou-shan), 353–412. Interagency *ad hoc* Sedimentation Work Group

Chang, H. H. (1993) *FLUVIAL-12—Mathematical Model for Erodible Channels. Users Manual.* San Diego State University, San Diego, California.

Cunge, J. A., Holly, F. M. Jr & Verwey, A. (1980) *Practical Aspects of Computational River Hydraulics.* Pitman Publishing, London.

Einstein, H. A. (1959) The bed-load function for sediment transportation in open channel Flows. *US Dept Agric. Soil Conservation Service Tech. Bull. no. 1*, 1026, 1–17.

Engelund, F. & Hansen, E. (1967) *A Monograph on Sediment Transport in Alluvial Streams.* Teknisk Forlag, Copenhagen, Denmark.

Gee, D. M. (1988) Status of HEC-6: scour and deposition in rivers and reservoirs. In: *Twelve Selected Computer Stream Sedimentation Models Developed in the United States* (ed. by F. Shou-shan), 39–64. Interagency *ad hoc* Sedimentation Work Group.

Graf, W. H. (1970) *Hydraulics of Sediment Transport.* McGraw-Hill, New York.

Habersack, H. (1997) Raum-zeitliche Variabilitäten im Geschiebehaushalt und dessen Beeinflussung am Beispiel der Drau (Spatio-temporal variabilities in the bed-load regime and its management at the River Drau). *Wr. Mitteilung No. 144.*

Habersack, H. & Nachtnebel, H. P. (1997) Changes in sediment transport and river engineering concepts, case study of the River Drau in Austria. *UNESCO IHP-V: Technical Documents in Hydrology no. 10*, 277–286.

Holly, F. M. Jr (1988) Charima and sedicoup codes for riverine mobile-bed simulation. In: *Twelve Selected Computer Stream Sedimentation Models Developed in the United States* (ed. by F. Shou-shan), 263–352. Interagency *ad hoc* Sedimentation Work Group.

Hunziger, R. P. (1995) Fraktionsweiser Geschiebetransport (Fractionwise bed-load transport). *Mitt. VAW Zürich 138.*

Kabala, Z. J. & Milly, P. C. D. (1990) Sensitivity analysis of flow in unsaturated heterogeneous porous media: Theory. Numerical model and its verification. *Wat. Resour. Res.* **26**(5), 593–610.

Karim, M. F. & Kennedy, J. F. (1982a) IALLUVIAL: a computer-based flow- and sediment-routing for alluvial streams and its application to the Missouri River. *Iowa Inst. Hydraul. Res. Report no. 250.* The University of Iowa, Iowa City, Iowa, August.

Karim, M. F. & Kennedy, J. F. (1982b) IALLUVIAL: computer-based predictors for sediment discharge and friction factor of alluvial streams. *Iowa Inst. Hydraul. Res. Report no. 242.* The University of Iowa, Iowa City, Iowa, December.

Li, R.-M., Mussetter, R. & Grindeland, Th. R. (1988) Sediment-routing model HEC2SR. In: *Twelve Selected Computer*

Stream Sedimentation Models Developed in the United States (ed. by F. Shou-shan), 413–462. Interagency *ad hoc* Sedimentation Work Group.

Meyer-Peter, E. & Müller, R. (1948) Formulas for bed-load transport. In: *The Second Meeting of the International Association for Hydraulic Sructures*, appendix 2, 39–44.

Parker, G. (1990) Surface-based bed-load transport relation for gravel rivers. *J. Hydraul. Res.* **28**(4), 417–543.

Phillips, B. C. & Sutherland, A. J. (1979) Modelling alluvial channel bed transients. *J. Hydraul. Div. ASCE* **105**(HY3), March.

Sereinig, N. (1995) Anwendung und Adaptierung eines Geschiebetransportmodells an der Oberen Drau (Application and adaptation of a sediment transport model at the upper Drau). Master Thesis at the Institute for Water Management, Hydrology and Hydraulic Engineering, Universität für Bodenkultur, Wien.

Shou-shan, F. (ed.) (1988) *Twelve Selected Computer Stream Sedimentation Models Developed in the United States*. Interagency *ad hoc* Sedimentation Work Group, 353–412.

Smart, G. M. & Jäggi, M. N. R. (1983) Sedimenttransport in steilen Gerinnen (Sediment transport in steep channels). *Mitt. VAW Zürich 64.*

Söhngen, B. & Kellermann, J. (1997) 1D-Morphodynamische Modellierung großer Flußstrecken (1D-morphodynamic modelling of long river reaches). *Wasserbau-Mitteilungen der Technischen Hochschule Darmstadt 98.*

Spanzel, M. (1997) Einfluss sedimentologischer Parameter auf die numerische Simulation des Geschiebetransportes (Influence of sedimentological parameters on the numerical simulation of the bed-load transport). Master Thesis at the Institute for Water Management, Hydrology and Hydraulic Engineering, Universität für Bodenkultur, Wien.

Strand, R. I., Orvis, C. J. & Randle, T. J. (1988) Sediment transport and river simulation model. In: *Twelve Selected Computer Stream Sedimentation Models Developed in the United States* (ed. by F. Shou-shan), 95–148. Interagency *ad hoc* Sedimentation Work Group.

Thomas, W. A. (1982) Mathematical modelling of sediment movement. Chapter 18 in: *Gravel-Bed Rivers* (ed. by R. D. Hey, J. C. Bathurst & C. Thorne). John Wiley, Chichester, UK.

Toffaleti, F. B. (1968) *A Procedure for Computation of the Total River Sand Discharge and Detailed Distribution. Bed to Surface*, 1–29. Committee on Channel Stabilization, US Army Corps of Engineers.

Yang, C. T., Molinas, A. & Song, C. C. S. (1988) GSTARS—generalized stream tube model for alluvial river simulation. In: *Twelve Selected Computer Stream Sedimentation Models Developed in the United States* (ed. by F. Shou-shan), 149–178. Interagency *ad hoc* Sedimentation Work Group.

Yang, C. T. (1973) Incipient motion and sediment transport. *J. Hydraul. Div. ASCE* **99**(HY10), 1679–1704.

Yang, C. T. (1984) Unit stream power equation for gravel. *J. Hydraul. Div. ASCE* **110**(HY12), 1783–1797.