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Sediment transport functions and their evaluation using data from large alluvial rivers of Bangladesh

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Abstract An investigation was carried out to test the reliability of various commonly used sediment transport models against data from selected large alluvial rivers of Bangladesh. Another purpose of conducting the evaluation was to test the viability of a new sediment transport equation proposed by the first author. Hydrological and morphological data were collected from various organizations for the Brahmaputra, Ganges, Gorai, Surma and Teesta rivers covering a number of years. These data were utilized to estimate the sediment load in these rivers at selected stations using the well-known sediment discharge formulae proposed by Engelund–Hansen, Ackers–White, Van Rijn, Yang, Shen–Hung and Hossain. The performance of the selected sediment discharge formulae was tested on the basis of accuracy of predicting the observed sediment discharge within a certain discrepancy ratio. The evaluation revealed that Hossain's equation is not only comparable with equations widely used for sediment transport functions, but in many cases, superior in terms of achievement for prediction of sediment load against data for large alluvial rivers.

NOTATION

A, a	constant, coefficient or as defined
B, b	average width of the channel, constant or as defined
C, c	coefficient of sediment transport function
C_t	total average sediment concentration in
D, D_{50}	sediment diameter
D_{gr}, D_*	dimensionless grain diameter
g	acceleration due to gravity
g_s	sediment transport (weight)
H, d	average depth of flow
K	a coefficient
m, n	exponent
Q, Q_c	measured and assessed water discharge respectively
q_b, q_s	volumetric bed load and suspended load transport respectively
R, R_b	hydraulic radius
S, s	water surface slope and specific gravity respectively
u, v, V	mean flow velocity
u_*, v_*, V_*	shear velocity
u_{cr}, V_{cr}	critical velocity
X	sediment transport, mass flux per unit mass flow rate
α	coefficient in rough turbulent equation
γ, γ_s	specific weight of water and sediment respectively

ω, ω_r fall velocity of sediment and representative sediment ($D_{50} = 0.15$ mm) particles
τ_0 bed shear stress
ν kinematic viscosity

INTRODUCTION

How much sediment flows through the rivers or which is the most reliable sediment transport formula that can be recommended, are the key questions usually faced by the scientists and engineers who are engaged in various sediment control projects. The design and execution of flood control schemes are chiefly governed by the peak flood levels, which in turn depend upon the scour and deposition of sediments. Due to heavy siltation, natural rivers used for navigation are silted and thus reduce the clear depth required for navigation. Sediment deposited in rivers and harbours may sometimes require costly dredging. Since each situation is unique in its combination of physical phenomenon, and there are a number of existing sediment transport formulae, it is extremely difficult for an engineer to choose the appropriate one. This problem is even more complex for large alluvial rivers of Bangladesh which are transboundary in nature and carry widely varying discharge and sediment loads over the year. It is thus desirable to know which of the available sediment formulae give reasonable results against the data of large transboundary rivers flowing through Bangladesh. The selection of the present equations was based on the findings of some previous studies by White et al. (1975), Yang (1977) and Hossain & Barr (1988). In the present study sediment transport formulae of Engelund & Hansen (1967), Shen & Hung (1971), Ackers & White (1973), Van Rijn (1984) and Hossain (1992) were selected to test their efficiency. The efficiency is tested on the basis of the guidelines provided by the Task Committee (1971) of ASCE, Yang (1977) and Hossain & Barr (1988).

REVIEW OF SEDIMENT TRANSPORT EQUATIONS

The rate of sediment transport in rivers depends on many variables such as water discharge, average flow depth, flow velocity, energy slope, shear stress, stream power, particle size and gradation as well as temperature. It is very difficult to simultaneously incorporate all these variables and to develop one sediment transport function. A number of such transport functions has been put forward and details can be found in the literature. In the following, the six equations tested are briefly presented.

Hossain's formula

Based on the concept of dimensional analysis and similitude argument Hossain (1987, 1992) proposed that sediment concentration in a stream of steady water and sediment flow is a power function of: (a) the product of Froude number and slope of energy gradient, (b) the settling velocity ratio and (c) the discharge ratio. The functional form of the equation could be expressed as follows:

$$C_t = A\left[X^a Y^b Z^c\right]$$

$A = 6.496 * 10^5$ for $B/H < 500$; $A = 6.496 * 10^6$ for $B/H > 500$; $X = V\,S/\sqrt{gH}$ and $a = 0.745$; $Y = \omega_r/\omega$ and $b = 0.633$; $Z = Q/Q_c$ and $c = 0.50$ and $Q_c = [(2.15 + KB/H)H\,(gs)^{1/5}]^{2.5}$. Settling velocities of the sediment particles have been computed using Rubey's equation. K is proposed to be equal to 0.05 – Q * 10^{-6} for $Q > 5000$ m^3 s^{-1} and 0.03 – $Q * 10^{-7}$) for $Q < 5000$ m^3 s^{-1}.

Engelund–Hansen equation

Engelund–Hansen's(1967) equation is based on the shear stress approach. The equation can be written as:

$$g_s = 0.05\ \gamma_s V^2 \sqrt{D_{50}/\{\ g(\gamma_s/\gamma - 1)\ \}}\ \left[\tau_0/(\gamma_s - \gamma)D_{50}\right]^{3/2}$$

Yang's equation

Yang (1973) proposed a sediment transport formula based on the concept of unit stream power, which can be utilized for the prediction of total bed material concentration transported in sand bed flumes and rivers. The formula is as follows:

$$\log C_t = 5.435 - 0.286\log(\omega\ D_{50}/\upsilon) - 0.457\log(u_*/\omega) +$$
$$\{1.799 - 0.409\log(\omega\ D_{50}/\upsilon) - 0.314\log(u_*/\omega)\}\log(V\,S/\omega - V_{cr}\,S/\omega)$$

The value V_{cr}/ω is given by:

$$\frac{V_{cr}}{\omega} = \frac{2.5}{\log\dfrac{u_* D}{\nu} - 0.06} + 0.66, \quad 0 < \frac{u_* D}{\nu} < 70$$

$$\frac{V_{cr}}{\omega} = 2.05, \qquad 70 < \frac{u_* D}{\nu}$$

Van Rijn formula

Van Rijn (1984) developed an analytical relationship for sediment load transport in terms of the saltation height, particle velocity and bed load concentration. The transport equation can be expressed in a simplified form when only the mean velocity, flow depth and particle size are known was given as:

$$\frac{q_b}{ud} = 0.005\left(\frac{u - u_{cr}}{\left[(s-1)gD_{50}\right]^{0.5}}\right)^{2.4}\left(\frac{D_{50}}{d}\right)^{1.2}$$

$$\frac{q_s}{ud} = 0.012\left(\frac{u - u_{cr}}{\left[(s-1)gD_{50}\right]^{0.5}}\right)^{2.4}\left(\frac{D_{50}}{d}\right)(D_*)^{-0.6}$$

$$u_{cr} = 0.19(D_{50})^{0.1}\log\left(\frac{12R_b}{3D_{90}}\right) \qquad \text{for } 0.1 \le D \le 0.5 \text{ mm}$$

$$u_{cr} = 8.5(D_{50})^{0.6}\log\left(\frac{12R_b}{3D_{90}}\right) \qquad \text{for } 0.5 \le D \le 2.0 \text{ mm}$$

Ackers–White equation

Ackers & White(1973) applied the advantages of dimensional analysis technique, but used the physical arguments to express the mobility and transport rate of sediment in terms of some dimensionless parameters. The mobility number is denoted by F_{gr}, and a general definition is:

$$F_{gr} = V_*^{\,n} / \sqrt{gD(s-1)}\left[V/\sqrt{32}\,\log(\alpha\, d/D)\right]^{1-n}$$

Dimensionless grain diameter is applicable to coarse, transitional and fine sediments and is the cube root of the ratio of immersed weight to viscous forces, i.e.

$$D_{gr} = D\left[g(s-1)/\upsilon^2\right]^{1/3}$$

A general dimensionless sediment transport function can be expressed as:

$$G_{gr} = \mathrm{f}(F_{gr}, D_{gr})$$

$$G_{gr} = C\left(\frac{F_{gr}}{A} - 1\right)^m$$

in which $G_{gr} = \dfrac{Xd}{sD}\left(\dfrac{F_{gr}}{A} - 1\right)^n$ and A, C, m and n can be obtained as follows:

$$n = 1.00 - 0.56\log D_{gr}$$

$$A = 0.23/\sqrt{D_{gr}} + 0.14$$

$$m = 9.66/D_{gr} + 1.34 \text{ and}$$

$$\log C = 2.86\,\log D_{gr} - (\log D_{gr})^2 - 3.53$$

For coarse sediments i.e. $D_{gr} > 60$; $n = 1.00$, $A = 0.17$, $m = 1.50$, $C = 0.025$.

Shen–Hung equation

Shen & Hung (1971) assumed that sediment transport is so complex that no single Reynolds number, Froude number, or combination of these can be found to describe sediment transport under all conditions. Instead of trying to find a dominant variable which dominates the rate of sediment transport, they recommended a regression equation based on 587 sets of laboratory data in the sand size range. Their regression equation is:

$$\log C_t = -107\,404.459 + 324\,214.747{*}Y - 326\,309.589{*}Y^2 + 109\,503.872{*}Y^3$$

where $Y = \left[V\,S^{0.57}/\omega^{0.32}\right]^{0.0075}$

DATA

Data collected for the present study are discharge, velocity, width, cross-sectional area, mean daily discharge, observed suspended sediment load etc. for the Brahmaputra, the Ganges, the Surma, the Gorai and the Teesta rivers. The data were collected from the Bangladesh Water Development Board and Surface Water Modelling Centre. A summary of data used in the study is presented in Table 1. It may be noted that the Brahmaputra, the Ganges, the Surma and the Teesta are transboundary rivers and their basins spread over China, Bhutan, India, Nepal and Bangladesh. The basin of the Gorai wholly lies within Bangladesh.

Table 1 Summary of data used in the present study.

Parameters	Brahmaputra	Ganges	Gorai	Surma	Teesta
No. of data	63	112	89	46	229
Slope (cm km^{-1})	7.14	5.14	4.12	4.0	2.97
D_{50} (mm)	0.17	0.15	0.10	0.20	0.22
Q_{max} ($m^3 s^{-1}$)	68 700	57 200	5560	2100	2530
Q_{min} ($m^3 s^{-1}$)	3750	1140	53	120	51
Q_{avg} ($m^3 s^{-1}$)	19 600	11 700	1150	549	840

METHODOLOGY

The various hydraulic, geometric and sediment transport parameters needed in the computations were collected and processed on a personal computer. Data were arranged on the river basis. Sediment loads were computed for the individual data sets of each river using the selected formulae. These were then compared with those of the measured values. The discrepancy (ratio of calculated value to measured value) for each set of data was considered for comparison of performance. The percentage of data coverage between accepted lower and upper limits of the discrepancy ratio and their statistical properties was taken as the criteria of the goodness of fit. For the present evaluation, the final accuracy order was prepared on the basis of data coverage between the discrepancy ratio of one-half and two. In addition, the calculated values were plotted against the observed values for the same data cases, so that the scatter about the perfect agreement line can also be considered. This later basis of comparison is achieved by plotting of discrepancies against percentage data coverage. The shape of the curves generally resembles those of the normal distribution curve and gives an indication of the degree of bias. This was also considered in recommending the most reliable equations.

RESULTS AND DISCUSSION

Table 2 summarizes the data coverage between the selected discrepancy ratio range for the equations against the data of various rivers. Data coverage after adjustment has also been shown for the same range of discrepancy ratio. This has been done to achieve an alternative and unbiased comparison. The discrepancy ratios were adjusted by multiplying with appropriate correction factors so as to bring the mean discrepancy ratio to unity for each formula tested. It may be seen from Table 2 that on the basis of data falling within the discrepancy ratio range from 0.5 to 2.0, Hossain's equation was the best, the data coverage being 73% , 63.4%, 62.9%, 62.9% and 58.0% respectively for the Brahmaputra, Ganges, Surma, Gorai and Teesta rivers. This was closely followed respectively by the Yang, Engelund–Hansen, Ackers–White, Shen–Hung and Van Rijn equations. The equations were then adjusted and a new degree of achievement of prediction was assessed for the same discrepancy ratio range. The adjustment brought significant improvements to Engelund–Hansen, Ackers–White, Shen–Hung and Van Rijn equations. In the case of the Ganges River, the Engelund–Hansen equation was found to be the best followed by the Hossain and Yang equations. In the case of the Teesta River the adjustment also improved the predictive performance of the Engelund–Hansen, Shen–Hung and Van Rijn equation significantly and this may be observed in Table 2.

The technique of comparison adopted by the ASCE Task Committee ((1971) was based on plotting the predicted and measured sediment discharge as a function of water discharge. Typical comparisons based on this technique are shown in Figs 1 and 2 for the rivers Brahmaputra and Ganges. The figures show that the Hossain equation predicts the observed sediment load more closely and is followed by Engelund–Hansen and Yang's equations. Data coverage for various selected range of discrepancy ratios was also assessed and plotted for discrepancy ratio against percentage of data coverage. One such plotting is shown in Fig. 3 for the case of the Brahmaputra . This may be called a bias plot as it depicts the distribution of predictive performance about the mean lines of unit discrepancy. Careful examination of the plotting revealed that Hossain's equation has the best distribution, followed by the Yang and Engelund–Hansen equation. Hossain's equation thus shows its superiority over other equations when tested against large alluvial rivers of Bangladesh. This may be due to conditions for experimental series on which this particular equation is based provided a good representation of the conditions found in the field.

Table 2 Summary of performances of various sediment transport equations against observed data in relation to data coverage between discrepancy ratio of 0.5 and 2.0.

Equations	Brahmaputra:		Ganges:		Surma:		Gorai:		Teesta:	
	unad	ad	unad	ad	unad	ad	unad	ad	unad	ad
Hossain's	73.0	-	63.4	-	62.9	-	62.9	-	58.0	-
Yang's	68.3	-	58.9	-	57.3	-	57.3	-	40.2	-
Engelund–Hansen	11.1	69.9	33.0	74.1	39.3	51.7	39.3	51.7	6.10	76.9
Ackers–White	42.9	61.9	12.5	51.8	36.0	46.0	36.0	46.0	41.1	60.3
Shen–Hung	33.3	50.8	42.9	-	25.8	40.5	25.8	40.5	3.06	54.2
Van Rijn	23.8	41.3	35	-	38.2	-	38.2	-	-	-

unad = unadjusted and ad = adjusted.

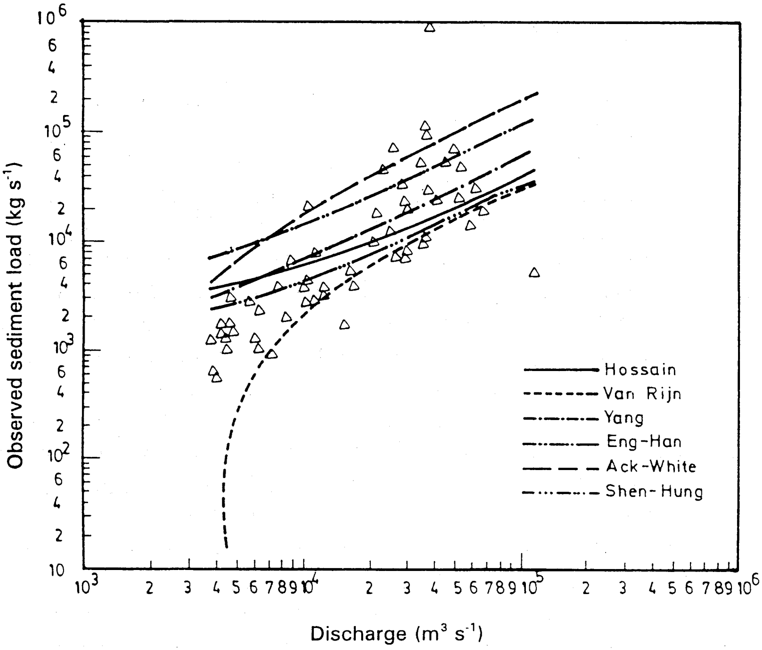

Fig. 1 Comparison of transport formulae: Brahmaputra River.

CONCLUSIONS

Based on the data coverage for testing the order of accuracy of transport equations, the Hossain's model is considered to be the best, followed by Yang's and Engelund–

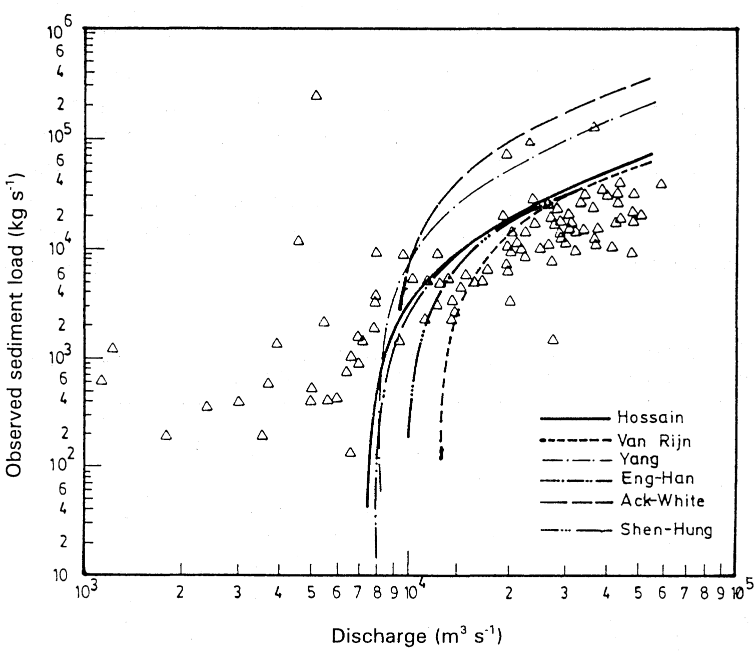

Fig. 2 Comparison of transport formulae (Ganges River).

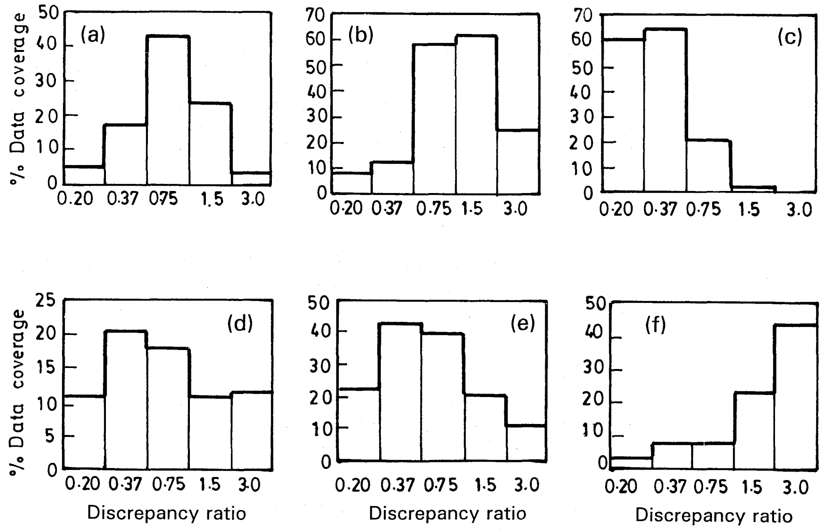

Fig. 3 Distribution of bias of different models on the River Brahmaputra: (a) Hossain, (b) Yang, (c) Engelund–Hansen, (d) Van Rijn, (e) Shen–Hung and (f) Ackers–White.

Hansen's models. The degree of bias was considered to be a further measure of the degree of accuracy. In this case also, Hossain's model showed the best performance followed by Yang's. With the exception of Hossain's equation, adjustment of discrepancy ratio brought significant improvement in terms of data coverage to other well-known sediment transport equations tested in the present study.

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