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Hydrophysical model of soil erosion: a basic equation and influence of bed load and suspended sediment on soil detachment by shallow water flow

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Abstract The hydrophysical model is based on the three premises. According to the first and the second premises soil detachment is proportional to the cubed flow velocity in general. Within the range where flow velocity is higher than 0.4 u_0 and less than 1.6 u_0 the detachment of soil particles has a probabilistic nature. The experimental data are described well by the equation formed from these premises. The third premise runs as follows: the detached soil particles, which move in the water course of an overland flow as bed load and suspended sediment, in some way influence the detachment of new soil particles. The influence of natural and artificial bed load produced from different materials on wash out of chernozem samples are described satisfactorily by the exponential function. The suspended sediment corks up the pores of soil. Therefore it leads to a strengthening of the cohesion forces between soil particles and aggregates and consequently to a decrease of erodibility.

Key words bed load; erosion modelling; influence; soil detachment; soil erosion; suspended sediment

INTRODUCTION

The analyses of well known erosion models show that all of them contradict the field and experimental data (Larionov *et al.*, 2003). According to numerous plot studies soil loss is proportional to the sine of slope angle and slope length raised to an exponent which is significantly less than one. The equations of soil detachment written as a function of slope and slope length show the following. According to the models which are based on the assumption that soil detachment is the function of actual shear stress expressed either as shear stress minus the critical value (Foster, 1982) or the ratio of squared flow velocity to squared critical value minus one (Mirtskhulava, 1970), the soil detachment rate is proportional to the sine of slope angle and slope length, both raised to 0.666. Models which rely on a stream power to characterize the capability of flow to detach soil (Rose, 1985; Hairsine, 1992) indicate that soil loss is proportional to the sine of slope angle and slope length.

Nearing (1991) proposed a probabilistic model of soil detachment. One of the aims of the model was to overcome an apparent inconsistency in terms of the orders of magnitude differences between soil strength and flow shear stresses. According to his data soil strength is of the order of kPa, while flow shear stresses are of the order of Pa. It is known that local shear stresses associated with turbulent burst-events are much greater than the average flow shear stresses. The average shear stress of the burst-events is 150 times greater than the average flow shear stress. This fact is only one part of the explanation. Nearing (1991) suggests that the detachment process is determined by the overlapping tails of two distributions (burst-event shear stresses and soil particle resistance to detachment). The final equation of soil detachment written as a function of slope and slope length is a nonlinear function. The detachment rate is proportional to the sine of slope raised to 1.333 and slope length raised to 0.333.

The model developed by Gendugov *et al.* (1997) proceeds from the assumption that flow velocities below the critical value also produce soil detachment. According to this model soil detachment is proportional to the sine of slope angle raised to 0.88–0.58 and slope length raised to 0.56–0.44.

Analysis shows that the above mentioned models are partly or completely contrary to the facts. The models of Foster (1982), Mirtskhylava (1970) and Gendugov *et al.* (1997) underestimate both the slope steepness and slope length factors. The model of Rose (1985) greatly overestimates the slope length factor. The model proposed by Nearing (1991) overestimates the slope steepness factor.

In order to overcome the discrepancy between the field soil loss data collected at North Caucasus and modelled data using the second edition of USLE (Larionov & Krasnov, 1993), a new soil detachment model was developed. The new model contains a probabilistic element and is based on the most general laws of nature. The model describes existing flume experiment data from literature well and was confirmed by the experiments designed to study soil detachment by high velocity flow. The aim of this paper is to demonstrate the main features of the basic equation and to quantify the influence of bed load and suspended sediment on soil detachment.

THE EQUATION OF SOIL DETACHMENT

Parameterization and validation

The main equation of soil detachment is deduced from two premises: (a) erosion is the work of water flow, which is responsible for the detachment; (b) particles are detached by the flow streams if instantaneous flow velocities exceed the threshold. The simple mathematical construction based on the first premise leads to the conclusion that soil detachment is proportional to the power of flow:

$$D_r \propto k_r \gamma u^3 \tag{1}$$

where D_r is the rill detachment rate, k_r is the rill erodibility of soil, γ is the specific weight of water, and u is the average flow velocity. It should be mentioned that in the case of shallow flow, water power is proportional to the cubed average flow velocity.

According to the second assumption the detachment of soil particles has to have a stochastic nature if the average flow velocity is close to the critical value. In that case detachment events depend on overlapping of two probability distributions. The first is the distribution of instantaneous flow velocity; the second is the distribution of resistance of soil particles to detachment. Probability of events falling within a certain limit is usually determined from the special tables but this is inconvenient for our case. It is easier to use the cumulative curve of probability expressed by a logistic function. Then the probability density function (P_u) of instantaneous flow velocity may be approximately expressed as:

$$P_u = \left[1 + 10^{a(1-u/u_0)}\right]^{-1} \tag{2}$$

where u is the average flow velocity, u_0 is the critical velocity and a is the coefficient depended on the instantaneous flow velocity dispersion. According to measurements of instantaneous flow velocity (Mirtskhulava, 1967) coefficient a equals approximately 4. Conveniently, the probability of soil particle tensile stress should be described by a logistic equation with the same variables as in the case of flow velocity. Because the stress exerted on soil particles by water flow is proportional to flow velocity raised to the second power, the average relative value of soil tensile strength may be expressed as squared critical flow velocity. Then the probability (P_s) of soil particle tensile strength may be written as:

$$P_s = \left[1 + 10^{b(1-u^2/u_0^2)}\right]^{-1} \tag{3}$$

where b is the coefficient depended on the dispersion of soil particles tensile strength. Other variables are already mentioned above.

Recalling equation 1 soil detachment (D_r) by clear water should have a form:

$$D_r = k_r \gamma u^3 \left[1 + 10^{a(1-u/u_0)}\right]^{-1} \left[1 + 10^{b(1-u^2/u_0^2)}\right]^{-1} \tag{4}$$

Equation (4) of soil detachment by clear water flow was adjusted and verified using existing literature experimental data (Kuznetsov & Glazunov, 1985; Nearing *et al.*, 1981; Larionov & Krasnov, 1997) obtained under laboratory conditions over a wide range of flow velocities (0.21–1.97 m s^{-1}). According to the consequence of the first and the second premises the relationship between the soil detachment and the cubed flow velocity should be linear if $u \geq 1.6u_0$. This is confirmed by Fig. 1. It also indicates that detachment takes place in the zone where flow velocity is much less than the critical magnitude. In this case the relationship between soil detachment and cubed flow velocity is linear too. The two segments of straight line are connected by the *s*-shaped line. This part of the graph corresponds to the zone where the soil detachment process has a probabilistic nature. This peculiarity of the relationship between soil detachment and cubed flow velocity is indicated by equation (4). It should be added by the term designated to describe the soil detachment in the zone where flow velocity is less than the critical value. Results of parameterization and validation of equation (4) adding the above mentioned term are presented in Table 1 (Fig. 1). The flow velocity at the height of roughness elements and average flow velocity at the near-bed 1 cm layer were used as variables for the soil detachment equation. The results were good enough in both cases. The coefficients of determination are between 0.98 and 0.99. This is slightly higher than described by Nearing *et al.* (1991). However, from a practical point of view the flow velocity at the near bed of 1 cm layer is much more reasonable than flow velocity at the height of roughness elements. Firstly, the magnitude of erodibility, k_{1r} and k_{2r}, is proportional to the tensile strength of soils. This in turn is inversely proportional to the weighted average soil aggregate diameter according to the data of six soil materials used by Nearing *et al.* (1991). The flow velocity at the height of roughness elements does not indicate such regularity. Secondly, for all sieved samples the magnitude of the critical velocity, in the case of using the flow velocity in the near-bed 1 cm layer, is almost the same, contrary to the flow velocity at the height of roughness elements. Thirdly, in the case of flow velocity at the near bed layer the constant value of coefficient b can be applied for soils with aggregates of uniform size. The equation of soil detachment by clear shallow water flow is written in the form:

$$D_r = 10^{-6} \gamma u_s^3 \{k_{1r}\left[1+10^{-4(1-u_s/u_0)}\right]^{-1} + k_{2r}\left[1+10^{4(1-u_s/u_0)}\right]^{-1}\left[1+10^{b(1-u_s^2/u_0^2)}\right]^{-1} \tag{5}$$

where D_r is the soil loss per unit area and unit time (g m^{-2} s^{-1}); γ is the specific weight of water, (kg m^{3}); u_s is the flow velocity at the 1-cm near bottom layer, (m s^{-1}); k_{1r} and k_{2r} are the soil rill erodibility (m^{-1} s^{2}); a and b are dimensionless coefficients.

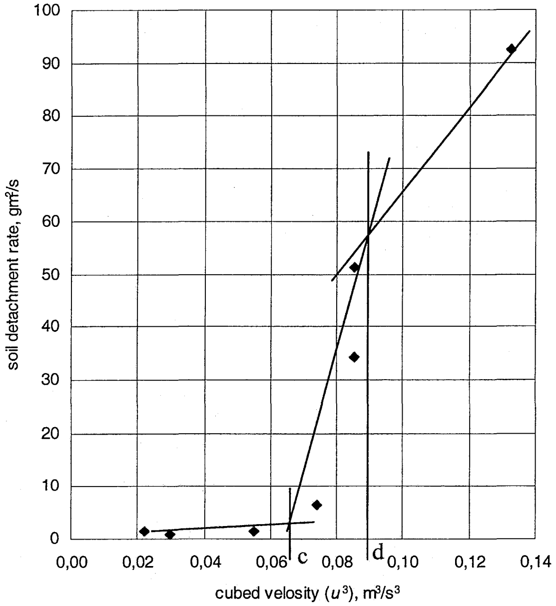

Fig. 1 Relationship between cubed flow velocity and soil detachment rate.

Table 1 The magnitude of parameters and coefficients of equation (3).

Parameters	SOIL							
	Russell			Paulding			Chernozem	Loam
	Aggregate size. mm							
	0.47	1.022	2.065	0.596	1.428	2.719	2.4	1.5
Flow velocity calculated at the height of roughness elements								
U_0. m s^{-1}	0.33	0.427	0.536	0.357	0.49	0.578	0.374	–*
K_1. m^{-1} s^{2}	50	30	45	40	18	16	0.015	–*
K_2. m^{-1} s^{2}	1500	850	1250	700	590	980	62	67
A	4	04	4	4	4	4	4	4
B	10	22	18	10	5	10	2	2
C	–4	–4	–4	–4	–4	–4	–4	–4
R^2	0.976	0.989	0.996	0.990	0.986	0.992	0.984	0.900
Average error. %	24.19	17.4	19.49	8.24	16.38	7.86	7.84	15.12
Average flow velocity at 1 cm near bottom layer								
U_0. m s^{-1}	0.770	0.772	0.770	0.772	0.773	0.767	0.53	–*
k_1. m^{-1} s^{2}	2.8	5.7	18.0	4.0	5.0	8.0	6.2	–*
k_2. m^{-1} s^{2}	115	138	440	69	132	410	29	17
A	4	4	4	4	4	4	4	4
B	14	14	14	14	14	14	2	2
C	–4	–4	–4	–4	–4	–4	–4	–4
R^2	0.980	0.984	0.994	0.991	0.994	0.996	0.981	0.898
Average error %	26.52	22.27	19.98	10.88	17.56	21.10	7.47	26.52

* The tests were carried out under flow velocities which were much greater than u_0.

The erodibility k_{1r} works if flow velocity in the standard layer is less than 0.4 u_0. The logistic expression in the first rectangular brackets in equation (5) turn to zero if flow velocity $\geq u_0$. The value of a equals 4 for the upland flows as it is shown above. The coefficient b depends on the dispersion tensile strength of soil particles or aggregates. Coefficient b equals 14 for soils of uniform aggregate size. The coefficient becomes 2 for ploughed and undisturbed soil. The threshold velocity (u_0) can be calculated taking c and d from Fig. 2:

$$u_0 = \sqrt[3]{\frac{c+d}{2}} \tag{6}$$

The average flow velocity (u_s) in the near bed layer of 1 cm may be estimated by the equation of Izbashas & Haldre (1959) which is proposed for rough bed flows. It has the form:

$$u_s = u\,D^{-1.333} \tag{7}$$

where D is the flow depth (cm); u is the average flow velocity (m s^{-1}).

Within the framework of this paper it is impossible to give the exhaustive answer on the question, why the cohesion forces which are initially of order of K_p, under the energy of water flow decrease so greatly that the maximum values of the fluctuating bed shear stresses which are of the order of the first 100 Pa (Nearing *et al.*, 1991) attain the capacity to produce soil detachment. Probably this may be conditioned by two causes: The first one is believed to be due to the weakening of the cohesion forces of soil particles and due to the hydration of soil contacted water flow and ions exchange between the surface layer of soil and water. This is in conformity with the fact that the slow process of soil detachment takes place under flow velocities below the critical value. It may be possible that in that case the cohesive forces in the upper layer of soil become negligibly small and the water flow acts only against the force of gravity.

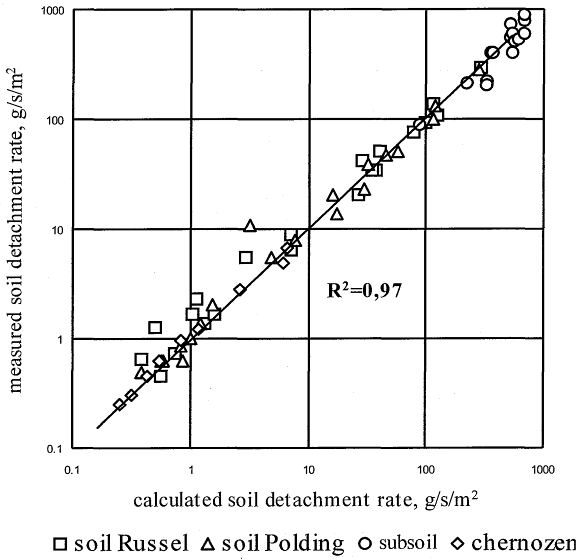

Fig. 2 Predicted detachment rates using equation (5) *vs* measured detachment rate.

If flow velocity is strong enough to impart to soil particles protruded into water reciprocal movement (trembling) the fatigue-limit of soil particle can be overcome by pulsating shear stresses which magnitudes are much less than tensile strength. Mirtskhulava (1970) was the first who emphasized the role of fatigue destroying in the process of soil detaching.

Influence of bed load and suspended sediment on soil detachment

The simple mathematical construction based on the third premise shows that bed load should decrease the detachment capacity of water flow. Furthermore the magnitude of influence is strongly affected by soil aggregate stability. The best known erosion models take the influence of sediment load on the process of soil detachment into account. According to Foster (1982) the detachment of soil particles and its further transportation are produced by the free energy of water flow. The quantity of energy required for transportation of sediment is much less than for detachment. Thereafter the free energy of water flow is used in the first place on sediment transport. Rose (1985) suggests that the main reason of the decrease of the soil detachment rate along the rill is due to deposition the flow bed by the settled soil particles. But Merten *et al.* (2001) have shown experimentally that the magnitude of sediment which influences the soil detachment is less than predicted by Foster. With regard to the suggestion of Rose (1985) the sediment layer at the bed of rills is usually absent. Thus it is widely accepted that sediment influences soil detachment but there is no common opinion on the reason and mechanisms of this process.

Experiments were carried out in order to understand and quantify the influence of bed load and suspended sediment on soil detachment. The circular type of flume was used for this study. Their length and width are 2069 cm and 20 cm, respectively. The volume of the water circulated in the flume is approximately 40 l. Pieces of porolon (5 × 5 × 5 mm), rubber (3 × 3 × 3 mm) and rubber coated copper conductor (3 mm in diameter, 3.5–4.5 mm in length) were used as soft bed load. Gravel of 1–2 mm size was used as material for the hard bed load. The suspended sediment was imitated by silty loam. The soil used in this study for the detachment tests, is a loamy chernozem dried and sieved to obtain the 1.5–2 mm size fraction. Pre-wetted soils for the detachment tests were formed in containers of 7–2 cm cross section under constant pressure to obtain the desired density (1.1–1.3 g cm^{-3} in bed load experiments and 1.1; 1.2 and 1.3 g cm^{-3} in suspended sediment experiments). The samples of soil were placed in the pan with thin (approximately 0.5 cm) layer of water to saturate the soil from the bottom. The saturated samples were placed into the chamber for 10–12 h before testing.

The sample was mounted in the flume so that the surface of the sample was flash with the flume bed. The sample squeezed out from the container during the experiment so that the soil surface remained at the level of the flow bed. Each treatment was replicated 5–10 times.

The results of the bed load experiments indicate that the influence of the soft bed load particles and gravel are absolutely different. The bed load produced from soft material significantly decreases the soil detachment rate. The gravel particles strongly increase the soil detachment. The influence of bed load on the soil detachment rate can be expressed by an exponential function

$$D_{rl} = D_r e^{ax_h + bx_s} \tag{8}$$

where D_{rl} is the soil detachment rate produced by the sediment loaded water flow; D_r is the soil detachment rate produced by clear flow; a and b are the coefficients which describe the influence the soft and hard bed load the soil detachment; x_h and x_s are the amounts of hard and soft particles transported as bed load above the flow bottom, units bed load particles over 1 m^2 of flow bed. Equation (8) indicates satisfactory results (Fig. 3). The magnitude of coefficients is shown in Table 2. The influence of hard particles on soil detachment was justified by the corrosion effect. This confirmed by multitude elongated craters which can be easily seen by the naked eye on the surface of the treated soil sample. The reason for the influence of soft bed load particles on the soil detachment process is not so clear. Two suggestions can be made. On the one hand the soft particles protect the soil from scouring of water flow at the point where particles contact the soil surface. On the other hand bed load reduces the turbulence of water flow and thus diminishes the magnitude of instantaneous shear stresses. The experiments can not give clear answers to this problem.

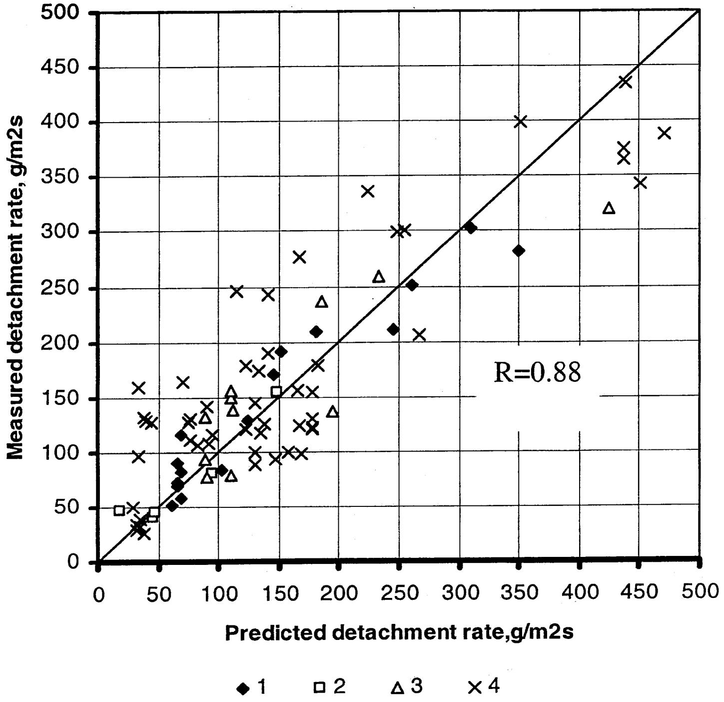

Fig. 3 Soil detachment rate predicted by equation (8) *vs* measured soil detachment rate. Bed load material: (1) porolon; (2) rubber; (3) rubber coated conductor; (4) quartz gravel.

Table 2 Influence of bed load on soil detachment.

Material	Bulk density, g cm^{-3}	Coefficient of Eq. (8)
Porolon	≈1.00	–0.00063
Rubber	1.21	–0.00048
Rubber coated conductor	3.36	–0.00016
Quartz gravel	2.65	0.00034

Table 3 Influence of suspended sediment on the soil erodibility.

Bulk density, g cm^{-3}	1.1				1.2				1.3			
Sediment concentration, g l^{-1}	0	6	12	24	0	6	12	24	0	6	12	24
Without infiltration												
Erodibility	243.1	135.4	108.1	–	123.5	74.5	67.8	–	10.8	11.9	5.3	–
Infiltration takes place												
Erodibility	370.8	85.38	58.2	57.1	126.1	43.3	25.9	17.8	15.4	7.6	3.9	1.8

The suspended sediment leads to an apparent decrease of the soil detachment rate (Table 3) which is believed to be due to the siltation of inter-aggregate pores. The light mass of sediment particles between dark aggregates can be seen easily on the break of treated samples. The soil samples with predominant point contact connections between aggregates turn into the more homogeneous soil body wherein tensile strength is sufficiently higher compared to the same soil with unsilted pore space. Thereafter the erodibility increases during the process of pore siltation. The degree of pore space siltation depends on the sediment concentration and the water infiltration rate. It explains why the decrease of detachment is higher in the tests that conducted under free infiltration of water and relatively high sediment concentration.

CONCLUSION

The proposed soil detachment model based on the three self evident premises show that soil detachment is proportional to cubed average flow velocity if it meets the condition $u \geq 1.6u_0$ and the probabilistic function is in the domain where flow velocities ranged from $u = 0.4u_0$ up to $u = 1.6u_0$. If flow velocity is less than $0.4u_0$ the rate of soil detachment is small, but proportional to the cubed velocity of flow too. The model explains well existing literature data of soil detachment..

The bed load effects strongly on the soil detachment rate but the mechanics of influence of soft bed load material is not yet clear. The suspended sediment decreases the soil erodibility due to siltation of the interaggregate porous space. The study should contribute to a better understanding of the mechanisms of the influence of sediment on soil detachment and a further quantification of this process.

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REFERENCES

Gendugov, V. M., Kuznetsov, M. S., Khalilov, M. S. & Ivanyta, C. C. (1997) Novyi podhod k ocenke vliyaniya erozii na pochvy (A new approach to evaluation of flow erosion effect on soil). *Vestnik Mosk. Un-ta. Cer. 17. Pochvovedenie* **2**, 37–41 (in Russian).

Foster, G. R. (1982) Modeling the erosion process. In: *Hydrologic Modeling of Small Watersheds* (ed. by C. T. Haan *et al.*). *ASAE Monogr.* **5**, 296–380.

Izbash, S. V. & Haldre, Kh. Y. (1959) *Gidravlika perekrytiya rusel rek* (The hydraulics of river damming). Gosenergoizdat, Moskva-Leningrad, USSR (in Russian).

Hairsine, P. B. & Rose, C. W. (1992) Modeling water erosion due to overland flow using physical principles: rill flow. *Water Resour. Res.* **28**, 245–250.

Kuznetsov, M. S. & Glazunov, G. P. (1985) *Eroziya pochv* (Soil Erosion). Izdatel'stvo Mosk. Universiteta, USSR (in Russian).

Larionov, G. A. & Krasnov, S. F. (1997) Gydrofizicheskaya koncepciya erozii pochv (A hydrophysical concept of soil erosion). *Pochvovedenie* **5**, 85–93.

Larionov, G. A., Dobrovol'skaya, N. G., Krasnov, S. F. & Yuan, Lu B. (2003) Novoe uravnenie factora rel'efa dlya statisticheskih modelei vodnoi erozii (New equation of relief factor for statistic models of water erosion). *Pochvovedenie* **10**, 1239–1247 (in Russian).

Merten, G. H., Nearing, M. A. & Borges, A. L. O. (2001) Effect of sediment load on soil detachment and deposition in rills. *J. Soil Sci. Am.* **65**, 861–868.

Mirtskhulava, Ts. E. (1967) *Ochistka kanalov i metod ocenki ikh stabil'nosti* (Scouring of channels and method of evaluation of its stability). Moscow, Kolos, USSR (in Russian).

Mirtskhulava, Ts. E. (1970) *Inzhenernye metody rascheta i prognoza vodnoi erozii* (Engineering Methods of Soil Loss Prediction). Gidrometeoizdat, Moscow, USSR (in Russian).

Nearing, M. A. (1991) A probabilistic model of soil detachment by shallow turbulent flow. *Trans. of ASAE.* v. 34, **1**. 81–85.

Nearing, M. A., Bradford, J. M. & Parcker, S. C. (1991) Soil detachment by shallow flow at low slopes. *Soil Sci. Soc. Am. J.* **55**. 339–344.

Rose, C. W. (1985) Development of soil erosion and deposition models. *Adv. Soil Sci.* **2**, 1–163.