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# The effects of inaccurate input parameters on the modelling of deposition of suspended sediment

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**Abstract** Modelling sediment transport processes, in particular the deposition and erosion of suspended material in reservoirs, is a very complex task for two reasons: (a) the St. Venant and the sediment transport equations require a large set of parameters (flow and transport parameters, initial and boundary conditions, etc.) which are frequently estimated or calibrated with few observations of the response quantities of the model; and (b) most of the parameters can not be described as deterministic constants or functions due to their high variability and irregularity. Therefore, one has to describe them as realizations of stochastic variables (or fields) and the state variables, such as velocity, water level, bed-level, etc. as stochastic response quantities. Consequently the validation of almost all models is impossible in the sense that the response quantities will never be exactly determined, and fluctuations in the model output have to be accepted. In this paper, the authors are interested in the latter problem, namely the stochastic nature of the data and the results of the simulation. A simple one-dimensional non-steady flow and transport model depending on many stochastic parameters was studied. The objective function is the quantity of deposition (resp. erosion) which is now a stochastic response variable. Its mean and the relative sensitivities to the input parameters are under consideration. For this purpose, the well known first order reliability method (FORM) as well as Monte Carlo simulations are applied.

## INTRODUCTION

Engineering predictions of sediment transport in open-channel systems, and especially unsteady flow conditions, is fraught with uncertainties due to spatial and temporal variability, measurement errors, limited sampling of the parameters, boundary and initial conditions, and sink/source terms. During recent years many sophisticated mathematical models have become available to calculate unsteady flow in one, two or three dimensions. These models will be coupled with transport equations for bed load or suspended load to calculate, for example, long term behaviour of morphological changes of rivers, sedimentation of reservoirs, transportation of pollutants, and so on.

The problem is not only the accurate calibration of the model but also the acquisition of realistic values for the various input parameters. For most of the parameters one has to consider their stochastic nature, and therefore they have to be described as random variables or spatio-temporal random fields. The results obtained depend on the stochastic nature of the input variables. It is important to know how simultaneous inaccuracies of the various input parameters influence the result and to identify those parameters which determine the process most.

The well known first order reliability method (FORM) is a useful tool to assess the influence of stochastic input variables on the calculated result. In this paper we want to show how this method can be applied to judge quantitatively the influence of different parameters. We applied this method to solutions for deposition and erosion of suspended material under unsteady flow conditions. For comparison we also performed corresponding Monte Carlo simulations.

## HYDRODYNAMIC MODEL

The hydrodynamic model consists of a simple one-dimensional, nonlinear model for prediction of water level and vertically averaged horizontal velocity (Meselhe & Holly, 1997). The model is based on the depth averaged continuity equation:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{1}$$

and the depth averaged momentum equation:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}+g\,I_1\right)+gA(S_f - S_0)=0 \tag{2}$$

where $A$ = wetted cross-sectional area; $Q(x,t)$ = discharge; $g$ = gravitational acceleration and $I_1$ = the hydrostatic-pressure force term that can be expressed as:

$$I_1=\int_0^{h(x,t)}(h-\eta)\frac{\partial A}{\partial \eta}(x,t)\mathrm{d}\eta \tag{3}$$

where $h(x,t)$ = water depth. Finally, $S_0$ = d$z$/d$x$ is the bottom slope with $z$ = bed level, and $S_f$ is the friction slope which can be defined by the following expression:

$$S_f=\frac{n^2 Q|Q|}{A^2 R^{4/3}} \tag{4}$$

where $R$ = hydraulic radius (ratio of the cross-sectional area to the wetted perimeter) and $n$ = Manning's friction coefficient.

## SEDIMENT TRANSPORT MODEL

If one-dimensional conditions do exist, it has generally been assumed that the transport and deposition of suspended sediment could be represented by the convection-diffusion equation:

$$\frac{\partial}{\partial t}(CA)+\frac{\partial}{\partial x}(uAC)-\frac{\partial}{\partial x}\left(k_x A\frac{\partial C}{\partial x}\right)+S_d-S_e=0 \tag{5}$$

where $C$ = concentration of the transported solute; $u$ = $Q/A$, the cross sectional mean flow velocity; $k_x$ = longitudinal dispersion coefficient and $S_d$ and $S_e$ =

sink/source terms for deposition and entrainment of sediment, respectively. The deposition rate $S_d$ can be expressed as:

$$S_d = p v_s C \quad \text{kg m}^{-2}\ \text{s}^{-1} \tag{6}$$

where $p$ = probability of deposition or bed incorporation, because only a part of sediments that settle onto the sediment bed will be incorporated into the bed. The settling velocity $v_s$ of the grain class size is calculated using the formula of Cheng (1997):

$$v_s = \frac{\nu}{d_m}\left(\sqrt{25 + 1.2 D_*^2} - 5\right)^{1.5} \quad \text{m s}^{-1} \tag{7}$$

$$D_* = d_m \left( \frac{(\rho_F - \rho_W) g}{\rho_W n y^2} \right)^{1/3} \tag{8}$$

$$\nu = \frac{0.00178}{(1 + 0.0337T + 0.00022T^2)\rho_W} \quad \text{m}^2\ \text{s}^{-1} \tag{9}$$

where $\nu$ = kinematic viscosity of fluid; $T$ = fluid temperature; $D_*$ = dimensionless particle diameter; $\rho_W$ and $\rho_F$ = the density of water and sediment, respectively.

The probability $p$ of particles remaining deposited is given by Krone *et al.* (1977):

$$p = \begin{cases} 1 - \dfrac{\tau_b}{\tau_d}; & \tau_b \le \tau_d \\ 0; & \tau_b > \tau_d \end{cases} \tag{10}$$

where $\tau_d$ = critical shear stress for deposition and $\tau_b$ = bottom shear stress. The change of the bed level $dz_d$ by deposition of sediment at every time step $dt$ is calculated by:

$$dz_d = S_d \frac{(\rho_F - \rho_W)}{\rho_F(\rho_B - \rho_W)} dt \quad \text{m} \tag{11}$$

where $\rho_B$ = bulk density of the deposited material.

For the entrainment model $S_e$, a formula due to Ariathurai & Arulanandan (1978) is used:

$$S_e = \begin{cases} E\left(\dfrac{\tau_b}{\tau_e} - 1\right); & \tau_b \ge \tau_e \\ 0; & \tau_b < \tau_e \end{cases} \quad \text{kg m}^{-2}\ \text{s} \tag{12}$$

where $E$ = an erodability coefficient and $\tau_e$ = critical erosion shear stress. Erosion occurs when $\tau_b > \tau_e$ and erodable sediment exists at the bottom. The potential decrease of the bed level caused by erosion of sediment during a time interval $dt$ is calculated by the formula:

$$dz_e = S_e \frac{(\rho_F - \rho_W)}{\rho_F (\rho_B - \rho_W)} dt \quad \text{m} \tag{13}$$

If the available thickness of the deposited sediment layer is less than $dz_e$ then all of the sediment is entrained.

The bottom shear stress $\tau_b$ can be calculated by the formula

$$\tau_b = \frac{\rho_w g n^2 u^2}{h^{1/3}} \quad \text{N m}^{-2} \tag{14}$$

where $g$ = acceleration due to gravity and $n$ = Manning's friction coefficient.

## HYPOTHETICAL ENGINEERING PROBLEM

To illustrate a solution for uncertainty of deposition caused by stochastic input variables, a simple one-dimensional hypothetical engineering problem was defined, which was not intended to calculate a real situation. The purpose was only to show how uncertain input parameters could influence the calculated result.

For analysis we assumed a rectangular channel with a length of 10 km, an initial bed slope of 1/1000 and a channel width of $B$ = 100 m. A barrage with a height of $H_w$ = 8 m was defined as the downstream end of the reach. The channel's length of 10 km was divided into 100 computational reaches of $\Delta x$ = 100 m each. The initial condition of the backwater was calculated for a constant inflow of $Q$ = 298 m$^3$ s$^{-1}$ and a Manning friction coefficient of $n$ = 0.03 s m$^{-3}$. The discharge over the weir was calculated by the formula:

$$Q = \tfrac{2}{3}\sqrt{2g}\mu B(h + z - H_w)^{3/2} \quad \text{m}^3\ \text{s}^{-1} \tag{15}$$

with a discharge coefficient $\mu$ = 0.7. For $h + z - H_w < 0$ the discharge $Q$ was set to zero (Fig. 1).

The objective of our analysis was to calculate the time dependent amount of the deposited sediment for a certain inflow hydrograph (Fig. 2) and to check the propagation of the uncertainty of the calculated amount due to stochastic input parameters and to identify the relative sensitivity of the underlying input parameters.

For calculation of the sediment inflow into the reservoir a sediment rating curve is needed. The relation between flow discharge $Q$ and suspended load concentration $C$ at the inflow was estimated by the formula:

$$C = s\left(\frac{Q}{100}\right)^r \quad \text{kg m}^{-3} \tag{16}$$

where $s$ and $r$ are empirical constants.

## NUMERICAL SOLUTION OF THE PARTIAL DIFFERENTIAL EQUATIONS

The system of the coupled and nonlinear equations (1)-(14) with suitable initial

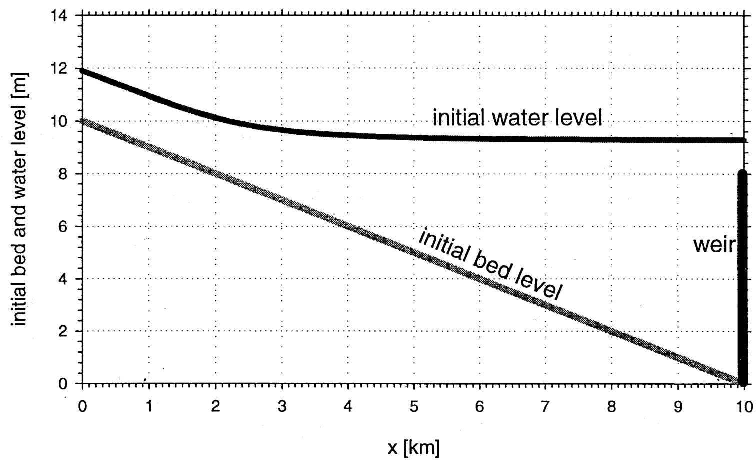


**Fig. 1** Sketch of the initial conditions.

conditions has to be solved numerically. The system is discretized in time, decoupled and then discretized in space. We suppose that the state variables and the parameters of the system are known at the time $t_n$; we compute these variables for the time $t_n + dt$ using the following steps:

The hyperbolic equations (1) and (2) combined with the corresponding boundary conditions (15) and (16) are first solved to compute the hydrodynamic quantities. The numerical solution makes use of an implicit finite difference scheme (MESH) developed by Meselhe & Holly (1997). The algorithm is based on a two step, predictor-corrector procedure. The predictor step consists of a forward sweep that carries the effect of upstream boundary conditions (16) followed by a backward (corrector) sweep that propagates the effect of downstream boundary conditions (15).

Using the solutions of $u$ and $h$, we solve the sediment transport equation (5) with an implicit non-centred finite difference scheme. The results of (5) are used to compute the quantity of deposition or erosion and the bed level $z$ given by equations (6)–(14).

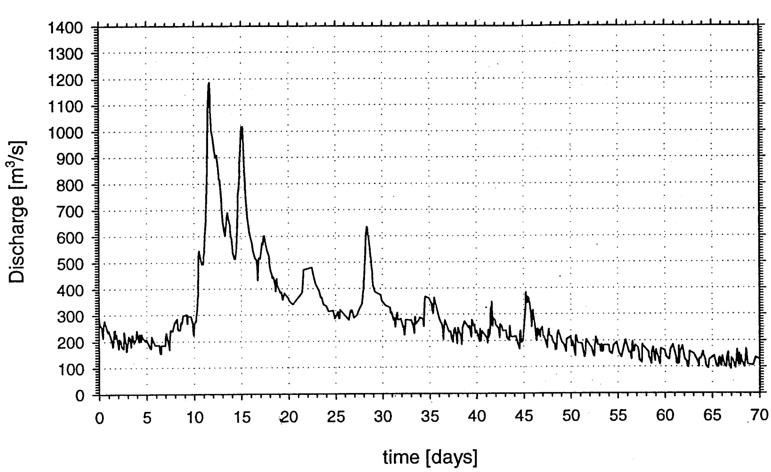


**Fig. 2** Inflow hydrograph.

In our scheme, we allow the transport equation (5) to have different (in general bigger) time steps than the one needed for the hydrodynamic equations since the variation in the bed level is in our case small compared to the variation in $u$ and $h$. The time step used for the solution of the hydrodynamic equations is of the order of 90 s while the one used for the transport equation is 360 s. In addition, to minimize artificial diffusion when solving the transport equation (5), we refine the space discretization used to solve the hydrodynamic equations (100 intervals) by dividing each interval in to 10 subintervals.

## FIRST ORDER RELIABILITY METHOD

The first order reliability method, which has been used in the field of structural reliability for some time can be applied, in principle, to stochastic problems incorporating any amount of probabilistic information, including first and second statistical moments, marginal distributions, partial joint distributions, and the full joint distribution. The method is suitable for use with either numerical or analytical solutions. A great advantage of this method is that information concerning the sensitivity of the result to variations of input parameters and their statistical moments is an integral part of the first order reliability solution. The method is described in detail in the literature (Kitikawa & Der Kiureghian, 1980; Sitar *et al.*, 1987). Therefore we will give only a brief description of the method here.

In a general reliability formulation a so called performance function is defined as $Z = g(\mathbf{X})$, where $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$ is a vector of random variables describing the uncertain parameters of the problem. In the $n$-dimensional space of $\mathbf{X}$, the hypersurface $g(\mathbf{X}) = 0$ denotes the boundary between "safe" and "unsafe" regions. Once a performance function is formulated, the probability of failure is given by:

$$p_f = P[g(\mathbf{X}) \le 0 = F_Z(0) = \int_{g(X)\le 0} f_x(x)\mathrm{d}x \tag{17}$$

in which $f_x(x)$ is the joint probability density function (pdf) of $\mathbf{X}$ and the $n$-fold integral is over the unsafe region. The exact solution of equation (17) is only possible in special cases.

With the first order reliability method however, an approximate solution of equation (17) can be achieved by transforming the density functions of the input variables $\mathbf{X}$ into independent standard normal variables. By an optimization procedure the so called β-point (Hasofer & Lind, 1974) is calculated. This is the shortest distance of the performance function $g(\mathbf{X}) = 0$ from the origin. The performance function is linearized in that point and the probability is estimated now by using the standard normal integral.

Sensitivities with respect to variations of the variables are given through the gradient vector of β. To standardize the variations the preceding gradient vector can be scaled by the diagonal matrix of standard deviations $\mathbf{D}$ (Der Kiureghian & Liu, 1985). The unit sensitivity vector is therefore defined as:

$$\gamma = \frac{(\Delta_x \beta)\mathbf{D}}{|(\Delta_x \beta)\mathbf{D}|} \tag{19}$$

This vector gives an indication of the importance of each variable $X_i$.

A simple algorithm to compute $\beta$ and $\gamma$ is given in Sitar *et al.* (1987) and presented here. The vector of random parameters $\mathbf{X}$ is transformed to a vector of standard normal variables as:

$$\mathbf{Y} = \Gamma\,\mathbf{D}^{-1}(\mathbf{X} - \mathbf{M}) \tag{20}$$

where $\mathbf{M}$ is the vector mean of $\mathbf{X}$ and $\mathbf{D}$ is the diagonal matrix of the standard deviations of each component of $\mathbf{X}$. The matrix $\Gamma = \mathbf{L}^{-1}$ and $\mathbf{L}$ is the lower-triangular matrix of the Cholesky decomposition of the correlation matrix of the random variables $\mathbf{X}$. If we define:

$$G(\mathbf{Y}) \equiv g(\mathbf{D}\,\mathbf{L}\,\mathbf{Y} + \mathbf{M}) = g(\mathbf{X}) \tag{21}$$

then the unsafe region is also defined by $G(\mathbf{Y}) \le 0$. The correlation matrix of the vector random variables $\mathbf{Y}$ is deduced from the correlation matrix of $\mathbf{X}$ (Der Kiureghian & Liu, 1985).

To find the shortest distance of the performance function $G(\mathbf{Y}) = 0$ from the origin in the standard space we make use of the following iteration:

- choose an initial Vector $\mathbf{Y}$, for example $\mathbf{Y} = 0$;
- compute the gradient vector:

$$\Delta_y G(\mathbf{Y}) = \left(\frac{\partial G(y)}{\partial y_1}, \dots, \frac{\partial G(y)}{\partial y_n}\right) \tag{22}$$

  and the unit vector

$$\alpha = -\left\{\frac{\Delta_y G(\mathbf{Y})}{|\Delta_y G(\mathbf{Y})|}\right\} \tag{23}$$

- compute the new point

$$y_{i+1} = \left[\alpha_i y_i + \frac{G(y_i)}{|\Delta_{yi} G(y_i)|}\right]\alpha_i^T \tag{24}$$

- if convergence is not achieved set $\mathbf{Y} = \mathbf{Y}_{\text{new}}$ and compute the new gradient vector $\nabla_y G(\mathbf{Y})$.

In general the algorithm converges in a few iterations. The distance of the solution to the origin is $\beta = |\mathbf{Y}|$ and the failure probability and the sensitivities are given by equations (18) and (19). This algorithm is implemented and tested on simple examples given in Der Kiureghian & Liu (1985) and in Sitar *et al.* (1987).

## UNCERTAINTY OF THE INPUT PARAMETERS

Reliability analysis is based on the knowledge of the underlying parameter uncertainty. The input parameters for calculation of sediment transport with equations (1)–(17) are known to contain considerable uncertainty. However careful evaluation of temporal or spatial variability and of the level of uncertainty for these

**Table 1** Assumed stochastic input parameters.

| Parameter | Probability density function | Expected value | Variation coefficient resp. lower and upper bounds |
|---|---|---|---|
| $s$ | normal | 0.2 | 20% |
| $r$ | normal | 1.5 | 10% |
| $d_m$ | log. normal | $0.03 \times 10^{-3}$ m | 30% |
| $\rho_B$ | normal | 1700 kg m$^{-3}$ | 10% |
| $\tau_d$ | uniform | 1 N m$^{-2}$ | 0.5–1.5 |
| $\tau_e$ | uniform | 2 N m$^{-2}$ | 1.5–2.5 |
| $E$ | uniform | $2 \times 10^{-4}$ kg m$^{-2}$ s$^{-1}$ | $1 \times 10^{-4}$–$3 \times 10^{-4}$ |
| $k_x$ | uniform | 40 m$^2$ s$^{-1}$ | 0–80 |
| $n$ | normal | 0.03 m$^3$ s$^{-1}$ | 20% |

variables has not been conducted. For a preliminary analysis we used the stochastic input parameters listed in Table 1.

The empirical parameters $s$ and $r$ describe the relationship between sediment concentration $C$ and water discharge $Q$ (equation 16). These parameters have to be measured. For our analysis we chose normal variates with mean values of 0.2 and 1.5 and a coefficient of variation (CV) of 20% and 10% respectively.

The settling velocity $v_s$ of the given size fraction was determined by the characteristic grain diameter $d_m$ which we assumed to have a logarithmic normal distribution. The bulk density $\rho_B$ was set to 1700 kg m$^3$ with a CV value of 10%.

Determination of realistic values for the deposition and erosion terms $S_d$ and $S_e$ is very difficult, because natural sediments are subject to large variability. The critical deposition shear stress $\tau_d$ and the critical erosion shear stress $\tau_e$ as well as the specific erodability coefficient $E$ have to be measured. However, variation of $\tau_d$, $\tau_e$ and $E$ for different flow regimes and different types of sediment are inadequately understood and data are sparse. Therefore we estimated these values assuming a rectangular distribution with upper and lower bounds listed in Table 1.

For the longitudinal dispersion coefficient $k_x$ very different values are reported in literature (Singh *et al.*, 1987). We assumed as a first approximation lower and upper bounds of 0 to 80.

Manning's hydraulic friction coefficient $n$ is the mean parameter for determination of the bottom shear stress $\tau_d$ (equation (14)). Gates & Al-Zahrani (1996) reported variation coefficients from 6% to 51%. Here we assumed a mean value of $\mu(n) = 0.03$ s m$^{-3}$ and a coefficient of variation CV($n$) = 20%. The boundary and initial conditions are treated as deterministic values. In a future refined analysis the uncertainty of the parameters will be based on real field data, involving stochastic boundary conditions and temporal and spatial correlations.

## UNCERTAINTY OF PREDICTED DEPOSITION

For the inflow hydrograph (Fig. 2) we calculated a sediment inflow of 3 098 201 t after 70 days. The outflow of sediment was 94%, and 6% was deposited. Figure 3 shows the time dependent deposition/erosion rates calculated with the mean values of

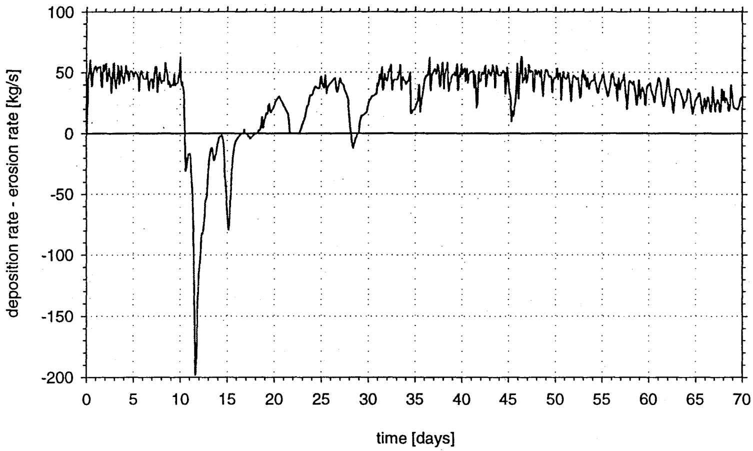


**Fig. 3** Deposition and erosion rates.

the input parameters.

Figures 2 and 3 show the strong relationship between the rate of deposition (erosion response) and the boundary condition discharge $Q$. Each time the discharge is high, the erosion rate is high, too and the deposition rate is small.

The integration in time of the combination of erosion and deposition is shown in Fig. 4 together with the mean value of the deposit and with the 16% and 84% quantiles and the 2.5% and 97.5% quantiles (these correspond to the mean plus and minus one and two standard deviations respectively, if the distribution were normal) calculated by a Monte Carlo simulation with 5882 realizations of the input parameters.

The calculated volume of the deposit shows a very high uncertainty. The 95% confidence interval for the deposit after 50 days, for example, has a lower value of 5251 $m^3$ and an upper value of 498 781 $m^3$. The bounds for the 68% confidence interval are 26 252 $m^3$ and 228 283 $m^3$, respectively.

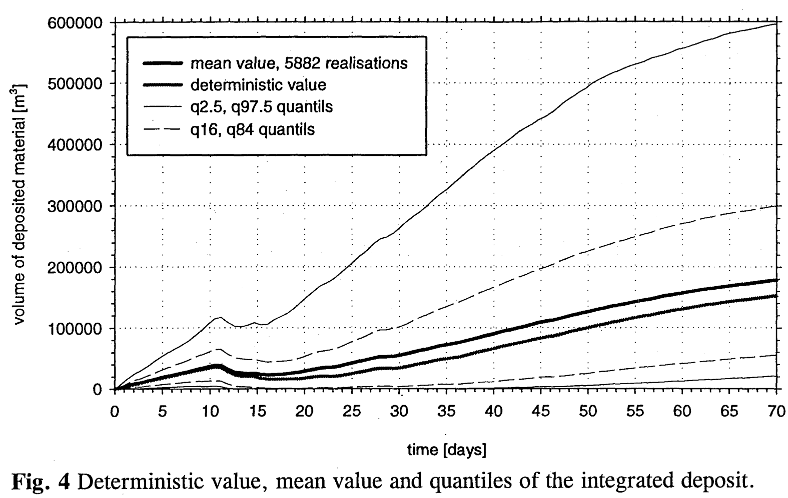


**Fig. 4** Deterministic value, mean value and quantiles of the integrated deposit.

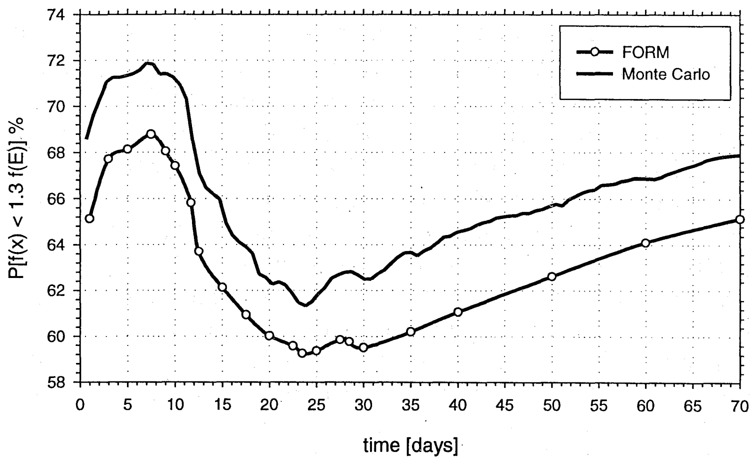


**Fig. 5** Monte Carlo and FORM calculations of the probability of exceeding a target value $D_0 = 1.3f(\mathbf{X})$.

Figure 5 shows the probability for the deposit of exceeding a target value $D_0$ which is 30% greater than the deposit calculated with the mean values of the input parameters.This probability is computed using two different methods, namely Monte Carlo simulation (5882 realizations) and FORM. For the first order reliability analysis we used a performance function for the deposit of $Z = g(\mathbf{X}) - D_0$ where $D_0$ is a deterministic target value and $\mathbf{X}$ is the vector of input variables as shown in Table 1. The difference is to our opinion due to the non-linearity which is not taken in to account by FORM while the Monte Carlo simulation is rather exact. This difference is small (2% to 3%) which gives credibility to FORM. With increasing deposition the probability increases and when erosion occurs the probability decreases due to the fact that we have to consider the two uncertain erosion parameters $\tau_e$ and $E$.

The normalized sensitivity of the stochastic parameters is given in Fig. 6. The legend of Fig. 6 represents the parameters approximately in their order of

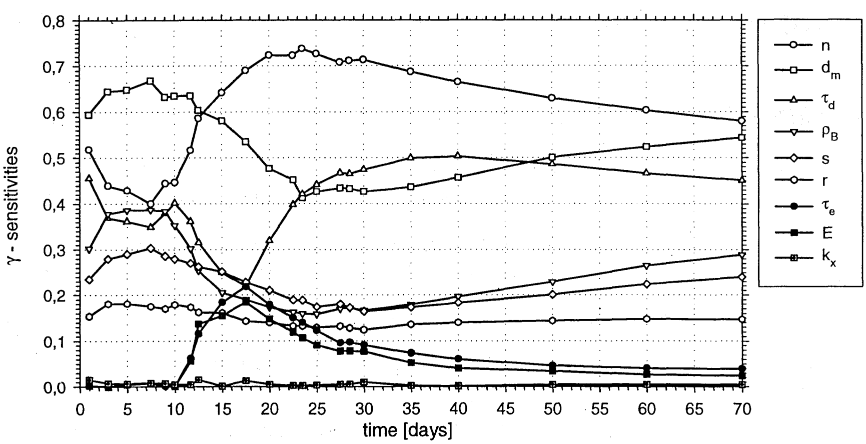


**Fig. 6** γ-sensitivities of the input parameters.

sensitivities. Variability in the three parameters $n$, $d_m$ and $\tau_d$ has a rather big influence on the deposition and its variation. The result is less sensitive to the parameters $\rho_B$, $s$, $r$ and in general the two parameters $E$ and $\tau_e$ have a very small influence. This is different in the period between day 10 and day 20. There the discharge $Q$ is high and therefore the erosion parameters $\tau_e$ and $E$ have a bigger influence. After the period of erosion, the influence of these parameters decreases. It is apparent from Fig. 6 that the diffusion coefficient $k_x$ has almost no influence on the total quantity of the deposition and its variation.

These results are only valid for the assumed uncertainties of the input parameters. More complex problems will arise when considering the spatial and temporal variability of the parameters and the stochastic boundary and initial conditions, and will result in different sensitivities. In future investigations we will use the first order reliability method for interpretation of more complex and realistic probabilistic problems based on real field data.

## SUMMARY AND CONCLUSIONS

A first order reliability approach to numerical solutions for one-dimensional deposition and erosion of suspended sediment using stochastic input parameters for unsteady flow conditions has been presented. We analysed a hypothetical one-dimensional problem. Nine stochastic input parameters were considered and the probability of entering a particular limiting state for the deposit for a special inflow hydrograph was estimated. We showed that the outcome of the first order analysis closely matches corresponding Monte Carlo simulations. With this method we were able to evaluate the relative importance of each input parameter at every time for our special problem.

The first order reliability method is a practicable alternative to Monte Carlo simulations, especially for long time period simulations where Monte Carlo calculations require very long computer runs. As the next phase of our research, we are planning to apply the first order reliability method together with Monte Carlo simulations for more complex problems based on real field data, involving stochastic boundary conditions and temporal and spatial correlations of the input parameters.

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