The Role of Erosion and Sediment Transport in Nutrient and Contaminant Transfer (Proceedings of a symposium held at Waterloo, Canada, July 2000). IAHS Publ. no. 263, 2000.

Modelling cohesive sediment transport in rivers

BOMMANNA G. KRISHNAPPAN

Aquatic Ecosystem Protection Branch, National Water Research Institute, Burlington, Ontario L7R 4A6, Canada

e-mail: krish.krishnappan@cciw.ca

Abstract A new model is proposed for the transport of fine sediment in the Athabasca River. The model is based on empirical relationships developed from laboratory experiments in a rotating circular flume using Athabasca River sediment. The model takes into account the differences in the critical conditions for erosion and deposition of fine sediment and treats the erosion and deposition processes as mutually exclusive. Therefore, under constant shear stress, the fine sediment will undergo either erosion or deposition but not both simultaneously. Such a treatment reflects the true nature of the cohesive sediment transport mechanism. The model may be easily incorporated into existing models of contaminant transport, fate and bioaccumulation.

INTRODUCTION

Many of the existing models of transport, fate and bioaccumulation of toxic substances in surface waters assume that the transport characteristics of fine-grained cohesive sediment are analogous to those of the coarse-grained non-cohesive sediment and adopt sediment transport theories that were developed for the latter to treat the former. Among the many differences between the two types of sediments, the most crucial is the difference in the critical condition for initiation and cessation of sediment motion in a flowing medium. In the case of coarse-grained sediment, the critical conditions for initiation and cessation merge into a single criterion, which means that coarse sediments undergo simultaneous erosion and deposition while being transported under a constant bed shear stress. In the case of cohesive sediment, on the other hand, two distinct critical conditions were identified in the literature; one for erosion and a different one for deposition (see, e.g. Partheniades & Kennedy, 1966; Partheniades *et al.*, 1968; Mehta & Partheniades, 1975; Lau & Krishnappan, 1994). Therefore, for fine sediments, the simultaneous erosion and deposition is not possible and these sediments undergo either deposition or erosion when subjected to a certain bed shear stress.

A true representation of the erosion and deposition of fine sediment is important for contaminant transport models. If the simultaneous erosion and deposition is assumed, then the model will predict an enhanced dispersion of the contaminants, whereas a mutually exclusive erosion and deposition will result in the preservation of comparatively high concentrations of sediment bound contaminants over long distances from the source. In this paper, a new sediment transport algorithm is proposed, which allows the treatment of erosion and deposition processes as mutually exclusive ones. The new algorithm is based on erosion and deposition experiments that were carried out in a rotating, circular flume at the National Water Research Institute at Burlington, Ontario, Canada using sediments from the Athabasca River near Hinton, Alberta.

BASIS OF NEW ALGORITHM

Details of deposition and erosion experiments carried out in the rotating, circular flume for the Athabasca River sediment were described in Krishnappan & Stephens (1996). The deposition results of Krishnappan & Stephens (1996) are reproduced in Fig. 1. Figure 1(a) shows the variation of concentration of sediment in suspension as a function of time during deposition under different bed shear stresses. This figure shows that the sediment concentration drops gradually and reaches a steady-state value for all runs. The figure also shows that the steady-state concentration is a function of the bed shear stress. When the bed shear stress is higher, the sediment concentration of the suspension is also higher. For the lowest shear stress tested, the steady-state concentration was only one tenth of the initial concentration and a slightly lower shear stress would have produced a nil concentration in suspension. The shear stress at which the concentration in suspension becomes zero is termed the critical shear stress

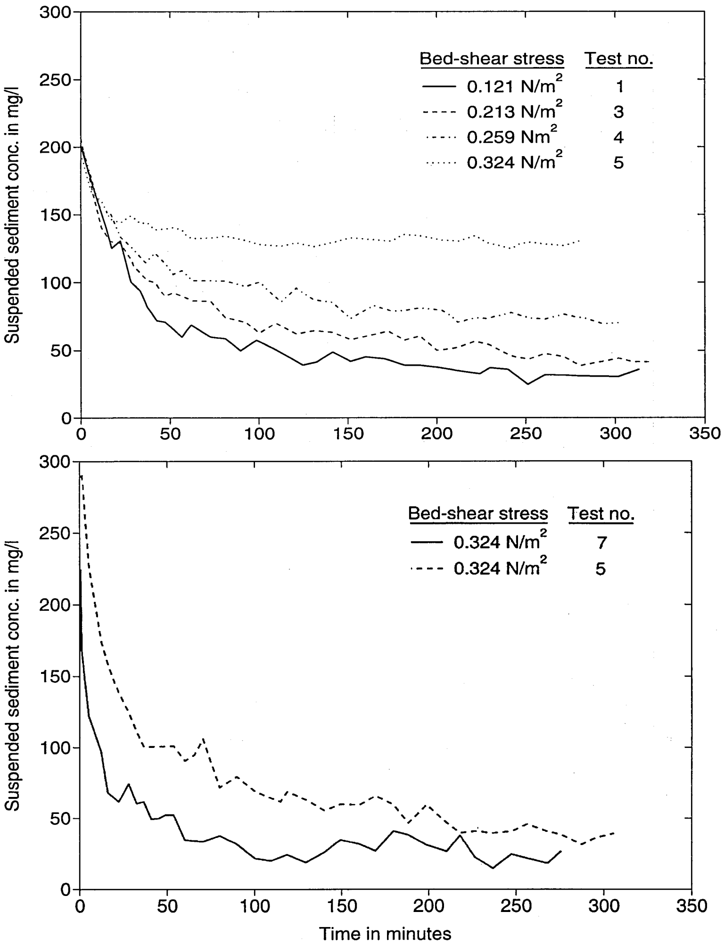

Fig. 1 Variation of concentration for (a) different shear stresses, and (b) different initial conditions.

for deposition. From the flume experiments of Krishnappan & Stephens (1996), the critical shear stress for deposition of Athabasca River sediment was estimated to be $0.10\ \mathrm{N\ m^{-2}}$.

Figure 1(b) shows the depositional characteristics of the sediment under constant bed shear stress but with different initial concentrations. It can be seen from this figure that the steady-state concentration which results during deposition is a function of the initial concentration. Such a behaviour is peculiar to fine-grained cohesive sediments and conforms to earlier studies on cohesive sediment transport (see Partheniades & Kennedy, 1966; Mehta & Partheniades, 1975; Lick, 1982). For a given sediment, the ratio between the steady-state concentration and initial concentration was found to vary only as a function of the bed shear stress. In other words, in the deposition process of fine sediment, the amount of sediment that would deposit under a particular bed shear stress is a function of the amount of sediment introduced to the system initially, and the fraction of the sediment that would deposit is constant as long as the bed shear stress is held constant. Such a conclusion allows one to establish a relationship between the fraction of the sediment that would deposit and the bed shear stress. The relationship established for the Athabasca River sediment is shown in Fig. 2.

In Fig. 2, the bed shear stress is expressed in terms of the critical shear stress for deposition (i.e. the shear stress below which all the initially suspended sediment would eventually deposit). From this figure, it can be seen that when the bed shear stress is at or below the critical shear stress for deposition, the fraction that would deposit takes a value of unity and the fraction deposited decreases as the bed shear stress increases. When the bed shear stress is about six times the critical shear stress for deposition, the fraction deposited becomes zero and all the initially suspended sediment stays in suspension. A power law relationship between the fraction deposited and the ratio of bed shear stress to the critical shear stress for deposition was fitted and is shown as solid line in Fig. 2. The relationship takes the following analytical form:

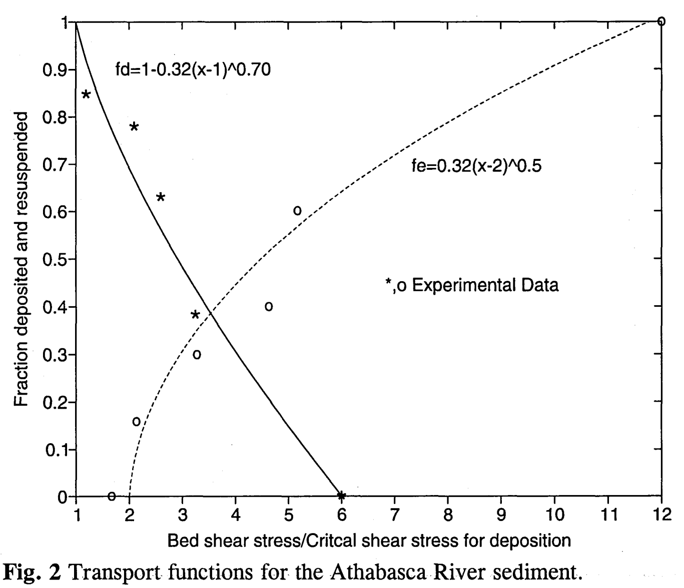

Fig. 2 Transport functions for the Athabasca River sediment.

$$
\begin{aligned}
f_d &= 1.0 - 0.32(\tau_0/\tau_{cd} - 1)^{0.70} && \text{for } 1 < \tau_0/\tau_{cd} < 6 \\
f_d &= 1.0 && \text{for } \tau_0/\tau_{cd} \le 1 \\
f_d &= 0 && \text{for } \tau_0/\tau_{cd} \ge 6
\end{aligned}
\tag{1}
$$

where f_d is the fraction deposited, τ_0 is bed shear stress and τ_{cd} is the critical shear stress for deposition.

The results of the erosion experiment that was carried out by Krishnappan & Stephens (1996) are shown in Fig. 3. In this figure, the concentration of eroded sediment and the applied shear stress are shown as functions of time. For the erosion test, the sediment was allowed to deposit and consolidate for a known period of time and then the shear stress was applied in steps as shown in Fig. 3. The sediment deposit was fully stable until the bed shear stress of 0.21 N m^{-2} was established. At this bed shear stress, the sediment concentration began to increase and attained a steady-state value of about 5 mg l^{-1}. Therefore, the critical shear stress for erosion is slightly lower than this shear stress. A value of 0.20 N m^{-2} was selected as the critical shear stress for erosion for the sediment that deposited at the critical shear stress for deposition. Note that the critical shear stress for erosion is two times the critical shear stress for deposition, confirming the earlier result that the fine sediments of the Athabasca River behave in a manner similar to that of the cohesive sediment.

From Fig. 3, it can be seen that, at each shear stress step, the variation of sediment concentration with time was similar where a steep increase was observed as the shear stress was applied followed by a gradual increase towards a steady-state concentration. The magnitude of the steady-state concentration at a particular shear stress step was lower than the steady-state concentration during the deposition experiment with the same bed shear stress. For the maximum shear stress (0.520 N m^{-2}) tested, not all the deposited sediment was resuspended. The maximum concentration reached was only about 60% of the total concentration that would have resulted from complete resuspension. For complete resuspension, a shear stress 12 times larger than the

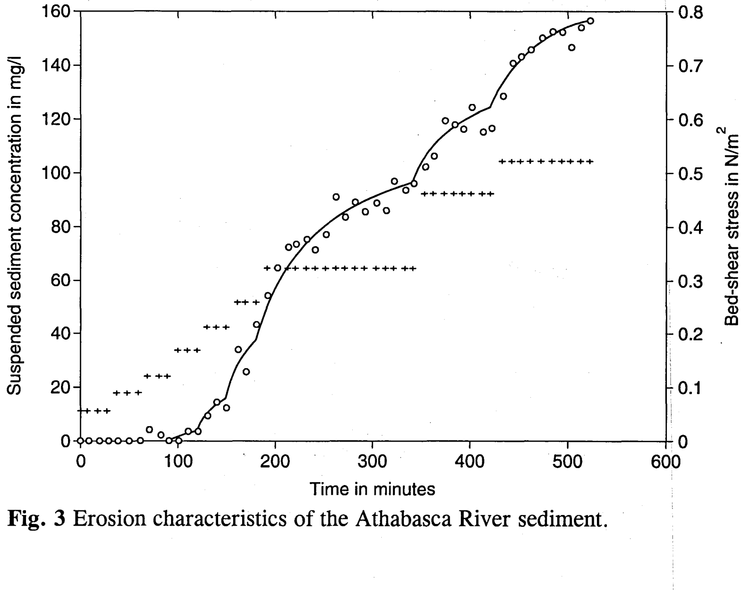

Fig. 3 Erosion characteristics of the Athabasca River sediment.

critical shear stress for deposition would have been required. From the erosion experiments, the fraction of resuspension was determined for various bed shear stresses and plotted in Fig. 2 as circles. A power law relationship was fitted through the experimental points. The form of the power law is shown below:

$$\begin{aligned} f_e &= 0.32(\tau_0/\tau_{cd} - 2)^{0.5} && \text{for } 2 < \tau_0/\tau_{cd} < 12 \\ f_e &= 0 && \text{for } \tau_0/\tau_{cd} \leq 2 \\ f_e &= 1 && \text{for } \tau_0/\tau_{cd} \geq 2 \end{aligned} \tag{2}$$

where f_e is the fraction of sediment re-suspended. For a sediment fraction that is deposited at a shear stress different from the critical shear stress for deposition, the above function can still be used with the following modifications: (a) the critical shear stress term has to be replaced by the shear stress at which the deposition occurred, and (b) the upper limit of the shear stress has to be maintained at the original value of 12 times the critical shear stress for deposition. Using the two functions given by equations (1) and (2), a new model is proposed for the transport of fine sediments of the Athabasca River. The details of the model are outlined in the next section.

DETAILS OF THE NEW MODEL

Basic information

The river reach is divided into a number of river segments shown schematically in Fig. 4. The flow rate in each segment is considered to be steady. Let the flow rate of the control segment be Q (m^3 s^{-1}). For a varying flow, a quasi-steady state is assumed and the flow hydrograph is approximated by a step function. The average bed shear stress (τ_0) has to be determined for each segment as a function of the flow rate. This can be done by employing friction factor relationships developed for uniform flows. The choice of the relationship would depend on the available flow and river geometry data. The relationship proposed for the Athabasca River is outlined in the Appendix.

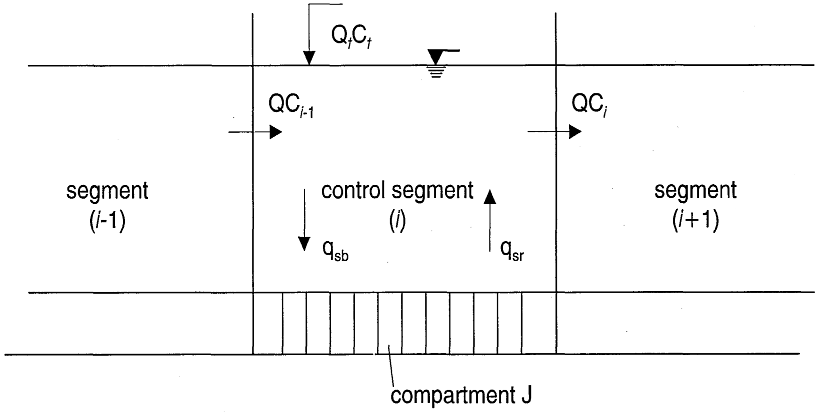

Fig. 4 Schematic representation of sediment mass balance.

For each segment, the range of the flow rates over an average year is established and the corresponding boundary shear stress range is calculated. This shear stress range is then divided into a fixed number of flow stages. Each flow stage is identified with its own average boundary shear stress which is the mean value of the shear stresses bounding the range. To be consistent with the sediment transport functions given by equations (1) and (2), the boundary shear stress is normalized using the critical shear stress for deposition.

Mass balance of sediment in a control segment

Sediment inflow The mass balance of the sediment in a control segment is calculated as follows: the sediment entering the control segment can be (a) from the upstream segment and (b) from tributary inflows. The amount of sediment (q_{su}) entering the control segment during a time interval of Δt can be expressed as follows:

$$q_{su} = QC_{i-1}\Delta t + Q_t C_t \Delta t \tag{3}$$

where C_{i-1} is the sediment concentration in the upstream segment, C_t is the concentration of sediment in the tributary inflow to the control segment, and Q_t is the tributary inflow rate.

Sediment depositing to the bed A portion of the incoming sediment will deposit depending on the prevailing flow conditions. The sediment quantity transported from the upstream segment will be deposited only when the shear stress in the control segment is lower than that in the upstream segment. If the shear stress in the control segment is equal to or greater than the upstream segment, then the sediment arriving from the upstream segment would have gone through the deposition process already and would have reached the steady-state concentration. Therefore, this sediment has to be routed straight through the control segment. The amount that would deposit in the control segment, therefore, can be calculated as:

$$\begin{aligned} q_{sd} &= (QC_{i-1}\Delta t) f_d + (Q_t C_t \Delta t) f_d & \text{if } \tau_I \le \tau_{I-1} \\ q_{sd} &= (Q_t C_t \Delta t) f_d & \text{if } \tau_I > \tau_{I-1} \end{aligned} \tag{4}$$

where q_{sd} is the amount deposited during the current time step.

The amount remaining in suspension (q_{ss}) becomes:

$$\begin{aligned} q_{ss} &= (QC_{i-1}\Delta t + Q_t C_t \Delta t)(1 - f_d) & \text{if } \tau_I \le \tau_{I-1} \\ q_{ss} &= Q_t C_t \Delta t (1 - f_d) + (QC_{I-1}\Delta t) & \text{if } \tau_I > \tau_{I-1} \end{aligned} \tag{5}$$

Sediment resuspension Sediment can also be resuspended from deposits that occurred in previous time steps. The mass of resuspended sediment can be calculated by keeping track of the amount of deposited sediment and the shear stresses at which deposition takes place. This can be done by schematizing the river bed to consist of different compartments and assuming that each compartment holds sediment deposited at a particular shear stress. For example, let us assume that there are N compartments in the control segment and each compartment is identified with an index, say, J.

Therefore, J varies from 1 to N. Compartment 1 is assumed to collect sediment deposited at shear stress equal to or less than the critical shear stress for deposition, τ_{cd}, and compartment 2 collects sediment deposited at shear stresses between τ_{cd} and $2\tau_{cd}$ and so on. Let the sediment deposited in each of the compartments from the previous time steps be P_J. For a given shear stress, the sediment resuspended from various compartments can be calculated by applying equation (2) for each compartment with the appropriate τ_{cd} value as follows:

$$q_{sr} = \sum_{J=1}^{N} q_{srJ} = \sum_{J=1}^{N} P_J f_{eJ} \tag{6}$$

where q_{sr} is the total amount of resuspended sediment and f_{eJ} is the erosion function for the compartment J.

Knowing the amount of sediment resuspended from and deposited to the bed, the concentration of the suspended sediment and the amount of sediment in the various compartments in the bed of the control segment at the end of the current time step can be calculated as follows:

Sediment concentration at the end of the current time step The suspended sediment concentration in the control segment at the end of the current time step is:

$$C(i) = \frac{(q_{ss} + q_{sr})}{(Q\Delta t)} \tag{7}$$

Amount of sediment in various bed compartments at the end of the current time step The amount of sediment left behind in the control segment at the end of the current time step is:

$$\sum_{J=1}^{N} P_J^* = \sum_{J=1}^{N} P_J(1 - f_{eJ}) + \delta(J,K) \cdot q_{sd} \tag{8}$$

where P_J^* is the updated value of P_J at the end of the current time step. The function $\delta(J, K)$ takes a value of unity, when $J = K$, and zero when $J \neq K$. The term K denotes the compartment which receives the deposited sediment during the current time step.

The concentration $C(i)$, as calculated by equation (7), is routed to the downstream segment and the calculations outlined above for the control segment are repeated for the downstream segment. The process is continued until all the segments are encountered. The calculations are then repeated for the next time step until the simulation period is covered.

APPLICATION OF THE ALGORITHM

The above algorithm was incorporated into the WASP5 model and applied to the Athabasca River to predict contaminant transport and fate in the river system as part of the Northern River Basin Study. The results are described in a report by Golder Associates Ltd (1996). The report concluded that the incorporation of the sediment transport algorithm improved the prediction of the WASP5 model.

SUMMARY

A new algorithm for the transport of fine sediments of the Athabasca River is developed based on laboratory experiments in a rotating circular flume. The algorithm is formulated for steady flows and may be used for gradually varied flow by considering the flow rate as quasi-steady. This algorithm is an improvement over the existing sediment transport models as it accounts for the differences in the critical conditions for erosion and deposition processes of fine sediment transport. Incorporation of the algorithm in a contaminant transport model (WASP5) has improved the performance of the model.

REFERENCES

Golder Associates Ltd (1996) Final Report on implementation of a new algorithm for simulation of fine grained sediment transport in the Athabasca River. Northern River Basins Study, Calgary, Alberta, Canada.

Krishnappan, B. G. & Stephens, R. (1996) Critical shear stresses for erosion and deposition of fine suspended sediment from the Athabasca River. *Northern River Basins Study Project Report no. 85*, Northern River Basins Study, Edmonton, Alberta, Canada.

Lau, Y. L. & Krishnappan, B. G. (1994) Does reentrainment occur during cohesive sediment settling? *J. Hydraul. Engng, ASCE* **120**(2), 236–244.

Lick, W. (1982) Entrainment, deposition and transport of fine-grained sediments in Lakes. *Hydrobiologia* **91**, 31–40.

Mehta, A. J. & Partheniades, E. (1975) An investigation of the depositional properties of flocculated fine sediments. *J. Hydraul. Res.* **13**, 361–381.

Partheniades, E. & Kennedy, J. F. (1966) Depositional behaviour of fine sediment in a turbulent fluid motion. In: *Proc. Tenth Conf. on Coastal Engineering* (Tokyo, Japan), 707–724. Am. Soc. Agric. Engrs, New York.

Partheniades, E., Cross, R. H. & Ayora, A. (1968) Further results on the deposition of cohesive sediments. In: *Proc. Eleventh Conf. on Coastal Engineering* (London, UK), vol. 2, 723–742. Am. Soc. Agric. Engrs, New York.

APPENDIX

Calculation of bed shear stress

Assuming that the hydraulic parameters such as the flow rate Q ($m^3 s^{-1}$), the flow cross-sectional area A (m^2), the wetted perimeter, P (m) and the representative bed material size D_{65} (m) are known, we can calculate the bed shear stress using the logarithmic friction factor relation as follows:

$$U_* = \frac{(Q/A)}{2.50\ell n[11.0(A/P)/(2.50D_{65})]} \tag{A1}$$

where U_* is the shear velocity in m s^{-1}.

Knowing U_*, the bed shear stress τ_0 in N m^{-2} can be calculated using the following conversion:

$$\tau_0 = 1000U_*^2 \tag{A2}$$

The equation (A1) assumes that the flow is in rough turbulent regime which may be a reasonable assumption for the Athabasca River. When the flow is ice-covered, a 50% reduction in U_* values can be assumed as a first approximation. Further refinement can be carried out if it is warranted.