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The use of relative celerities of bed forms to compute sediment transport in the Paraná River

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Abstract In this work the problem of modelling movable bed flow is partially addressed by considering the relative magnitude of the celerities at which disturbances on the water surface and on the bed propagate. Part of the interest in computing bed forms celerities is to estimate the total sediment load transported by the Paraná River. To this end, the relative celerities implied by different transport formulae are determined by a linear stability analysis of the Exner approximation to the shallow-water equations. Then, nonlinear celerities are calculated numerically by perturbing the sediment load flowing into a uniform channel of similar slope and roughness as those encountered in the Paraná River. Finally, by matching numerical results with recorded field data of dune migration, a representative value of the total sediment load of the Paraná River is obtained.

INTRODUCTION

With the advent of the free trade agreement known as MERCOSUR, the Paraná River has been subject to increasing commercial navigation. In particular, the maintenance of the navigation channels of the commercial block has become a key issue in the past few years.

A quick way to determine the necessary volumes to be removed by dredging is to resort to a computer program such as HEC-6 (USACE, 1993) which is able to simulate changes in the river profile due to scour and deposition of sediment. The code is a one dimensional numerical model aimed at simulating the long term behaviour of predominantly sandy rivers. The sediment transport capacity of the stream is estimated with a formula depending on the hydraulic condition at each cross-section (USACE, 1993; Havis *et al.*, 1993). The lack of representative data made it difficult to select a proper solid discharge formula for the Paraná River. In particular, preliminary calculations made in waterways of the river showed that one of the most sensitive parameters was the inflowing sediment-discharge rating curve. Therefore, analysis of a one-dimensional model, based on the same underlying principles as those of HEC-6, conveying bed load perturbations over moderate periods of time is still a challenge.

Since the Paraná River is a very wide natural stream relative to its local depth, any numerical simulation based on a proper representation of the shallow-water

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equations seems to be appropriate to treat the problem. However, once coupled to a mobile bed this system of governing equations is significantly more difficult to solve than the equivalent fixed bed system. If the ratio between the celerities at which infinitesimal disturbances at the bed and water surface perturbations propagate is small, then it is natural to follow the quasi-steady state approximation of Exner for the water flow (Leliasky, 1966). A case in point is given for the Paraná River. While the river flows at a mean rate of 16 000 m^3 s^{-1}, it may require about 100 000 s to transport an equivalent amount of sand. In consequence, when the long term behaviour of sandy rivers is sought, it is sometimes impractical or even prohibitive to use a fully coupled model to achieve the necessary level of simulation time. This assumption makes the solution easier to compute and preserves the essential coupling between the flow and the mobile bed. Such approximation is the basis for the movable bed open channel flow model HEC-6 and the one used in the present work.

MATHEMATICAL MODEL

The flow is treated as one-dimensional in the classical hydraulic approximation where quantities are depth averaged. The bed shear stress is modelled with the square of the flow velocity with a constant friction factor. The bed consists of non-cohesive material; water and sediment properties are taken as constant. Then, under the assumption that some scales of motion can be anticipated *a priori*, the dimensionless equation for the fluid mass, the momentum and the solid mass conservation for shallow water in a broad, slightly inclined channel, are given by:

$$\psi h_t + (uh)_x = 0 \tag{1}$$

$$\psi u_t + u u_x = -F_0^{-2}(h_x + \varsigma_x) + c_f\left(1 - \frac{u^2}{h}\right) \tag{2}$$

$$\varsigma_t + \frac{1}{(1-p)}(q_s)_x = 0 \tag{3}$$

where the sub-indexes x or t denote partial derivative (i.e. $h_1 = \partial h/\partial t$). As shown in Fig. 1, x is the distance measured in the stream direction, t is the so-called slow time defined below, u is the local flow velocity, h is the local depth, ζ is the bed departure from a plane inclined at an angle S_0 from the horizontal, q_s is the total sediment load per unit of channel width, p is the bed material porosity, and c_f is the

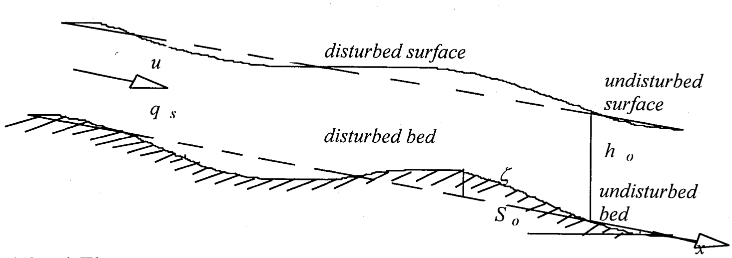

Fig. 1 Flow geometry

friction factor. The length, velocity and time scales chosen for the above dimensionless formulation are $(L, U, T) = (h_0, u_0, h_0^2/q_{s0})$ the undisturbed flow depth, the flow velocity and the morphological characteristic time, respectively. Here, $\psi = q_{s0}/u_0 h_0$ is the ratio of sediment load to liquid load transported by the uniform stream, and $F_0^2 = u_0^2/gh_0 = S_0/c_f$ is the Froude number based on the undisturbed flow quantities that set up when friction and gravity are in perfect balance. The above equations must be supplemented with proper initial and boundary conditions and a bed load relationship, which in general can be taken as a power function of either the local flow velocity or the local bed shear stress.

The quasi-steady state approximation of Exner considers that for $\psi << 1$, the transient term in the flow equations can be dropped (equivalent to taking the limit $\psi \to 0$):

$$(u h)_x = 0 \tag{4}$$

$$u u_x = -F_0^2 (h_x + \varsigma_x) + c_f (1 - \frac{u^2}{h}) \tag{5}$$

LINEAR STABILITY ANALYSIS

De Vries (1965) presented the major conclusions that can be drawn from a linear stability analysis of the system of equations (1)–(3) (see for example, Jansen *et al.*, 1979). Reynolds (1965), who clarified some aspects of an earlier work by Kennedy (1963), illustrated the failure of the shallow water equations to predict linear bed instability. Similar findings were reported years later by Gradowczyk (1968).

In summary, even though the linear stability analysis of the proposed system can be considered fairly complete, some useful information can still be obtained if the focus is restricted to the prediction of the celerities at which flow and bed perturbations propagate. Morris & Williams (1996) addressed this issue while studying hyper-concentrated flows. Moreover, the field has regained interest lately with the apparently dispersive behaviour of the full system of equations (Cui & Parker, 1997).

Since bed instability is not of primary interest in this work, the flow conditions considered are well above the Shields threshold value for incipient motion of bed material. Therefore, the critical condition either in terms of shear stress or mean flow velocity is ignored in all the transport formulae analysed hereafter.

In what follows, a starred variable means a dimensional quantity. Thus, assuming a sediment transport relationship of the type $q_{s_*} = \kappa u_*^{\alpha}$ (De Vries, 1965), which in dimensionless form becomes:

$$q_s = \frac{q_{s_*}}{q_{s_0}} = \frac{\kappa\, u_*^{\alpha}}{\kappa\, u_0^{\alpha}} = u^{\alpha} = f(u) \tag{6}$$

it is easy to establish that the system of equations (3), (4) and (5) can be combined into a well-known kinematic wave model (Reynolds, 1965):

$$\zeta_t + \frac{f_u\, u}{(1-p)\, h\, (1-F^2)}\zeta_x = \frac{f_u\, u\, c_f}{(1-p)\, h\, (1-F^2)}(F_0^2 - F^2) \tag{7}$$

where $F^2 = u^2 F_0^2 / h$ is the local Froude number and f_u is the Jacobian of the sediment transport equation $\partial f/\partial u)$.

Perturbing now the base flow with small disturbances $(u,h) = (1 + u', 1 + h')$, the above equation can be reduced, after neglecting terms involving products and powers of the primed variables, to:

$$\varsigma_t + c_0 \varsigma_x = c_0 c_f F_0^2 (h' - 2u') \tag{8}$$

where $c_0 = \alpha/((1 - \mathrm{p})(1 - F_0^2)$ is the celerity of the above linear hyperbolic model. This equation, supplemented with its linearized counterpart of the flow equations, provides a coupled system of linear equations that can be solved according to the normal mode technique. In this case, the form of the perturbation takes the simple form:

$$(u', h', \varsigma) = (\hat{u}, \hat{h}, \hat{\varsigma})\, e^{ik(x-ct)} \tag{9}$$

where $k = 2\pi/\lambda$ is a real valued quantity that defines the wave number of the perturbation of dimensionless wavelength λ, c is a complex valued quantity $(= c_r + ic_r)$ whose real part denotes wave speed and whose imaginary part denotes rates of growth or decay of the perturbation depending on the sign of kc_i and $(\hat{u}, \hat{h}, \hat{\varsigma})$ are the amplitude of the disturbances. Substitution of equation (9) into the linearized form of the governing equations yields

$$\frac{c_r}{c_0} = 1 - \frac{3\, c_f^2\, F_0^4}{9 c_f^2\, F_0^4 + k^2 (1 - F_0^2)^2}, \ldots \quad \frac{c_i}{c_0} = \frac{-k\, c_f\, F_0^2 (1 - F_0^2)}{9 c_f^2\, F_0^4 + k^2 (1 - F_0^2)^2} \tag{10}$$

It should be noted that the dependence of the celerity upon the frequency, $c_r = c_r(k)$,

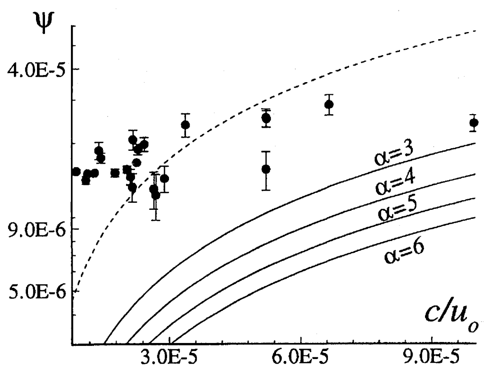

Fig. 2 Dimensionless bed forms celerities *vs* the sediment transport parameter ψ. Solid lines: analytical result given by equation (12). Dashed line: least square fit to the field data (circles).

Table 1 Exponent of sediment transport formula $q_s = u^{\alpha}$ according to several authors (Przedwojski *et al.*, 1995).

Formula	α
Meyer-Peter & Müller (1948)	3
Bagnold (1956) (after Low, 1989)	3
Du Boys (1879), (after Graf, 1971)	4
Smart (1984), (after Low, 1989)	4
Engelund & Hansen (1967), (after Jansen *et al.*, 1979)	5
Inglis & Lacey (Inglis, 1968)	5
Engelund & Fredsøe (1976)	5
Shields (1936), (after Low, 1989)	5
Einstein & Brown (Brown, 1950)	6

gives the dispersive-like type of behaviour of the system (Cui & Parker, 1997). Here, the focus is on wave propagation instead. Then, for the inviscid limit $c_f \to 0$, $c_r \to c_0$ is obtained.

Measured against the fast or flow time scale, the inviscid result can be restated in dimensional form by recalling that $c_{0_*} = c_0 L / T = c_0 q_{s_0} / h_0$. Referring this result with respect to the mean flow velocity, the dimensionless wave celerity is now

$$\frac{c_{0_*}}{u_0} = \frac{q_{s_0}}{u_0 h_0} c_0 = \frac{\psi\alpha}{(1-p)(1-F_0^2)} = \frac{\alpha\psi}{(1-p)} + O(F_0^2), \quad F_0^2 < 1 \tag{11}$$

This equation is plotted in Fig. 2 with $p = 0.4$ and $F_0 = 0.1$ for different values of α, where the star is omitted for brevity. The values of α can be associated with several transport formula, assuming for consistency normal flow conditions with a constant friction factor (Table 1). Figure 2 also plots the field data recorded by one of the authors while tracking dune migration in the Paraná River. The values of *celerity*/u_0 were obtained by comparing the dunes' locations at regular time intervals along fixed longitudinal profiles (Fig. 3). These profiles were previously placed along depth averaged streamlines determined with subsurface floats (Amsler, 1989). The corresponding values of the total load were estimated by combining measurements of suspended load with bed load values obtained with the method of bed forms displacement, which turned out to be of the order of 10% to 15% of the total load (Lima *et al.*, 1990).

It can be readily seen that for a fixed value of the transport parameter ψ, something that may well represent an equilibrium situation close to the normal flow condition here considered, the observed bed forms celerities are well below the

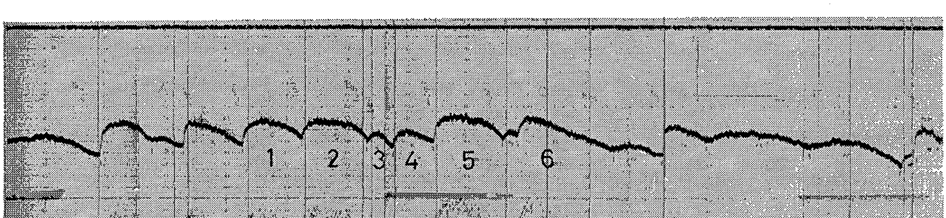

Fig. 3 Dune migration in the Paraná River recorded with an echo-sounder.

values predicted by several transport formulae. Fig. 3 also shows a linear least square fit of the field data, for which the value of α turns out to be very close to one ($\alpha = 1.044281$).

Finally, using the linear theory it is possible to draw a relationship between the linear celerity c_0 and the nonlinear celerity c, characteristic of the linear hyperbolic model and the kinematic wave model, respectively. Neglecting again the product of primed variables the following asymptotic result is obtained from equations (7) and (8):

$$\frac{c}{c_0} = \frac{u^{\alpha}(1-F_0^2)}{h-u^2F_0^2} \quad \rightarrow \quad \frac{1}{1-(1+\alpha)\hat{\varsigma}} \quad \text{as} \quad F_0 \rightarrow 0 \tag{12}$$

NONLINEAR STABILITY ANALYSIS

Numerical calculations were performed with a code based on the Galerkin procedure of the weighted residual approximation of the equations (4) and (5). In this way, once the flow was computed, the bed was moved one time step forward according to equation (3) by means of a Crank-Nicolson-Galerkin method, where the advection term was stabilized with a Petrov-Galerkin perturbation of the standard hat functions of the finite element method (Carey & Oden, 1986). The nonlinear celerities were calculated numerically by perturbing the sediment load flowing into an uniform channel of similar slope and roughness as those encountered in the Paraná River ($S_0 \cong 2.10^{-5}$, $c_f \cong 0.002$). The perturbation considered here was introduced as a jump discontinuity at the inlet section of the channel bed (Fig. 4). A comparison between linear prediction, equation (12), and the nonlinear calculation is shown in Fig. 5. This result confirms that as long as the perturbation remains small, the nonlinear effects are indeed negligible for α less than two which is within the range of the least

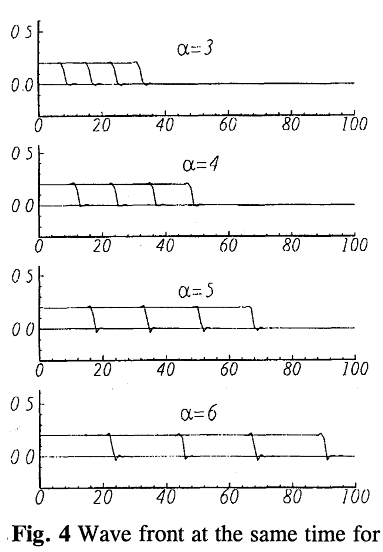

Fig. 4 Wave front at the same time for different transport exponent.

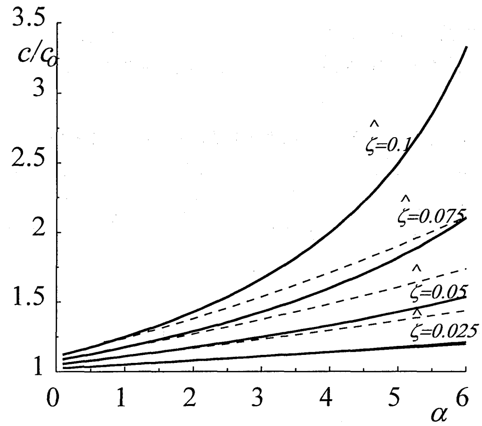

Fig. 5 Departure from the linear hyperbolic model, as predicted by the linear stability analysis (bold solid line), as computed with the kinematic wave model (dashed line).

square fit previously obtained for the Paraná River.

A final run was made with the fitted value of α with an independent set of field data. Fig. 6 shows the initial bed configuration recorded in Ago/92 and the final bed configuration predicted with the numerical model and the profile measured with an echo-sounder. The fit was obtained by matching the leading wave crests by trial and error. The computing elapsed time was 6.3 spanning 210 time steps (dimensionless). The transport parameter can be estimated, for mean values u_0 and h_0 to be equal to 1 m s^{-1} and 10 m, respectively, from the time scale previously defined

$$\psi = \frac{h_0\, t}{u_0\, t_*} = \frac{10\,\mathrm{m}\; 6.3}{1\ \mathrm{m\,s^{-1}}\, 42\ \mathrm{day}\ 86\,400\ \mathrm{s\,day^{-1}}} = 1.7\times10^{-5} \tag{13}$$

which is of the order of magnitude of the field data reported in this work.

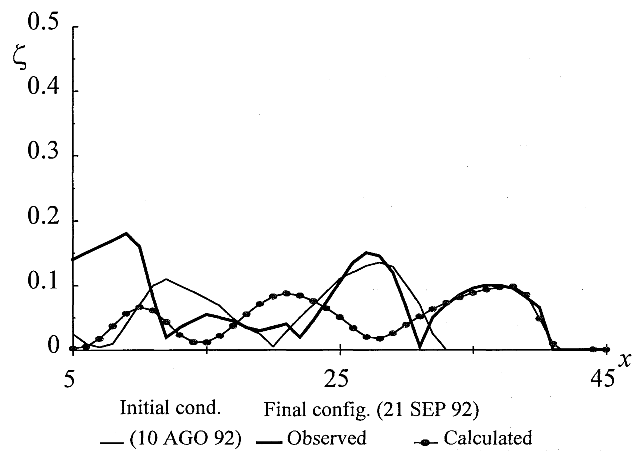

Fig. 6 Calculated values with the kinematic wave model *vs* observed amplitude of a dune train, starting from 10 August 1992 to 21 September 1992.

CONCLUSION

The problem of modelling movable bed flow was addressed in this work by considering the relative magnitude of the celerities at which disturbances on the water surface and on the bed are propagated. A linear stability analysis provides the natural framework to establish the comparison among transport formulae and field data. It was established analytically and numerically that for low Froude numbers, the bed forms celerities depend almost exclusively on the parameter ψ and the flow velocity exponent of the sediment transport formula. It was determined that bed forms in the Paraná River apparently follow a linear relation between bed load and flow velocity. Further research is needed to verify this number. In the meantime, a value of $\psi = O(10^{-5})$ seems to be a representative mean of the total sediment load transported by the Paraná River.

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