*Modelling Soil Erosion, Sediment Transport and Closely Related Hydrological Processes* (Proceedings of a symposium held at Vienna, July 1998). IAHS Publ. no. 249, 1998. 

# Sediment transport analysed by energy derived concepts

**WOLFGANG SUMMER**

*Austrian IAHS/ICCE Working Group at the Institute for Land and Water Management Research, Federal Agency for Water Management, A-3252 Petzenkirchen, Austria*

**WEI ZHANG**

*Department of Navigation and Ocean Engineering, Hohai University, Xikang Road 1, 210024 Nanjing, China*

**Abstract** The sediment transport in natural rivers may be estimated on the basis of the stream power concept. The rate of energy dissipation derives from an energy balance which adjusts to the river bed's roughness, sediment discharge, channel geometry and cross section. Under these assumptions the sediment transport is analysed under various flow conditions as well as changing turbulent kinetic energy. It is shown and discussed under what assumptions the concept of energy dissipation rate for sediment transport can be extended from steady open-channel flow to unsteady flow conditions.

## INTRODUCTION

Often the profitability of the investments into a reservoir for single or multiple water management purposes depends on the correct prediction of the life time of the reservoir. Hence, simulating sediment transport is an essential task in hydrological engineering. The sediment originating in the drainage basin will be transported in the river courses as bed and suspended load including wash load. Solids moving in suspension represent the major component of the above-mentioned silting processes. Therefore any accurate estimation of the relevant particle transport is of great importance in applied engineering practices.

Developed transport formulae present a relationship between sediment discharge and hydrological as well as hydraulic factors, such as flow discharge, slope, river morphometry, sediment characteristics, etc. As these sediment formulae were often gained from laboratory investigations under steady flow conditions, their basic applicability is usually limited to these flow conditions. But, in natural rivers most of the sediment displacement takes place during floods which represent unsteady flow. Hence, one should be aware of conceptual formula limitations and possible prediction errors.

This paper demonstrates how the sediment transport is derived from the physical principle of the power theories. Their applicability to the sediment prediction under unsteady open-channel flow is discussed.

## CARRYING CAPACITY FOR SEDIMENT LADEN FLOW

In open-channel flow the establishment of a relationship between the available potential energy, respectively the energy dissipation rate, and the sediment transport, is a very reasonable approach to this hydrodynamic process. Such a relationship, describing the sediment transport in open-channel flow, can be derived on the physical basis of a power concept. These concepts are listed in the following three paragraphs. They are described on their theoretical basis, including their derivation, as well as their linkage towards a new conceptual approach, which is briefly explained.

## Stream power

This concept was introduced and developed by Bagnold (1966, 1977). It is based on general physical principles arguing that the transport of grains depends on the efficiency of the available power. This concept was also firstly applied by Bagnold for the description of the bed-load transport, wherein he related the total power supply $P$

$$P = \tau \bar{u} = \rho g S_f h \bar{u} \tag{1}$$

($\rho$ = density of the fluid–sediment mixture, $g$ = gravity, $S_f$ = energy slope, $h$ = water depth, $\bar{u}$ = mean flow velocity, $\tau$ = shearing stress of the fluid on the river bed) to the unit bed area. In other words, it states that the rate of energy dissipation, used in grain transport, should be related to the rate of materials being transported.

## Unit stream power

If equation (1) is divided by ($gh$), the unit stream power (Yang, 1971, 1972) can be derived from the stream power. This slightly different theory relates the rate of potential energy dissipation to the unit weight of water. It can be defined as

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}Y}{\mathrm{d}x} = \bar{u}S \tag{2}$$

where $Y$ denotes the potential energy per unit weight of water, $x$ the longitudinal distance, $t$ the time and $S$ the energy or water surface slope. Based on this concept, Yang (1979) developed several sediment transport formulae from the analysis of relevant laboratory data as well as field observations in natural streams.

## Gravitational power

In addition to the previous two theories, the gravitational power theory (Velikanov, 1954) suggests that the energy dissipation rate for the transport of the fluid–sediment mixture can be divided into:

- the power required to overcome the transport resistance and
- the power necessary to keep sediment particles in suspension.

Based on this, Velikanov understands that energy allowances for the sediments transport have to be made in an energy balance. In terms of the solid friction of an entire volume of a disperse medium, the sediment concentration is given by

Velikanov as:

$$C_v = K\frac{\bar{u}^3}{\rho h\omega} \tag{3}$$

where $C_v$ denotes the volumetric concentration, $K$ represents a coefficient related to the energy dissipation rate assigned to the sediment transport and $\omega$ is the fall velocity of the sediment particle. These last two parameters result from laboratory investigations with limited natural conditions.

### Power concept for the sediment concentration and transport capacity

Under the assumptions of energy separation and potential sediment transport, a suitable equation can be gained on the basis of the available power, explaining the linkage between a power concept and the sediment transport rate. The relevant theory for the derivation of this concept can be expressed as follows, where the left-hand side of equation (4) represents the energy dissipation for suspended sediment transport and the right-hand side, the stream power:

$$(\rho_s - \rho_w)gC_v\omega = \eta\rho_w g\bar{u}S \tag{4}$$

$\rho_s$ and $\rho_w$ denote the gravity of the grains respectively of the water and $\eta$ denotes a coefficient that describes the actual portion of energy dissipation used for sediment transport. It is similar to Velikanov's $K$-value. Rearranging equation (4) leads finally to equation (5) which is the basis for the discussion on the sediment transport analysis under steady as well as unsteady flow conditions in open channels:

$$C_v = \eta\frac{\rho_w\bar{u}S}{\omega\left(1-\frac{\rho_w}{\rho_s}\right)} \tag{5}$$

Equation (5) shows that the sediment concentration depends directly on the available stream power ($\bar{u}S$) as well as on a factor determining the energy dissipating proportion for different flow and transport characteristics such as turbulence, river bed roughness, sediment properties, etc.

## TRANSPORT ANALYSIS UNDER STEADY FLOW CONDITIONS

The potential energy as the only source for work performance drives the flow of the water–sediment mixture from a location of higher altitude to a location situated lower down, overcoming all transport resistance, respectively, fluid shear resistance. It is a turbulent flow situation where the intensity of the turbulence depends strongly on hydraulic and geometrical parameters such as the river bed friction, as well as the overall river morphology. The internal forces of the turbulence themselves also use up a certain amount of the available potential energy so that the effective power available to transport is reduced.

Assume that the considered water volume consists of a mixture of water and suspended sediment particles as well as bed-load grains. The internal portion of the energy dissipation rate causes the continuous chaotic displacement of these particles. Nevertheless, repeating turbulence pattern related to the flow characteristics can be observed by the visualization of moving particles. The referring eddy structures develop on the river bed. In this area, close to the viscous sub-layer, turbulent motion is manifested in the form of sweep and burst events. They consequently spread periodically all over the flow field as large-scale turbulent eddies. The turbulence structures keep sediment particles in suspension as well as force bed-load grains into a part-time movement such as:

- an occasional rolling motion which includes the rarely observed sliding motion, or
- a saltation motion, which comprises a series of ballistics hops or jumps characterized by ascent from the river bed to a height of a few grain diameters.

The turbulent transport of the fluid–sediment mixture is therefore characterized by an intensive collision between grains as well as the disintegration of eddies. During all these processes, a certain amount of the dissipated kinetic energy is also transformed into another type of energy, such as heat and sound. Therefore the physical background of the power concept allows the consideration of, and consequently, the integration of different energy consuming processes into a simple formula, i.e. by the introduction of the already listed coefficients $K$ and/or $\eta$ which vary due to overall changing flow characteristics.

Nevertheless, the gravitational power concept, as the theoretical basis for the derivation of sediment transport formulae, probably was only thought suitable for steady flow situations. Under steady uniform flow ($v_{av} = \frac{1}{n} R_h^{2/3} S^{1/2}$) equation (5) leads to:

$$C_v = \eta \frac{\rho_w}{\omega\left(1 - \frac{\rho_w}{\rho_s}\right)} \frac{\bar{u}^3 n^2}{R_h^{4/3}} \tag{6}$$

and under the assumption of a wide river ($R_h = h$) equation (6) becomes

$$C_v = \eta \frac{\rho_w \bar{u}^3 n^2}{\omega \rho h^{4/3}} \tag{7}$$

Introducing the coefficient $\xi$ instead of $\eta$ equation (7) leads to

$$C_v = \xi \frac{\bar{u}^3}{\omega \rho h} \tag{8}$$

This last equation (7) is now identical to Velikanov's equation, where $\xi$ is an empirical coefficient, identical to $K$.

Depending on the internal available transport energy as well as on sedimentological factors (grain size, $\rho_s$) the sediment transport takes place in suspension and as bed material transport. The physical process behind equation (8) postulates that a certain amount of the available energy dissipates due to the transport of grains, others due to friction forces. However, the total transport capacity concentration is related to the energy dissipation as well as the intensity of the turbulence in the following manner:

- More available energy increases the rate of energy dissipation.
- An increased energy dissipation is directly related to a higher turbulence intensity as well as an increase in the sediment transport capacity.
- A certain amount of available energy can be dissipated in a variable portion—an increase in the turbulence intensity (i.e. increase in the bed roughness) means a decrease in the sediment concentration and *vice versa*; equation (8) follows this principle: an increased friction factor causes a decrease in the flow velocity and an increase in the water depth. They lead to a reduction in the sediment concentration. Whereas Celik & Rodi (1991) explain this phenomenon in such a way that the turbulence energy produced in the separated shear layers behind the roughness elements, needs to be convicted and diffused first to the region above the bed, and that during this process, energy is dissipated so that more energy is dissipated directly and a smaller part of the energy produced is available for suspension.

## TRANSPORT ANALYSIS UNDER UNSTEADY FLOW CONDITIONS

The following discussion refers to the assumption that the conceptional structure of equation (8) can also be applied to the estimation of the sediment transport capacity under unsteady flow conditions. Under natural conditions they cause most of the sediment displacement (Summer & Zhang, 1994). Therefore, unsteady processes are rather relevant in engineering practices than steady flow situations. Unsteady flow in an open channel is often modelled by the one-dimensional St Venant equations. The dynamic equation of these may be written as follows (Cunge *et al.*, 1980):

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\left(\frac{\partial h}{\partial x}-S_0\right)+gAS_f=0 \tag{9}$$

Rearranging equation (9) after substituting $Q/A$ by $\bar{u}$ leads to

$$S_f-S_0=-\frac{\partial h}{\partial x}-\frac{1}{g}\left[\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}\right] \tag{10}$$

where $S_0$ and $S_f$ denotes the bed slope and the energy slope respectively, $A$ the cross sectional area and $Q$ the discharge. According to the magnitude on the right-hand side of equation (10) the flood wave can be classified either as kinematic wave ($S_0$ = $S_f$) or diffusion/dynamic wave (Ponce, 1989).

Due to $S_0 = S_f$, the sediment transport for a kinematic wave situation can be expressed by equation (8). This has been proven by Suszka & Graf (1987) on the basis of laboratory investigations as well as by Nouh (1989) who monitored and analysed suspended sediment transport in ephemeral channels. Therefore, for rivers where the kinematic wave assumptions are valid, formulae obtained from steady flow can also be applied without additional errors due to hydraulic unsteadiness.

Looking at a diffusion/dynamic wave, where the right-hand side of (10) cannot be neglected, the energy slope at the rising stage of the flood is greater than the bed slope but smaller at the tail end. This has been indicated by Tu & Graf (1992), who compared the friction velocity under steady, uniform flow $u_{*S}$:

$$u_{*S} = \sqrt{ghS} \tag{11}$$

with the friction velocity $u_{*S\bar{u}}$

$$u_{*S\bar{u}} = \left\{ gh\left( S_0 - \frac{\partial h}{\partial x} - \frac{\bar{u}}{g}\frac{\partial \bar{u}}{\partial x} - \frac{1}{g}\frac{\partial \bar{u}}{\partial t} \right) \right\} \tag{12}$$

gained from the St Venant equation of motion. Figure 1 shows one of several relevant hydrograph investigations, indicating the true value of the friction velocity $u_{*S\bar{u}}$ during the passage of the hydrograph as well as the apparent friction velocity $u_{*S}$ under the assumption that $S_f$ equals $S_0$.

From these as well as from other investigations (e.g. Walling & Webb, 1988), it can be concluded that the maximum value for $S_f$ occurs at the rising stage of the discharge response during a storm event, so that the stream power should reach a maximum before the discharge peak. In detail, the concentration peak will be situated between the point of inflection on the rising side of the hydrograph, where the increase in the water table starts to decrease again, and between the peak discharge (Thomas & Lewis, 1993). In this case the application of the sediment transport equation (8) might lead to an increased error in the prediction—therefore an improvement in the formula concept is advisable.

## CONCLUDING REMARKS

An alluvial channel has the tendency to create a balance between flow controlling factors in order to adjust the sediment transport rate to the rate of potential energy expenditure. Such a concept is based on the power theory which can be expressed as stream power, unit stream power and/or gravitational power. These approaches to an understanding of the sediment transport process reject the concept that the rate of sediment transport depends mainly on one independent variable, such as discharge, average flow velocity, energy slope or shear stress. Based on these power concepts several suspended sediment transport equations were derived by different researchers (Ackers & White, 1973; Shen & Hung, 1972; Yang, 1972). The procedure of the derivation of such a sediment transport equation based on the gravitational power theory is presented. Its basic idea focuses on the energy balance of sediment transport, considering the fluid and sediment grains separately, where two forces occur, namely gravity (transporting the fluid along the slope) and resistance forces (tending to retard the flow movement). The relevant equation corresponds to a transport equation for steady open-channel flow given by Velikanov, gained in a simple way.

According to equation (8) it can be concluded that increased friction forces (i.e. increase in the river bed roughness causing higher turbulence) decrease the transport potential of particles. Therefore, the coefficients $\xi$ or $K$ denoting the energy dissipation ratio between sediment transport and friction resistances—i.e. turbulent eddy structures cause them; they have their origin on the river bed and gain their characteristics by the bed structure/roughness.

However, equation (8) can be extended to express the relationship between the sediment transport rate and unsteady open-channel flow. Considering flood discharges, their hydrographs can either rise slowly or rapidly. Consequently, this leads to two typical unsteady flow situations, which on the one hand can be classified as kinematic wave and on the other as diffusion and/or dynamic wave. With equation (8) it can easily be shown that during unsteady flow situations and under certain assumptions (sufficient suspended sediment supply, regular cross-section, etc.):

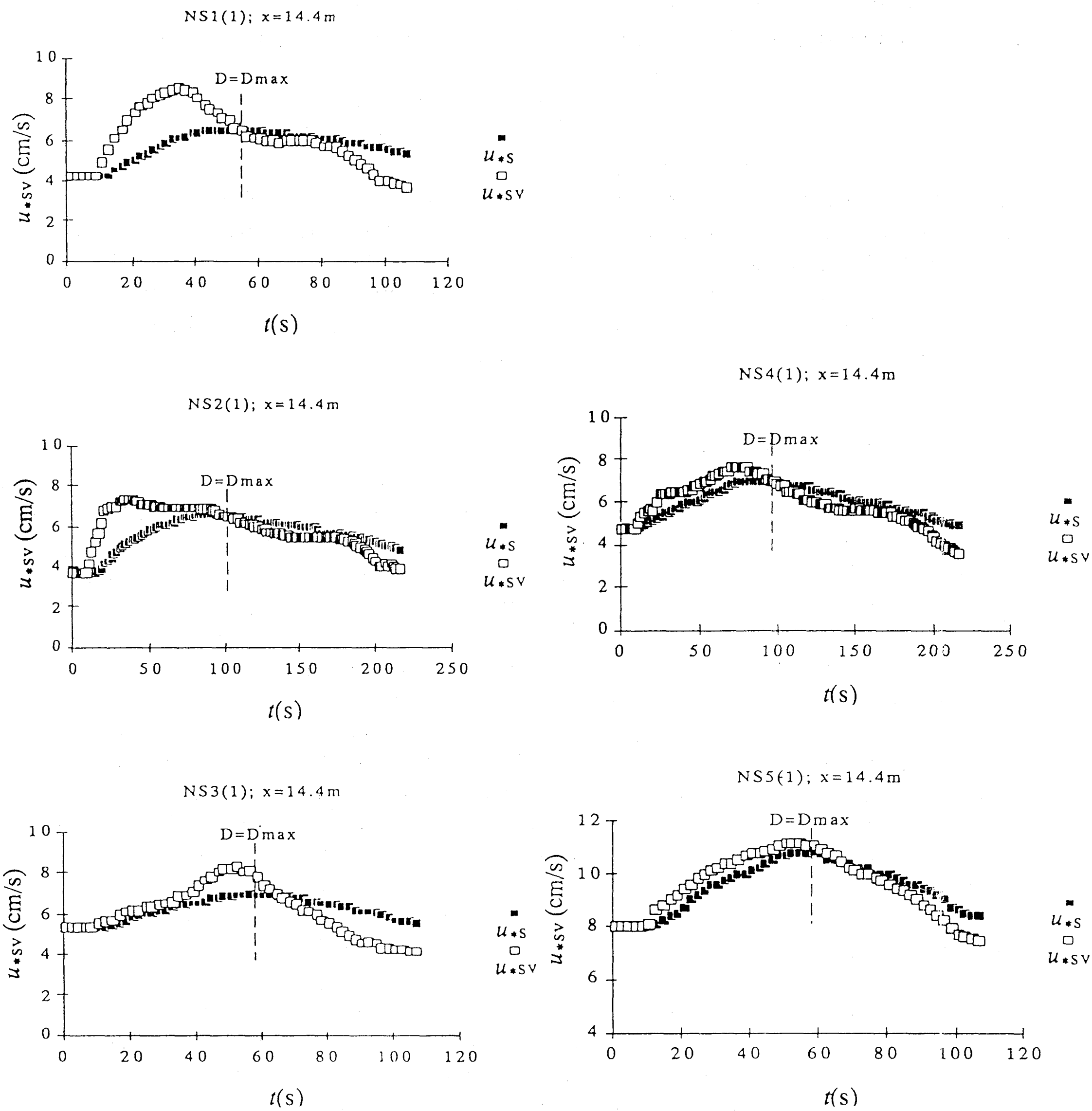


**Fig. 1** Friction velocity $u_{*S}$ and $u_{*S\bar{v}}$ for a hydrograph (Tu & Graf, 1992).

- the maximum suspended sediment concentration occurs at the rising stage of the hydrograph in the case of a dynamic wave,
- in the case of a kinematic wave (which in nature could be assumed for floods in big rivers or in steep channels) the concentration peak corresponds to the discharge peak.

When comparing equal discharges of a flood wave (diffusion/dynamic wave) it can be stated that more energy is available during the rise of a flood than during its decrease. This hydraulic principle leads to an energetic maximum at the rising branch of the storm flow. It also means that under the assumption of a direct relation between power and total sediment transport capacity the sediment concentration peak occurs on the rising flood stage. This physical/hydraulic phenomenon is one of the reasons for the hysteretic effect in a sediment discharge rating curve; hydrological factors have the second major impact on the creation of a looped rating curve.

## REFERENCES

Ackers, P. & White, W. R. (1973) Sediment transport: new approaches and analysis. *J. Hydraul. Div. ASCE* **99**, 2041–2060.

Bagnold, R. A. (1966) An approach to the sediment transport problem from general physics. *US Geol. Survey Prof. Pap. no. 422-I.*

Bagnold, R. A. (1977) Bedload transport by natural rivers. *Wat. Resour. Res.* **13**, 303–312.

Celik, I. & Rodi, W. (1991) Suspended sediment transport capacity for open channel flow. *J. Hydraul. Engng ASCE* **117**(2), 191–204.

Cunge, J. A., Holly, F. M., Jr & Verwey, A. (1980) *Practical Aspects of Computational River Hydraulics.* Pitman Advanced Publishing Program, London.

Nouh, M. (1989) Transport of suspended sediment in ephemeral channels. In: *Sediment* Budgets (ed. by. M. P. Bordas & D. E. Walling) (Proc. Porto Alegre Symp., December 1988), 97–106. IAHS Publ. no. 174.

Ponce, V. M. (1989) *Engineering Hydrology—Principles and Practices.* Prentice Hall, London.

Shen, H. W. & Hung, C. S. (1972) An engineering approach to total bed-material load by regression analysis. In: *Sedimentation Symposium to Honour Professor H. A. Einstein* (ed. by H. W. Shen), 14/1–14/7. Colorado State University, Fort Collins, Colorado.

Summer, W. & Zhang, W. (1994) An impact analysis of hydraulic and hydrological changes on the suspended sediment transport in the Austrian Danube. In: *Proc. XVII Donauländerkonferenz* (Budapest).

Suszka, L. & Graf, W. (1978) Sediment transport in steep channels at unsteady flow. In: *22nd IAHR Congress: Topics in Fluvial Hydraulics* (Lausanne, Switzerland), 166–170.

Thomas, R. B. & Lewis, J. (1993) A comparison of selection at list and time-stratified sampling for estimating suspended sediment loads. *Wat. Resour. Res.* **29**, 1247–1256.

Tu, H. & Graf, W. (1992) Friction in unsteady open-channel flow over gravel beds. *J. Hydraul. Res.* **31**, 99–110.

Velikanov, M. A. (1954) *Perenos Vzveshennykh Nanosov Turbulentnym Potokom.* Izd. AN SSSR, OTN no. 3.

Walling D. E. & Webb, B. W. (1988) The reliability of rating curve estimates of suspended sediment yield: some further comments. In: *Sediment* Budgets (ed. by. M. P. Bordas & D. E. Walling) (Proc. Porto Alegre Symp., December 1988), 337–350. IAHS Publ. no. 174.

Yang, C. T. (1971) Potential energy and sediment transport. *Wat. Resour. Res.* **7**, 312–322.

Yang, C. T. (1972) Unit stream power and sediment transport. *J. Hydraul. Div. ASCE* **98**(10), 1805–1826.

Yang, C. T. (1979) Unit stream power equations for total load. *J. Hydrol.* **40**(1/2), 123–138.