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Bayesian kriging and GLUE applied to estimation of uncertainty due to precipitation representation in hydrological modelling

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Abstract Hydrological modelling is subject to a set of well-known limitations, though these are rarely explicitly considered through an uncertainty analysis related to the results obtained. This work intends to emphasize the role of uncertainty analysis due to precipitation fields as applied to watershed flow simulation. For this purpose, after explaining the problem in detail, a methodology for uncertainty assessment through the Monte Carlo method is proposed, based on an estimation method called Generalized Likelihood Uncertainty Estimation and on Bayesian kriging. Emphasis is placed on precipitation fields and the effects that gauge network simplifications have on the output of a rainfall–runoff model. The hydrological model TOPMODEL was adopted for this analysis. The impact of uncertainty is evaluated through the analysis of the behaviour of the Iguaçu River watershed, located in the state of Rio de Janeiro, Brazil. The results reinforce the fact that a poor representation of precipitation fields in hydrological models is a considerable source of uncertainty.

Key words Bayesian kriging; precipitation estimation; rainfall–runoff models; uncertainty analysis

INTRODUCTION

The way hydrological models, whether conceptual or physically-based, represent natural processes at the watershed scale, leads to a considerable amount of uncertainty caused by many sources such as data acquisition, calibration issues and imperfect mathematical representation of these processes, as discussed by Rotunno (1995) and Beven & Freer (2001) among others. It could be said that the very nature of hydrological modelling is inherently imperfect, which leads to different levels of uncertainty whose magnitude cannot be easily estimated. The vast field in which hydrological models play a significant role makes the development of methods to estimate these uncertainties a very important task, so that these models can be used with a greater degree of confidence.

As proposed by Beven & Freer (2001), the Generalized Likelihood Uncertainty Estimation (GLUE) methodology offers a simple way to estimate the uncertainty in hydrological model output, by selecting a set of simulators of the watershed response that can be accepted as behavioural. The concept of behavioural simulators originates in the paradigm of equifinality: due to the inherent difficulties concerning the search of an optimal parameter set of a model, it is often only possible to find a number of parameter sets which simulate the system behavior equally well according to a quantitative measure of goodness of fit that must be defined *a priori*.

This work presents a methodology wherein the GLUE framework was used for the estimation of the hydrological model total uncertainty that comes from the approximation of a precipitation field from different configurations of a raingauge network. Given a number of raingauge measures, the precipitation field can be approximated using a geostatistical approach through the Bayesian kriging methodology, which combines the point measurements given by raingauges with the knowledge of the spatial precipitation structure, which can be estimated with remotely sensed images (e.g. radar images). It could be expected that the greater the number of these measures, the better the approximation of the real precipitation field would be, and thus the less significant the uncertainty contribution caused by the imperfect knowledge of the real precipitation field.

The framework proposed in this paper is illustrated with a case study which, although it uses synthetic rain data, generated by the turning bands method proposed by Mantoglou & Wilson (1982), demonstrates that it can be a useful tool to the estimation of the uncertainties in hydrological modelling caused by an approximate representation of precipitation. This procedure can also be used in the design of a new raingauge network, and for the evaluation of an existing one, a process whose main criterion should be the estimation of the approximated field such that an acceptable degree of uncertainty would be transferred to the model output if hydrological predictions are the main objective.

GEOSTATISTICS APPLIED TO PRECIPITATON FIELDS

According to Matheron (1962), geostatistics is the application of the formalism of random functions to the reconnaissance and estimation of natural phenomena. A natural phenomenon can often be characterized by the distribution in space of one or more variables, called regionalized variables.

The classical estimator of the semivariogram based on sampled data is presented by Journel & Huijbregts (1978) as follows:

$$\gamma^{-}(h) = \frac{1}{N(h)} \sum_{i=1}^{N(h)} \left[z(x_i) - z(x_i + h) \right]^2 \tag{1}$$

where $N(h)$ is the number of pairs of measurements separated by the distance h, $z(x_i)$ the measurement at point x_i, and $z(x_i+h)$ the measurement at point x_i+h.

As previously mentioned, this research work intends to pursue the geostatistical approach applied to the generation of precipitation fields. Recently developed methodologies in the geostatistical research area, such as Bayesian kriging, offer the possibility of integrating raingauge data (ground-truth) with structural correlation functions in a rigorous manner.

In this section, the Bayesian kriging methodology, introduced by Omre (1987) and used for example by Nobre (1992) and by Rotunno (1995), is presented as a new approach for fulfilling the objective of this paper, that is, the uncertainty assessment of rainfall–runoff models due to errors in precipitation field mapping at the watershed scale.

Bayesian Kriging Model

In the present research work, the precipitation information $z(x)$ within the domain at location x is assumed to be a realization of a stochastic function $Z(x)$. Soft data or subjective information is $m(x)$, whose underlying random function is $M(x)$. Its first two moments are assumed to be known *a priori* as $E[M(x)] = \mu_M(x)$ and $Cov[M(x+h), M(x)] = C_M(x+h,x)$, where h is the lag distance between two points.

As the covariance may be dependent on both support points $x + h$ and x, the corresponding variogram function for the soft data is defined as:

$$\gamma_M(x+h,x) = \frac{1}{2}[C_M(x,x) + C_M(x+h,x+h)] - C_M(x+h,x) \tag{2}$$

The expression for the conditional variogram of $Z(x)$ is:

$$Var\,[(Z(x+h) - Z(x)) \mid M(x)] = 2\gamma_{Z|M}(h) \tag{3}$$

Assuming $\{Z(x_i); i = 1,2,..., N\}$ is the set of observation, the Bayesian kriging model estimates the value at a point x_0 employing a linear estimator of the form:

$$Z^*(x_0) = \sum_{i=1}^{N} \lambda_i Z^T(x_i) + \mu_M(x_0) \tag{4}$$

where λ_i is a set of constant weights to be determined and $Z^T(x_i)$ is defined as:

$$Z^T(x_i) = Z(x_i) - \mu_M(x_i) \tag{5}$$

By applying the Lagrange multiplier technique, satisfying the non-bias condition and the minimum estimation variance, the following Bayesian kriging system is obtained:

$$\sum_{i=1}^{N} \lambda_i \left[\gamma_{Z|M}(x_i, x_j) + \gamma_M(x_i, x_j)\right] + \beta = \gamma_{Z|M}(x_0, x_j) + \gamma_M(x_0, x_j) \tag{6}$$

$$\sum_{i=1}^{N} = 1 \ \text{ with } j = 1,2, ..., N$$

where β is a Lagrange multiplier.

The error in the estimate at point x_0 is given by:

$$\sigma^2(x_0) = \sum_{i=1}^{N} \lambda_i \left[\gamma_{Z|M}(x_0 - x_i) + \gamma_M(x_0, x_i)\right] - \beta \tag{7}$$

According to Omre (1987), the estimation variance is ensured to be non-negative if both $\gamma_{Z|M}(.)$ and $\gamma_M(.,.)$ are conditionally positive definite functions. Therefore, $\gamma_{Z|M}(.)$

may be chosen from the class of variogram functions usually applied in Geostatistics and $\gamma_M(.,.)$ should be chosen from a larger family of functions. Omre (1987) suggests that an unbiased estimator for $\gamma_{Z|M}(h)$ for all lag distances h is:

$$\bar{\gamma}_{Z|M}(h) = \frac{1}{2N_h}\sum_{i=1}^{N_h}[(Z(x_i+h) - Z(x_i))^2 - (\mu_M(x_i+h) - \mu_M(x_i))^2 - 2\gamma_M(x_i+h, x_i)] \qquad (8)$$

GLUE METHODOLOGY

The Generalized Likelihood Uncertainty Estimation (GLUE) which was discussed and applied by Beven & Freer (2001) is a simple methodology to assess the total uncertainty of a model response. It is based on the *equifinality paradigm* as discussed earlier. In summary, it is assumed that it is not possible to find an optimal parameter set, but only a population of parameter sets equally considered as good (behavioural) simulators of the corresponding system.

The GLUE methodology consists in the search for these behavioural parameter sets. As a result of the application of GLUE, one can, after determining these behavioural sets, weight the predictions by the likelihood related to the parameter set that produces them. These simulations constitute a range of possible discharge values that are likely to occur, considering the input data and the initial conditions. The greater these ranges, the greater the uncertainty in the model response. GLUE is based on the following steps:

(a) definition of a likelihood function L and a threshold value L^*;
(b) generation of a set of N independent parameter sets $\theta = [\theta_1, \theta_2, \theta_3, ..., \theta_N]$ through sampling of the feasible parameter space;
(c) evaluation of the degree of acceptance of each parameter set considering the likelihood function L;
(d) choice of the behavioural N_B parameter sets taking into account the threshold value L^*;
(e) with the N_B behavioural parameter sets, constructing the likelihood distribution $L(M[\theta_i])$, which can be used to establish the range of behavioural model simulations.

As stressed by Beven & Freer (2001), the choice of the likelihood function and the acceptance criterion are subjective, and should be subjected to further discussion before its use within the GLUE methodology. The results will be conditioned by these choices, and also by the input data and the model structure.

CASE STUDY

The main idea of this study is to assess the impact of increasingly sparse representations of the original precipitation field, which was synthetically generated by the turning bands method. TOPMODEL, described in details by Beven *et al.* (1995), has been used to simulate the watershed response. This study has been carried out through the following steps:

(a) a spatial precipitation field was generated using the turning bands method, with a Gaussian structure; this field was considered as the ground-truth precipitation field; a semivariogram model was adjusted for the normalized experimental semivariogram of the generated precipitation field (soft data);
(b) a parameter set was chosen, which jointly with the ground-truth precipitation field, allowed the generation of the ground-truth streamflow record (4 months of hourly data);
(c) a fictitious dense network of 50 randomly distributed gauges was established;
(d) sparser networks with 30, 20 and 10 gauges were derived from this fictitious network; a semivariogram model was then adjusted for each one of the proposed scenarios (hard data); finally, using these semivariogram models jointly with the normalized experimental semivariogram (soft data), the Bayesian kriging method was used to estimate the correspondent precipitation fields;
(e) the GLUE methodology has been applied to each one of the derived precipitation fields; the discharge range was defined as containing 90% of the behavioural discharges.

RESULTS AND DISCUSSION

First, it should be noted that the Bayesian kriging methodology requires a semivariogram model for the normalized experimental semivariogram for the soft data, which in the present approach is assumed to be the spatial precipitation field generated by the turning bands method. In addition, it requires a semivariogram for the hard or ground-truth data, here considered to be the raingauge data. In fact, the study shows that, although not reported here, there is the presence of a correlation structure in the ground-truth raingauge dataset, as well as in the precipitation fields as depicted by the empirical omnidirectional semivariograms and by the empirical semivariograms defined for different directions. Additionally, the covariogram also indicates that, in fact, a relationship between raingauge data (hard data) and the data set provided by the simulated spatial precipitation fields (soft data) does exist. For the purpose of this work, the hypothesis of isotropy and the use of omnidirectional semivariogram were deemed acceptable. Secondly, it should be emphasized that the Bayesian kriging model uses *a priori* information. The raingauge data is used for this task.

Given the spatial precipitation structure, defined by the Bayesian kriging method, the results obtained using the GLUE approach showed that discharge ranges increased with the poorer representation of the ground-truth precipitation field (Figs 1 to 3). According to GLUE, the increase in discharge ranges represents an increase in uncertainty associated to the model response. As presented in Figs 1 to 3, the reduction of the initial sample used to generate the approximated precipitation field produced a greater degree of uncertainty in model response.

It can further be noted that at different time steps the observed discharges were not contained in the calculated ranges, which can indicate, following Beven & Freer (2001),

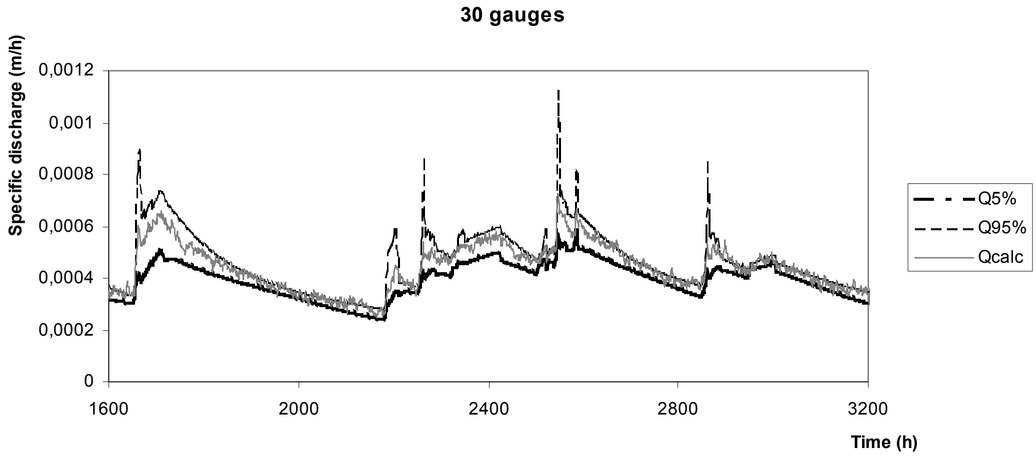

Fig. 1 Range[$Q_{5\%}$;$Q_{95\%}$] obtained for 30 gauges precipitation field.

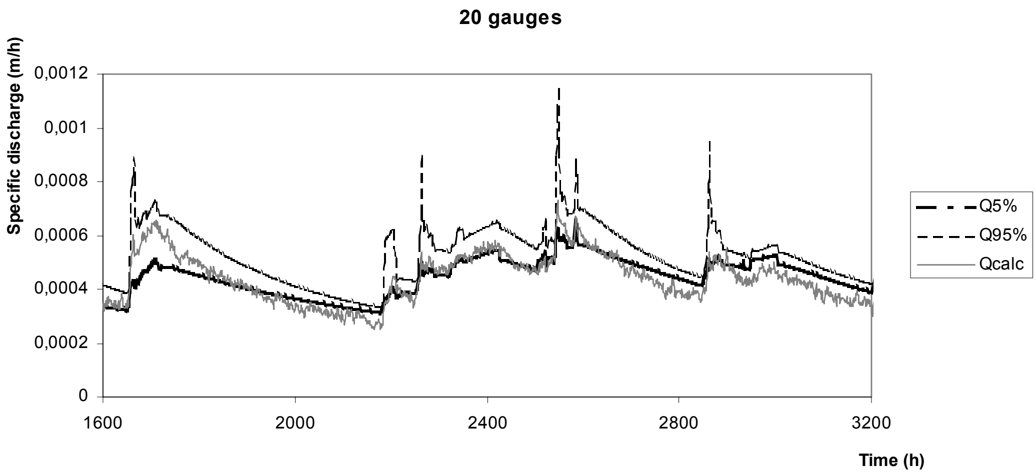

Fig. 2 Range[$Q_{5\%}$;$Q_{95\%}$] obtained for 20 gauges precipitation field.

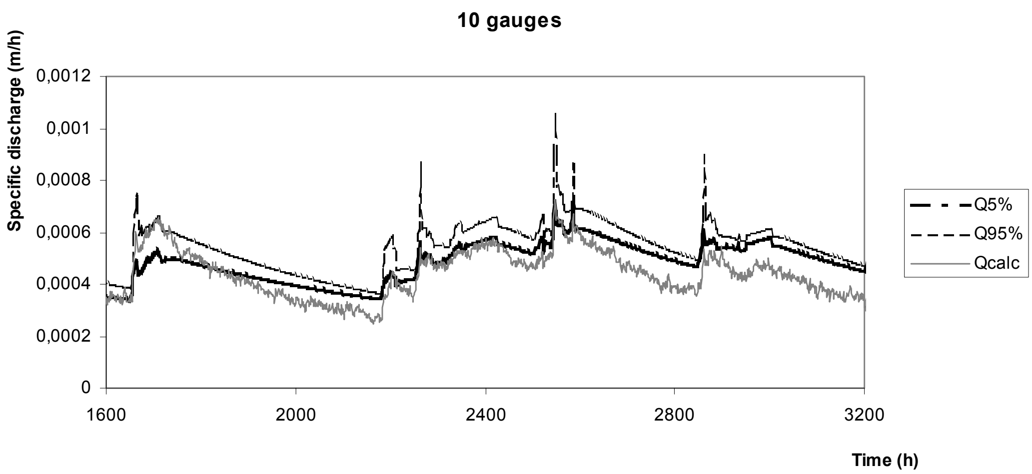

Fig. 3 Range[$Q_{5\%}$;$Q_{95\%}$] obtained for 10 gauges precipitation field.

errors due to structural problems of the model, but also errors in the observations. Moreover, with the reduction of the number of raingauges, an increasing overestimation of the calculated discharges was observed.

Additionally, the GLUE methodology allows performing a sensitivity analysis by means of evaluating the dispersion of the behavioural parameters. As the cumulative distribution function of the behavioural parameters differs from the 45° line, there is an increase in the sensitivity of the model parameter. Figures 4 and 5 indicate that the TOPMODEL presents high sensitivity to the *m*—exponential storage parameter, and low sensitivity to the *TD*—unsaturated zone time delay per unit storage deficit parameter.

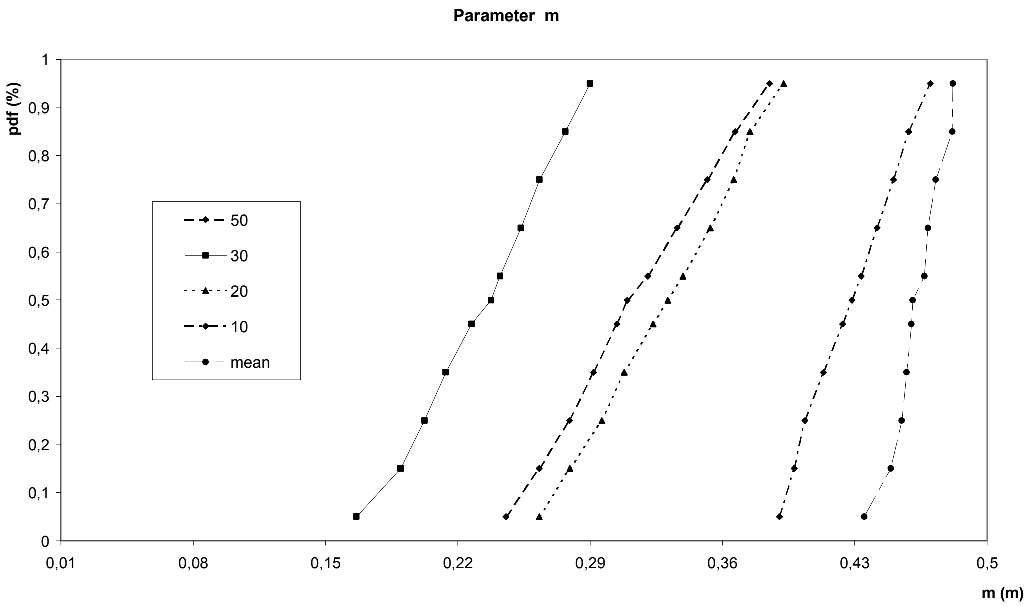

Fig. 4 Cumulative distribution function of parameter *m*.

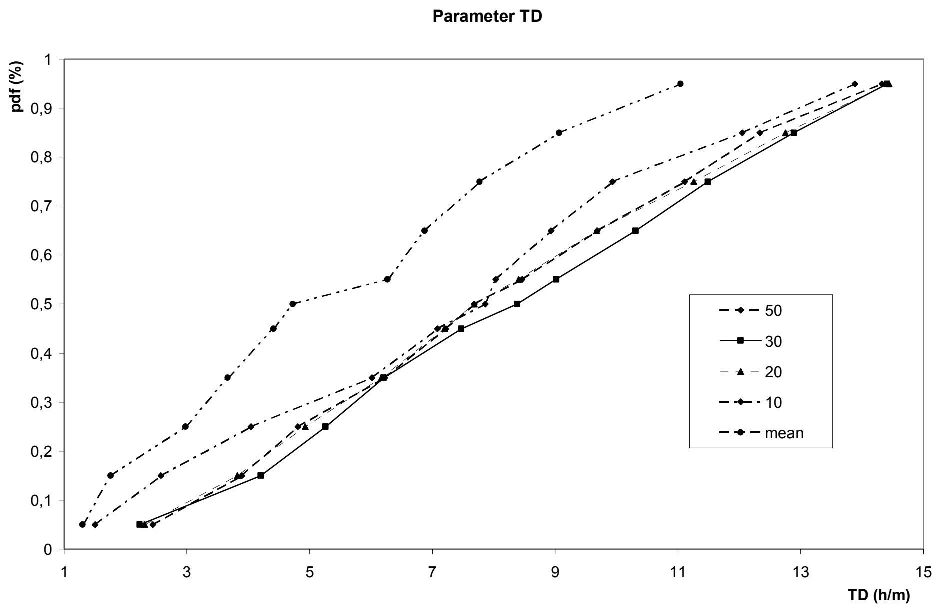

Fig. 5 Cumulative distribution function of parameter *TD*.

It can be noted that Fig. 4 shows a distribution shift in the feasible parameter space as different precipitation fields were used. Poorer representation of precipitation provided expected values of *m* which differ increasingly with respect to the ground-truth value used. This uncertainty caused a significant dispersion of the accepted simulations due to the great sensitivity of TOPMODEL to this parameter. Therefore, this effect can be confirmed by observing the increasing behavioural discharge ranges as presented in Figs 1–3. On the other hand, Fig. 5 depicts the similar behaviour of *TD* parameter pdf curves derived from the different samples due to the low sensitivity of the model output with respect to this parameter. The only exception was the curve obtained from the 30 gauge sample.

The precipitation field generated from the 30 measurements sample has led to a better approximation of the mean precipitation value, which produced simulations closer to the ground-truth streamflows. This fact explains the unexpected result showed in Figs 4 and 5 in the sense that the line for 30 gauges is actually left of those for 20 and 50 gauges. The reduction of the original 50 gauges sample has been done without a rigorous procedure, just selecting same points in a way that the remaining gauges could be reasonably equally distributed over the watershed. Important factors such as the local topography affect the precipitation formation, and therefore should be taken into account when defining the raingauge network.

As a final remark, it should be noticed that the results presented here should not be regarded as conclusive. Much more rigorous conditions (e.g. use of real precipitation and discharge data) have to be adopted to derive definitive conclusions of the impact of uncertain precipitation data on streamflows of a particular watershed. On the other hand, it should be emphasized that the presented framework is of general application, depending neither on a specific methodology to perform an uncertainty assessment nor on a particular hydrological model. The basis of this framework is the Bayesian kriging methodology, which can be applied to any watershed. The required data are streamflows, raingauge data and an estimation of the spatial correlation structure of precipitation distribution over the watershed, which can be obtained through the analysis of remotely sensed data, such as meteorological radar data.

CONCLUSIONS

This paper proposes a framework to analyse the total uncertainty in a model response due to the poor approximation of the precipitation field. Although its application does not involve great complexity, there are some important and not simple tasks that have do be done first, like the estimation of the precipitation semivariogram. After the choice of a hydrological model and a uncertainty assessment methodology, it can be used for evaluate the adequacy of an existing raingauge network, and it can guide the installation of new gauges.

The Bayesian kriging requires a semivariogram model for the normalized experimental semivariogram of the precipitation field which could be derived from satellite image or meteorological radar data (soft data). In addition, it requires a semivariogram for the hard data. From few raingauges, it might be difficult to derive the experimental semivariogram for the precipitation field. In this situation, our suggestion would be to use a normalized semivariogram model based on the spatial soft data. It could be assumed that the normalized semivariogram for the soft data and for the hard data are quite similar. Therefore, the effect would be to denormalize the semivariogram for the soft data based on the actual variance of the problem.

One advantage of this methodology is that, because of the usual scarcity of hard data, it allows the use of soft data, which in the case of meteorological radar are provided in abundance, for defining the structural correlation of the physical variable necessary for modelling or mapping. Another one is the possibility of including the variance maps produced by the Bayesian kriging in the sensitivity analysis through a second order approximation.

Finally, the authors' hope is that this work will bring new perspectives and insights for the assessment of uncertainty in hydrology.

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