A model of rill erosion by snowmelt

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Abstract Measurements of rill flow profiles, water discharges, sediment concentration, temperature of water, soil and air were conducted during spring snowmelt events on an experimental station located 100 km south of Moscow, Russia. The results indicate that: (a) the rill profiles have, as a rule, a triangular form; (b) the side-wall slope of a rill is close to the natural slope for non-frozen soils and depends on the water discharge; and (c) in general, the thawing of the soil surface occurs faster, than the soil particle detachment. As the knowledge of frozen soil erosion mechanics is limited, a number of assumptions have to be made for the model design. In detail, the Snow Melt Erosion Model (SMEM) includes the Chezy–Manning's equation, the Goncharov's equation to calculate bottom flow velocity, the Mirtskhulava's equation for estimation of soil particle detachment and the Kuznetsov's equation for critical bottom flow velocity. The model is tested with 7 years of data from two runoff plots located in the Central-Chernozem Zone of Russia (soil type is chernozem).

Key words rill erosion; snowmelt; erosion model; Russia

INTRODUCTION

Results of erosion studies in northern, central and eastern Europe indicate that the erosion rate during snow melt events can reach or even exceed the rainfall erosion rate. Rill formation is the fundamental erosion process during winter conditions. Understanding the nature of snowmelt erosion processes is essential for solving both the on-site and off-site problems and to deduce recommendations for management practices. Predictive modelling is an important tool in evaluating alternative technologies.

Recently, well known but often unadapted empirical equations have been applied for the assessment of soil losses during snowmelt periods (Wischmeier & Smith, 1978; Cheboterev *et al.*, 1979; Surmach, 1979; Edwards *et al.*, 1998). The main problem in the design of a snowmelt erosion model is connected to the characterization of soil detachment processes by snowmelt overland flow is one of the major problems to be solved. Additionally, the formation of a rill net by snowmelt overland flow is an open question. A physically based equation for particle detachment at frozen soil conditions was developed on the basis of laboratory experiments (Kuznetsov *et al.*, 1999, 2001; Kuznetsov & Demidov, 2002). However, extensive investigations have to be conducted to define the values of a number of relevant parameters. This partly restricts its application. The purpose of this paper is the presentation of a physically reasonable model for snowmelt rill erosion on hillslopes with a minimum of input parameters.

DATA AND MODEL

Rill profiles

The field station of the Institute of Basic Biological Problems (Russian Academy of Sciences) is located 100 km to the south of Moscow. The main agricultural practice of the predominant grey forest soils is autumn ploughing to a depth of 20–22 cm and winter wheat cultivation. The measurement of the following rill characteristics was conducted: water discharge, concentration of sediments, and cross section of a water flow. In addition to snow characteristics, air and soil temperature were measured. The results indicate that: (a) the rill profiles have in general a triangular form; (b) the slope of the side-wall of a rill depends on the water discharge and is close to the natural slope for non-frozen soils; (c) in general, the thawing of the soil surface occurs faster than the soil particle detachment. Also Gatto (2000) observed triangular rill profiles. These findings are of particular importance for the model development.

During 4 years of investigation 75 rill profiles for ploughed soils and 23 profiles for soils under winter wheat and correlated runoff characteristics were measured. Figure 1 presents representative cross sections of a rill on a fallow plot for different discharge values. The typical triangular shape of the rill cross section clearly indicates that rill incision is not limited by a frozen soil layer. Statistical analysis of the observed data show that the tangent of the angle α of the bank slope (Fig. 2) can be described with the following empirical relationship:

$$Tg(\alpha) = Tg(\alpha_{max}) - [Tg(\alpha_{max}) - Tg(\alpha_{min})] \exp(-\beta Q) \tag{1}$$

where Tg is tangent; Q is water discharge (l s^{-1}); β is stationary value (s l^{-1}); α_{min} and α_{max} are potential minimum and maximal angle α of the bank slope, respectively. The values of $Tg(\alpha_{min})$ of equation (1) and the average weighted relative deviation ($\varepsilon_{a.w}$) as measures for accuracy differ for fallow and winter wheat (Table 1). The received values $Tg(\alpha)$ are close to those that are recommended for amelioration of earthen channels.
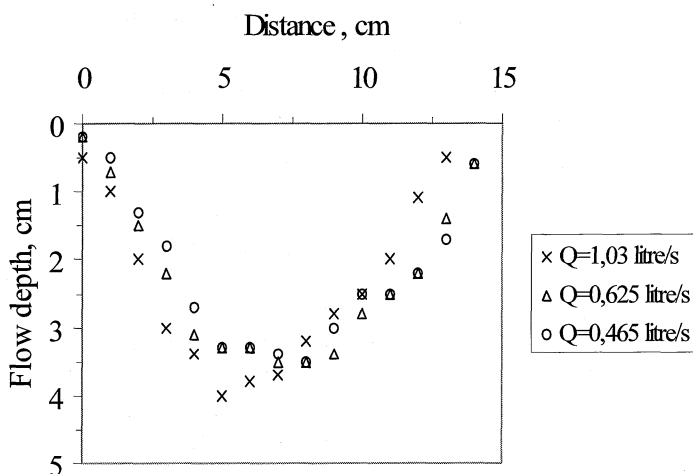

Fig. 1 Measured rill profiles for different discharges for a rill on a fallow plot, 31 March 1999.
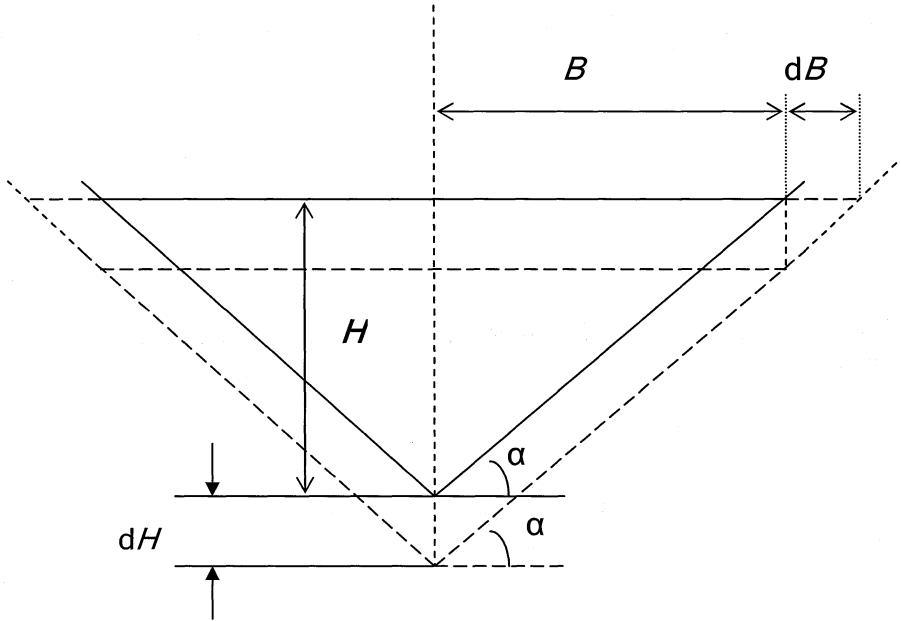
Fig. 2 Profiles of a rill at time t and $t + dt$.

Table 1 Parameters used in equation (1).

Parameter	Autumn ploughing	Winter wheat
$Tg\,(\alpha_{max})$	0.6	0.6
$Tg\,(\alpha_{min})$	0.1	0.2
$\beta\,(s\,l^{-1})$	3.5	3.5
$\varepsilon_{a.w}\,(\%)$	14.4	23.6

Soil detachment

The following assumptions are considered for a runoff plot with small length L and with the gradient i: (a) one rill is formed per plot; (b) the outlet discharge is known and the input of water in a rill normalized per unit length one will be identical for the entire plot; (c) for a small time increment the water discharge does not change practically; (d) the cross section of a water flow is determined by the water discharge at any time and at any distance from the top of a plot; (e) the soil particle detachment takes place for unfrozen soil conditions which are characterized by a minimum of coalescent force between soil particles; and (f) all detached soil particles are transported by water flow.

Let us consider profiles for a rill at time t and $t + dt$, where dt is small increment of time. In Fig. 2 for time t the profile is shown by a solid line, and for time $t + dt$ the profile is shown by a dashed line. As a result of erosion, the water flow was lowered by a quantity dH (m). The increment of cross sectional area of rill to within the small value dH will be evaluated:

$$dS_{rill} = 2\,h\,\text{Ctg}(\alpha)\,dH \qquad m^2 \tag{2}$$

where *Ctg* is cotangent; *h* is depth of flow (m). Division of both parts of equation (2) with d*t* results in:

$$\frac{dS_{rill}}{dt} = 2hCtg(\alpha)\frac{dH}{dt} \quad m^2 \, s^{-1} \tag{3}$$

Further:

$$\frac{dH}{dt} = \frac{q}{\rho_{soil}} \tag{4}$$

where *q* is the intensity of soil erosion (kg m^{-2} s^{-1}); and ρ_{soil} is the soil density (kg m^{-3}). From equations (3) and (4) follows that:

$$\frac{dS_{rill}(t,x)}{dt} = 2h(t,x)Ctg[\alpha(t,x)]\frac{q(t,x)}{\rho_{soil}} \tag{5}$$

where *t* is time (s); and *x* is the distance from the top of runoff plot (m). For any interval of time $(T_2 - T_1)$ the volume of the rill will increase:

$$Volume = \int_{T_1}^{T_2} dt \int_0^L \frac{dS_{rill}(t,x)}{dt}dx \quad m^3 \tag{6}$$

For this interval of time the rill erosion will be equal, thus:

$$RillErosion = \rho_{soil} \, Volume \quad kg \tag{7}$$

Further we use a series of simple equations:
—water discharge:

$$Q(t,x) = [QL(t)/L]\,x \tag{8}$$

where *L* is length of a plot (m); *QL* (*t*) is outlet discharge (m^3 s^{-1});
—the cross sectional area of water flow:

$$S_f = h^2 / Tg(\alpha) \quad m^2 \tag{9}$$

—hydraulic radius for the triangular form of the channel:

$$R = h\,Cos(\alpha)/2 \quad m \tag{10}$$

where Cos is cosine;
—Chezy-Manning equation:

$$V = R^{2/3}\, i^{1/2}/n \tag{11}$$

where *V* is flow velocity (m s^{-1}), *i* is channel slope (dimensionless), *n* is Manning's coefficient;
—water discharge:

$$Q(t,x) = S_f\, V \quad m^3\,s^{-1} \tag{12}$$

From the equations (9) to (12), it follows that :

$$h(t,x) = 2^{1/4}\, Q(t,x)^{3/8}\, [n\, Tg(\alpha)]^{3/8}\, [i^{-3/16}\, Cos^{-1/4}(\alpha)] \tag{13}$$

Thus, knowing the outlet discharge *QL (t)*, it is possible to apply equation (8) to calculate

$Q(t, x)$, and with equation (1) it would be possible to calculate an angle $\alpha(t, x)$. Further, with equation (13) it is possible to calculate depth of a flow $h(t, x)$, which enters equation (5).

For the dimension of q we use the method from Mirtskhulava for unfrozen soils (Mirtskhulava, 1970, 2000):

$$q = 1.1 \times 10^{-6} \omega D_{wsp} \rho_{paricle} \left(\frac{V_{\Delta}^2}{V_{\Delta,cr1}^2} - 1 \right) \quad \text{kg m}^{-2} \text{ s}^{-1} \tag{14}$$

where $\omega = 10$ s^{-1} is the frequency of pulsations of water flow; D_{wsp} is the average diameter of the water stable aggregates (m); $\rho_{particle}$ is the density of aggregates (kg m^{-3}); V_{Δ} is the bottom velocity (m s^{-1}); $V_{\Delta,cr1}$ is the first critical bottom velocity (m s^{-1}). If V_{Δ} is less than $V_{\Delta,cr1}$ then q will be zero. Goncharov's equation is applied for the calculation of bottom flow velocity (Goncharov, 1962):

$$V_{\Delta} = 1.25 \, V / \text{Log}_{10} \, (6.15 \, h / \Delta) \quad \text{m s}^{-1} \tag{15}$$

where Log_{10} is logarithm; Δ is the roughness of bottom rill (m). The bottom roughness can be expressed through the diameter of soil particles (Kuznetsov, 1981):

$$\Delta = 0.7 \, D_{wsp} \tag{16}$$

Hence, at a known water discharge $Q(t, x)$ it is possible to use equations (10), (11), (13), (15) and (16) to calculate the velocity of bottom flow V_{Δ} for any instance of time t and for any distance x from the top of a plot. The second critical velocity will be estimated without consideration of coalescence between particles following Kuznetsov (1981):

$$V_{\Delta,cr2} = 1.55 \sqrt{\frac{m_1 m_2 g}{\rho_{water} n_1} (1 - P) D_{wsp} (\rho_{mineral} - \rho_{water})} \tag{17}$$

where $V_{\Delta,cr2}$ is the second critical bottom velocity for fallow plots (m s^{-1}); for rill erosion m_1 = 1.4, m_2 = 1.0 and n_1 =2.3; g = 9.81 m s^{-2} is gravitational acceleration; P is the porosity of soil particles, (dimensionless); ρ_{water} is the density of water (kg m^{-3}); $\rho_{mineral}$ is the density of mineral (kg m^{-3}). The dependence between critical velocities is according to results from Mirtskhulava (1970):

$$V_{\Delta,cr2} = 1.4 \, V_{\Delta,cr1} \tag{18}$$

With information on the physical characteristic of a soil, it is possible to calculate the quantity of $V_{\Delta,cr1}$ by using equations (17) and (18), which enters into equation (14). Thus, it is possible to calculate the intensity of erosion at the bottom of a rill $q(t, x)$ and quantify dS_{rill}/dt with a defined quantity of water discharge $Q(t, x)$. The integration in formula (6) gives the rill volume and equation (7) allows the quantity of rill erosion to be estimated.

MODEL RESULTS

The first application of the snowmelt rill erosion model (SMEM) was conducted with data that were received from two runoff plots of the Niznedevitsk water-balance station (the Voronezh region, Russia). Both plots have identical length and width (100 m × 20 m), a slope of 5.5% and a northern exposure. The local chernozem soil has the following characteristics: ρ_{soil} = 0.91 g cm^{-3}, $\rho_{mineral}$ = 2.58 g cm^{-3}, D_{wsp} = 0.5 mm, P = 0.408. Table 2

Table 2 The agricultural management of the runoff plots.

Year	Plot 9	Plot 10
1962	Winter wheat	Fallow
1963	Winter wheat	Fallow
1964	Fallow	Fallow
1965	Fallow	Fallow
1966	Fallow	There were no measurements
1968	Fallow	Fallow
1969	Fallow	Fallow

presents the crop rotation during the years that were selected for modelling. Measurements of the water discharges $Q_{L,1}$, $Q_{L,2}$,... $Q_{L,N}$ and the sediment concentrations $C_{L,1}$, $C_{L,2}$... $C_{L,N}$ (where N is the number of measurements) were conducted during the daytime at instants of time t_1, t_2... t_N. The measurement data were used to calculate soil losses using the following equation:

$$SoilLosses = \sum_{i=2}^{N} \frac{1}{2}\left(Q_{L,i-1}C_{L,i-1} + Q_{L,i}C_{L,i}\right)\left(t_i - t_{i-1}\right) \tag{19}$$

To estimate the soil loss the length of each plot was divided into 10 equal parts (x_1, x_2... x_{10}, where x_j is distance from the top of a plot). According to data from Mirzkhulava, the first critical bottom velocity for winter grain $V_{\Delta,cr1,\text{Grain}} = 1.5\ V_{\Delta,cr1,\text{Fallow}}$ (Mirzkhulava, 2000). For each distance x_j and instant of time t_i (when the measurements were done) the values dS_{rill}/dt (equation 5) are calculated. Further, the values *Volume* (equation 6) and *RillErosion* (equation 7) were calculated for each day by applying a numerical integration. The estimated soil erosion varies between 32.6 kg year^{-1} for winter wheat of plot 9 and 66.5 kg year^{-1} for the fallow plot 10 kg year^{-1}. The deviation range was between 22.4% and 36.2%. In general, an overestimation of the model results compared to the measurement data can be observed (Fig. 3). Further analysis of the results shows that the average weighed relative deviation ($\varepsilon_{a.w}$) for rill erosion per day is $\varepsilon_{a.w} = 102\%$. The accuracy of the model increases for a longer period of 1 year to $\varepsilon_{a.w} = 62\%$.
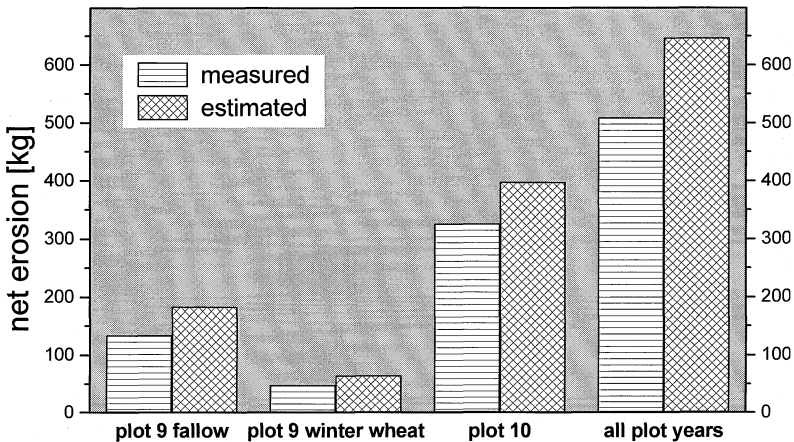

Fig. 3 Measured and modelled erosion from the two erosion plots.

CONCLUSIONS

The application of the snowmelt rill erosion model (SMEM) achieves results that are close to the data from erosion plot experiments. The accuracy of the modelling results increase with an increase in the modelling period. Hence, the basic assumptions can be used to develop a snowmelt rill erosion model that might be applied for the calculation of soil losses for longer periods. Further testing and analysis of parameter sensitivity has to be done to apply the model for single events and on a catchment scale.

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