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A particle tracking method to simulate sediment transfer over flood plains

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Abstract Deposition of contaminated sediments during high discharges is an important aspect on lowland river flood plains. Quantification of deposition often proceeds through modelling. However, raster-based sedimentation models in particular suffer from numerical dispersion in the sediment-transfer routine. To simulate conveyance of sediments over flood plains without introducing numerical dispersion, we developed a sediment-transfer model in which we implemented the particle tracking method "Method of Characteristics". This raster-based model uses input data such as initial concentrations, water levels, flow velocities and dispersion coefficients. In the future, we will use this model as part of a flood plain sedimentation model.

Key words flood plain; Method of Characteristics; modelling; numerical dispersion; particle tracking; Rhine River; sedimentation; suspended sediment

INTRODUCTION

Flood plains sequester large amounts of sediments and associated pollutants (e.g. Walling *et al.*, 1998; Middelkoop & Asselman, 1998; Owens *et al.*, 1999). To simulate the deposition patterns of these sediment-associated pollutants, several workers have developed sedimentation models. Some of these models have been based on a finite-element approach (e.g. Stewart *et al.*, 1998; Siggers *et al.*, 1999; Hardy *et al.*, 2000); other models have been based on a finite-difference approach (e.g. Nicholas & Walling, 1997, 1998; Middelkoop & Van der Perk, 1998). On the one hand, the finite-element models have the advantage of employing a mesh that can be optimized for local situations (i.e. a finer mesh in environments with higher gradients and *vice versa*) so as to minimize numerical errors. On the other hand, these models have the disadvantage that their output cannot be imported directly into a raster GIS for further processing, whereas this is relatively easy using finite-difference or raster-based models. Raster-based models, however, suffer from numerical dispersion (i.e. dispersion as an unwanted artefact of the numerical modelling technique), which may lead to erroneous results. Here we propose particle tracking as a method to cope with this problem in raster-based modelling.

An example of a particle tracking method is the "Method of Characteristics" (MoC). According to Garder *et al.* (1964) this method minimizes numerical dispersion. Konikow & Bredehoeft (1978) used MoC to simulate solute transport in groundwater. We modified MoC as used by Konikow & Bredehoeft (1978) to be applicable for surface water and implemented it in PCRaster (Wesseling *et al.*, 1996), a tool that aids in numerical modelling of environmental processes using rasterized input maps.

The purpose of this paper is to describe the modified MoC model, discuss the input data needed and give an example of the output. A stand-alone hydrodynamic model calculated the input data—such as water levels, x- and y-flow velocities—and we used empirical formulae to calculate dispersion coefficients.

MODEL CONCEPT

Flow equation

Konikow & Bredehoeft (1978) have developed a complete groundwater model that solves both the flow and transport equation. We, however, only considered the transport equation in their model, since we derived stationary flow fields from another model. The adapted transport equation is:

$$\frac{\partial(Ch)}{\partial t} = \frac{\partial}{\partial x_i}\left(hE\frac{\partial C}{\partial x_j}\right) - \frac{\partial}{\partial x}(hCu) - C'S \tag{1}$$

where C is suspended sediment concentration (SSC) (mg l^{-1}), t is time (s), x_i and x_j are Cartesian coordinates (m), h is water depth (m), E is hydrodynamic dispersion coefficient (m^2 s^{-1}), u is flow velocity (m s^{-1}), C' is SSC in a source or sink (mg l^{-1}), S is volume flux per unit surface area in a source or sink (m s^{-1}). The first term on the right-hand side is the dispersion term, the second one the convection term and the third one the source/sink term.

Konikow & Bredehoeft (1978) assumed that convective transport (transport of particles and solutes by flowing water) dominates solute transport and used a grid-based model. Under these circumstances, it is possible to define concentration changes for flowing reference particles that pass fixed nodes, rather than the conventional way of defining these changes for fixed reference nodes passed by flowing particles. MoC thus solves the transport equation taking flow lines as a basis instead of grid cells. Garder *et al.* (1964) called these flow lines "characteristic curves", hence the name Method of Characteristics. With this approach, equation (1) can be reduced to the following form:

$$\Delta C = \Delta t\left[\frac{1}{h}\frac{\partial}{\partial x_i}\left(hE\frac{\partial C}{\partial x_j}\right) + \frac{CS - C'S}{h}\right] \tag{2}$$

Note that the convection term in equation (1) has disappeared in equation (2).

Particle tracking

The first step in the calculation procedure involves placing a number of particles in each cell of the model grid (Fig. 1). All particles inherit the initial concentration from the grid cells in the input map. During each subsequent time step, all particles move a distance of, at most, one grid cell according to their characteristic curves (the curved lines in Fig. 1). After each time step, the model calculates a temporal average concentration C^* for each node on the basis of the new spatial arrangement of particles. Then, the change in concentration (ΔC) due to hydrodynamic or mechanical dispersion, divergence of velocity, sources/sinks and change in water depth is calculated using equation (2). The present version of the model does not yet

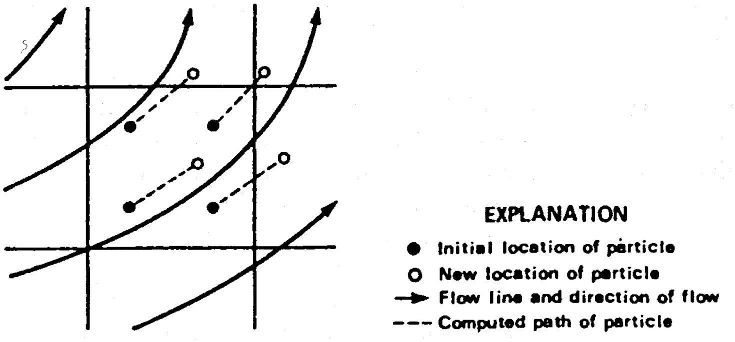

Fig. 1 The location of the particles in the raster cells and subsequent displacement by the model. Source: Konikow & Bredehoeft (1978).

consider deposition or resuspension of sediment, so the resulting concentration ($C^* \pm \Delta C$) is the new concentration at each node for the next time step. At the end of each time step, PCRaster provides the user with maps of this concentration.

Stability criteria and boundary conditions

The model treats the user-defined time step as the maximum time increment. If necessary, the model divides this time increment into smaller ones to ensure that particles cannot move more than one cell length at a time. The model bases this division on four stability criteria, involving the dispersivity values, water depth and flux, and flow velocities in the *x*- and *y*-direction, respectively.

The model uses two types of boundaries: no-flow boundaries and flux boundaries. The user has to define no-flow boundaries at the locations of river dikes or around high terrains in the flood plains that remain dry during high discharges. Next, the upstream and downstream boundaries of the study area have to be flux boundaries, with the flux being either constant or transient. The model considers upstream boundary cells as sources, while downstream boundary cells are considered to be sinks.

Model input

The model uses the following data as input maps: initial suspended sediment concentrations, flow velocities in *x*- and *y*-direction, water depth, flux, longitudinal and transverse dispersion coefficients. The size of the grid cell is a property of the input maps. The user also has to define the number of particles per cell (four, five or nine, while these numbers provide, at least for the first time step, a regular coverage of the grid cells) and the length of the time step.

EXAMPLE CALCULATION

We simulated the sediment dispersion for a river reach of the River Rhine during a period of high discharge that occurred in spring 2002, with a peak discharge of 8054 m^3 s^{-1} at the

Dutch-German border. In this simulation we used input maps with cell sizes of 50 × 50 m. We derived the input data in the following ways:

(a) Initial suspended sediment concentration, C' (mg l^{-1}): the *Infocentrum Binnenwateren* in Lelystad daily measures the *SSC* at Lobith (at the Dutch-German border). As initial *SSC* for all the flood plains in the model area we used the concentration at Lobith on the first day of the inundation: 30 mg l^{-1}. For the river channel, we adopted the average of the first five days of the inundation period, i.e., 53 mg l^{-1}.

(b) Water depth, h (m): we derived the water depth by subtracting the surface elevation from the water level. To define the surface elevation we constructed a Digital Elevation Model (DEM) from the base data provided by the *Adviesdienst voor Geo-informatie & ICT* (RWS-AGI) in Delft. RIZA-WST in Dordrecht calculated the water depth using the hydrodynamic model WAQUA (MX.Systems 2003). WAQUA solves the Saint-Venant equations in a curved grid, using as boundary conditions the river discharge at the upstream end and the water level at the downstream end of the modelled river stretch.

(c) Flow velocity, u (m s^{-1}): WAQUA provided the flow velocities in x- and y-direction (Fig. 2). From these values, we calculated the absolute flow velocities, which we later used in the calculation of the dispersion coefficients.

(d) Flux, S (m s^{-1}): we recalculated the river discharge entering and leaving the model area to a flux by dividing it by the cell area of the PCRaster grid.

(e) Transverse dispersion coefficient, E_T (m^2 s^{-1}): we used an equation proposed by Holley & Abrahams (1973):

$$E_T = c \cdot h \cdot u \tag{3}$$

(f) where E_T is the transverse dispersion coefficient (m^2 s^{-1}), c is a constant (–), h is the water depth (m) and u is the absolute velocity (m s^{-1}). Using tracer experiments in the model area on the Waal River, Holley & Abrahams (1973) found an approximate value of 0.03 for c. Since there are no data available for flood plains, we assumed this value to be valid for both the river channel and its flood plains. Still, the different character of the flood plains finds expression in lower E_T values because of their lower water depths and flow velocities.

(g) Longitudinal dispersion coefficient, E_L (m^2 s^{-1}): we used a formula provided by Fischer *et al.* (1985):

$$E_L = 0.011 \cdot u^2 \cdot w^2 / h \cdot u_* \tag{4}$$

where E_L is longitudinal dispersion coefficient (m^2 s^{-1}), 0.011 is an empirical constant, w is cell length (m) and u_* is shear velocity (m s^{-1}).

Model output

Figure 3 shows the model output for the middle reach of the Waal River, a distributary of the River Rhine. The flow is from right to left. The flood plain in the northeast of the model area receives a lot of suspended sediment, whereas the one in the south-central part receives far less because non-inundated areas block the inflow of river water. In the case of the flood plain in the northwest, the river water apparently cannot reach the far ends of it, leading to a pronounced north–south gradient in *SSC*.

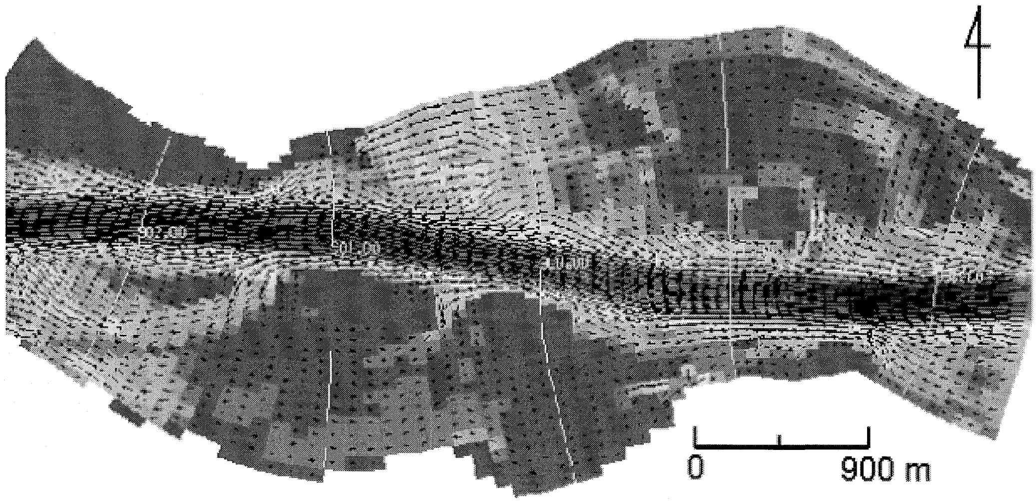

Fig. 2 Average flow velocities and flow directions in the study area as modelled by WAQUA. The length of the arrows indicates the relative size of the velocity. The lighter the colour, the higher the flow velocity. The channel is however dark due to the high number of arrows.

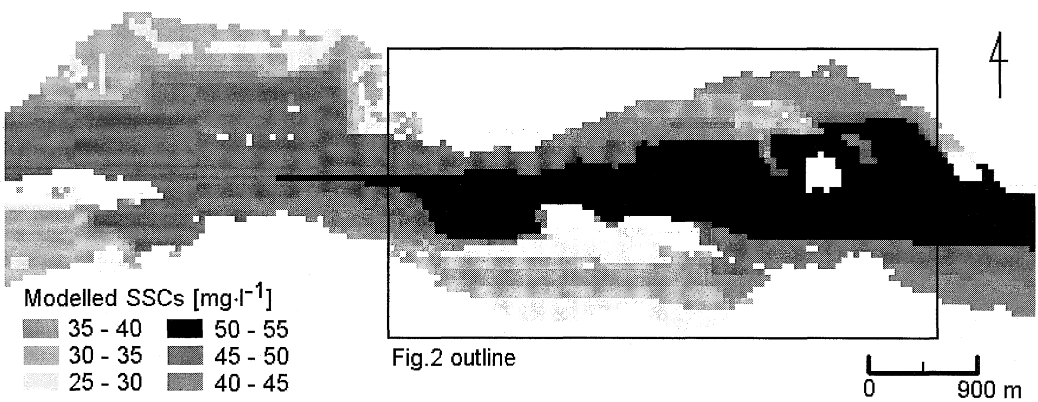

Fig. 3 Modelled suspended sediment concentrations for the middle reach of the Waal River. Flow is from right to left. The box denotes the area in Fig. 2.

DISCUSSION AND PERSPECTIVE

The implementation of the Method of Characteristics in a 2-D environmental modelling tool is promising and can aid in the prediction of the dispersion of contaminated sediments over flood plains. In the future, we will extend the model with a sedimentation module. We will calibrate this module with settling velocities reported in Thonon & Van der Perk (2003) and sedimentation data given in Middelkoop *et al.* (2003) and Thonon (2003).

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