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Influence of rating curve uncertainty on daily rainfall–runoff model predictions

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Abstract River discharge observations are usually affected by uncertainty, which is due to many concurrent causes and strongly affects the response of rainfall–runoff models. The present paper is aimed at studying the influence of imperfect rating curve knowledge on the uncertainty of the response of a daily conceptual linear-nonlinear rainfall–runoff model. To describe the impact of imperfect rating curve knowledge, simulations have been conducted using a conceptual rainfall–runoff model and continuous daily series of rainfall, air temperature and discharges recorded in a Sicilian catchment. The GLUE procedure was used to introduce the uncertainty of the rating curve in a classical rainfall–runoff model uncertainty analysis by randomly modifying the original rating curve. The final results show an increase of uncertainty with respect to the original rating curves for the majority of model predictions, but more significantly for overestimated rating curves than underestimated ones.

Key words GLUE; IHACRES; rating curve; rainfall–runoff model; uncertainty

INTRODUCTION

The knowledge of runoff in rivers is an essential requirement for the assessment of surface water resources. Runoff is commonly estimated indirectly by means of a curve relating water level (stage) to discharge. Actually, river discharge observations are never correct. As any other observed variable, they are affected by uncertainty which is due to many concurrent causes: uncertainty in measurements during field campaigns (unsteady flow, material in suspension, disturbance by wind, etc…), hysteresis effects, utilization of a simple form curve with a unique stable relationship between water level h and discharge Q, uncertainty arising from extrapolation of rating curves (Clarke *et al.*, 2000; Kuczera, 1996) and, mainly, complications caused by gradual or abrupt changes in rating curves over time. Now, using discharges calculated by means of a rating curve, for the calibration (validation) of a rainfall–runoff model, preferably includes knowledge regarding how this uncertainty propagates through the model structure and its influence especially on model parameters and an evaluation of model performances (Montanari, 2004). As a matter of fact, quantifying the uncertainty due to gradual or abrupt changes in rating curves over time is not easy; the goal of this study is to explore the possibility of using the GLUE framework to take into account the influence of the uncertainty of the rating curve on the model predictions.

INFLUENCE OF RATING CURVE UNCERTAINTY ON MODEL PREDICTIONS

An historical series of discharge observations is generally calculated by filtering hydrometric heights on a rating curve. For this purpose an "historical" rating curve can be obtained by fitting mean daily discharges and corresponding water levels (river stages), measured at specific river cross section. These empirical points are usually interpolated with a unique two-parameter power law which can cover the entire range of measured variables or with two or more curves each specified for the a particular interval. Uncertainty exists on the discharge values derived rating curve and a way to manage this uncertainty is to consider a "region" of the space (Q-h) in which the historical rating curve and the "uncertain" curves are most likely found (Fig.1(a)). The limits of this region can be seen as uncertainty bounds of the values of the historical rating curve and can be derived within a Monte Carlo framework, as it is rather difficult to know them *a priori* if all sources of uncertainty are considered.

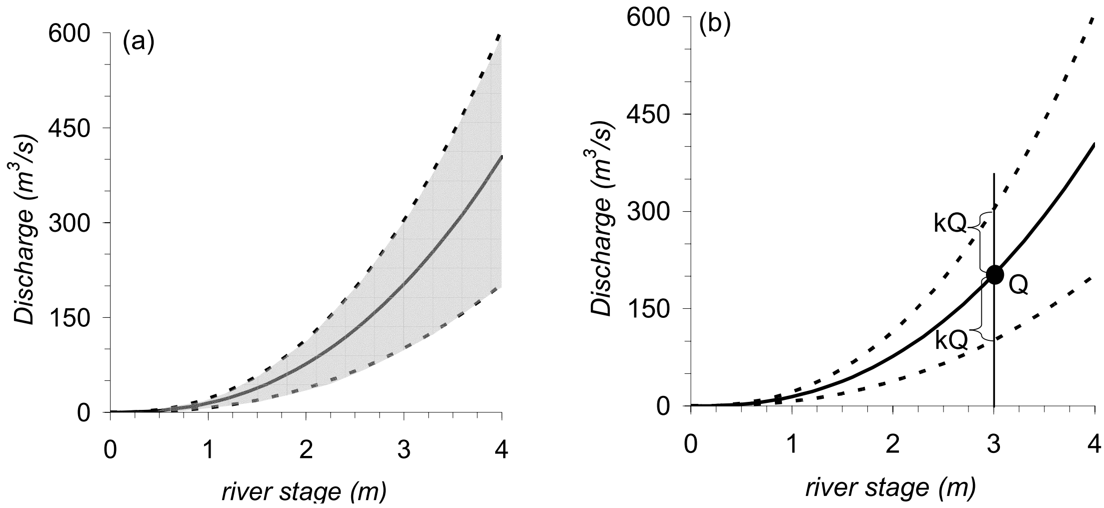

Fig. 1 (a) Uncertainty region for rating curve. (b) Definition of *k* parameter.

Thus, all the curves within this region can be derived in a simple way by perturbing the values of discharge for a given stage:

$$Q_{bounds}(h) = Q(h) + k\,Q(h) \qquad (1)$$

where k is a constant whose value is comprises between $-\infty$ and ∞, which defines the width of the uncertainty region (Fig. 1(b)).

Any new rating curve can be obtained by means of randomly picking values of Q within the region given a probability distribution of those values (i.e. uniform, normal, etc.).

Finally, the calibration of a rainfall–runoff model can be performed using the discharges derived from the different curves including their uncertainty which can be transferred to the model parameters and predictions. As matter of fact, the best way to carry out this analysis is to use a Monte Carlo calibration technique such as GLUE (Beven & Binley, 1992), which was developed as a methodology for the calibration and estimation of uncertainty of predictive models based on the idea of equifinality of models.

APPLICATION TO A CASE STUDY

In order to evaluate the influence of rating curve uncertainty on the hydrological response the proposed method has been applied to the San Leonardo River. The San Leonardo catchment, with an area of 520 km^2, is located in the northwestern part of Sicily, Italy. The catchment has a Mediterranean type climate with hot dry summers and a rainy winter season from October to April. The S. Leonardo catchment receives approximately 900 mm of precipitation annually with a mean annual runoff of 300 mm. This climate type is Dry-Subhumid, according to the Thornthwaite classification. The hydrological response of this basin is dominated by long dry seasons and following wetting-up periods. The catchment can be considered as geologically homogenous, the dominant rock type being clay, siltstone and dolomite with calcareous soils. The indigenous vegetation is characterized by weeds, tree-shrub formations (mainly, olive-trees) and large agricultural areas for wheat cultivation.

The hydrological response of the catchment has been modelled by means of the conceptual rainfall–runoff model IHACRES (Identification of Hydrographs And Components from Rainfall, Evapotranspiration and Streamflow Data) (Littlewood *et al.*, 1997). In IHACRES the rainfall–runoff processes are represented by two modules: (1) a nonlinear loss module transforming precipitation into effective rainfall, and (2) a linear module based on the classical IUH theory to derive the total streamflow.

The nonlinear loss module involves calculation of an index of catchment storage *s(t)* for time step *t*, based upon an exponentially decreasing weighting of precipitation and temperature conditions:

$$s(t) = \frac{r(t)}{c} + \left(1 - \frac{1}{\tau_w[T(t)]}\right) \cdot z^{-1} \cdot s(t) \tag{2}$$

$$\tau_w[T(t)] = \tau_w e^{[0.062 \cdot f \cdot (20 - T(t))]} \tag{3}$$

where $s(t)$ is the catchment storage index varying from 0 to 1, $\tau_w[T(t)]$ is a variable controlling the rate at which the catchment wetness index $s(t)$ decays in the absence of rainfall, τ_w is the value of $\tau_w[T(t)]$ at $T = 20$°C, c is a parameter chosen to constrain the volume of effective rainfall to equal runoff, f is a temperature modulation factor on the rate of temperature, z^{-1} is the backward shift operator. The effective rainfall $u(t)$ is computed as the product of total rainfall $r(t)$ and the storage index $s(t)$:

$$u(t) = s(t) \cdot r(t) \tag{4}$$

In this study, following Ye *et al.* (1997), two extra parameters l (initial water-holding storage) and p (exponent of a power-law) were used to generate $u(t)$ to take in account the strong nonlinearity caused by the impact of long dry periods on the soil surface of low-yielding ephemeral streams:

$$u(t) = [s(t) - 1]^{p} \cdot r(t) \qquad \text{if} \quad s(t) > 1$$
$$u(t) = 0 \qquad \text{otherwise} \tag{5}$$

The linear convolution of net rainfall with the total unit hydrograph is allowed to be for three parallel elements: a linear channel and two linear reservoirs each describing the three main runoff components: surface flow, subsurface flow and baseflow. The form of the impulse response derived from the combination of these two linear elements has been expressed as:

$$\left.\begin{array}{l} h(t) = x_0 \cdot \delta(t) + x_s \cdot \exp(-\lambda_s t) + x_q \cdot \exp(-\lambda_q t) \\ \text{with:} \\ x_0 + x_s + x_q = 1 \end{array}\right\} \tag{6}$$

The response of the quick component is expressed in the form of Dirac delta function $\delta(t)$, because the catchment time lag is sufficiently smaller than the time interval of data aggregation and is fed by a fixed percentage of x_0 of the effective rainfall. The subsurface flow and baseflow slow components are expressed with exponential decay laws characterized by the coefficients λ_s and λ_q respectively equal to the inverse of the time constant for the reservoirs τ_s and τ_q fed by a fixed percentage of x_s and x_q of effective rainfall.

Model input data are spatially averaged rainfall and air temperature daily series, collected from January to December 1974 over the catchment, and daily discharge data measured at the San Leonardo at Monumentale gauging station located near the river mouth. These discharges were derived, apart from the river stage observations, through a stage–discharge relationship (rating curve) which is known for the same year. It is well known that a rating curve can vary even within a single year, especially for semiarid catchments, characterized by an intermittent regime. Such eventuality led us to choose just one year (1974) for this study where discrete points of stage and contemporaneous discharge were interpolated through a two-parameter power law.

Finally, the Generalized Likelihood Uncertainty Estimation (GLUE) methodology of Beven & Binley (1992) is used here in the estimation of predictive uncertainty of the rainfall–runoff model.

RESULTS AND CONCLUSIONS

Firstly, the ("historical") rating curve, obtained by the interpolation of the point representing the water level and discharge observation for the year 1974, has been considered correct, or rather lacking in uncertainty, so that model input data were represented by these "correct" data. A total of 30 000 sets of parameters for the model were generated, each parameter value being drawn from ranges thought feasible for the San Leonardo catchment on the basis of previous experience (Table 1).

Simulations were performed, for each parameter set for comparison between generated series of discharge and measured series of discharge observation. For each of these simulations a performance index has been evaluated in the form of the Nash and Sutcliffe Efficiency Criterion (Nash & Sutcliffe, 1970):

Table 1 Parameter ranges of IHACRES model used in the Monte Carlo sampling.

Parameter	c (mm)	τ_w (days)	f (°C^{-1})	x_0	x_q	x_s	τ_q (days^{-1})	τ_s (days^{-1})	l	p
Upper limit	0.01	1	1	0	0	0	1	30	0	0.1
Lower limit	1	20	10	1	1	1	30	400	3	5

c, volume-forcing constant; τ_w, basin drying time constant; f, temp. modulation factor; x_0, channel quick flow volumetric throughput; x_q, reservoir quick flow volumetric throughput; x_s, reservoir slow flow volumetric throughput; τ_q, quick flow time constant; τ_s, slow flow time constant; l, initial water-holding storage parameter; p, power-law parameter.

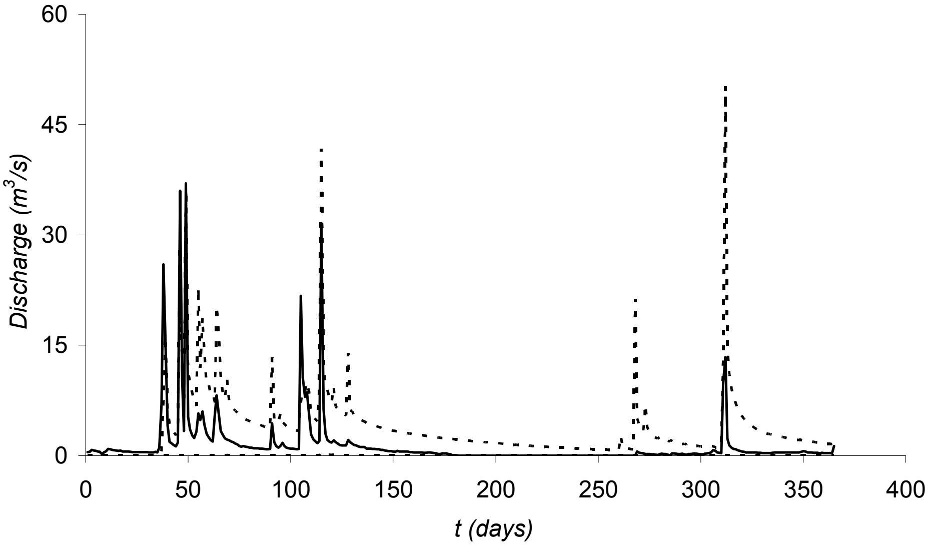

Fig. 2 Uncertainty bounds for 5% and 95% quantiles (dashed lines) for historical discharges (solid line).

$$L(\theta_i/Y) = (1-\sigma_i^2/\sigma_{obs}^2) \quad \sigma_i^2 < \sigma_{obs}^2 \tag{7}$$

where $L(\theta_i/Y)$ is the likelihood measure for the ith model simulation for parameter vector θ_i conditioned on the set of "historical" discharge observations Y, σ_i^2 is the associated error variance for the ith model and σ_{obs}^2 is the observed variance for the period under consideration.

From the cumulative distribution of discharges at each time step, the chosen discharge quantiles, 5 and 95%, have been calculated to represent the model uncertainty for the likelihood measure considered. Figure 2 shows the discharge prediction bounds based on the likelihood measure of equation (7) for the "historical" discharge series. Subsequently, the uncertainty in the rating curves has been considered according to the procedure described above and, particularly, choosing for the k_{max} coefficients in equation 1 the values of 0.5 and –0.5. Thus, within the region defined by these two bounds, 100 synthetic rating curves have been generated under the hypothesis of uniform distribution of the discharge values (Fig. 3(a)) and, hence, 100 new discharge series, containing the uncertainty from the rating curve (Fig. 3(b)).

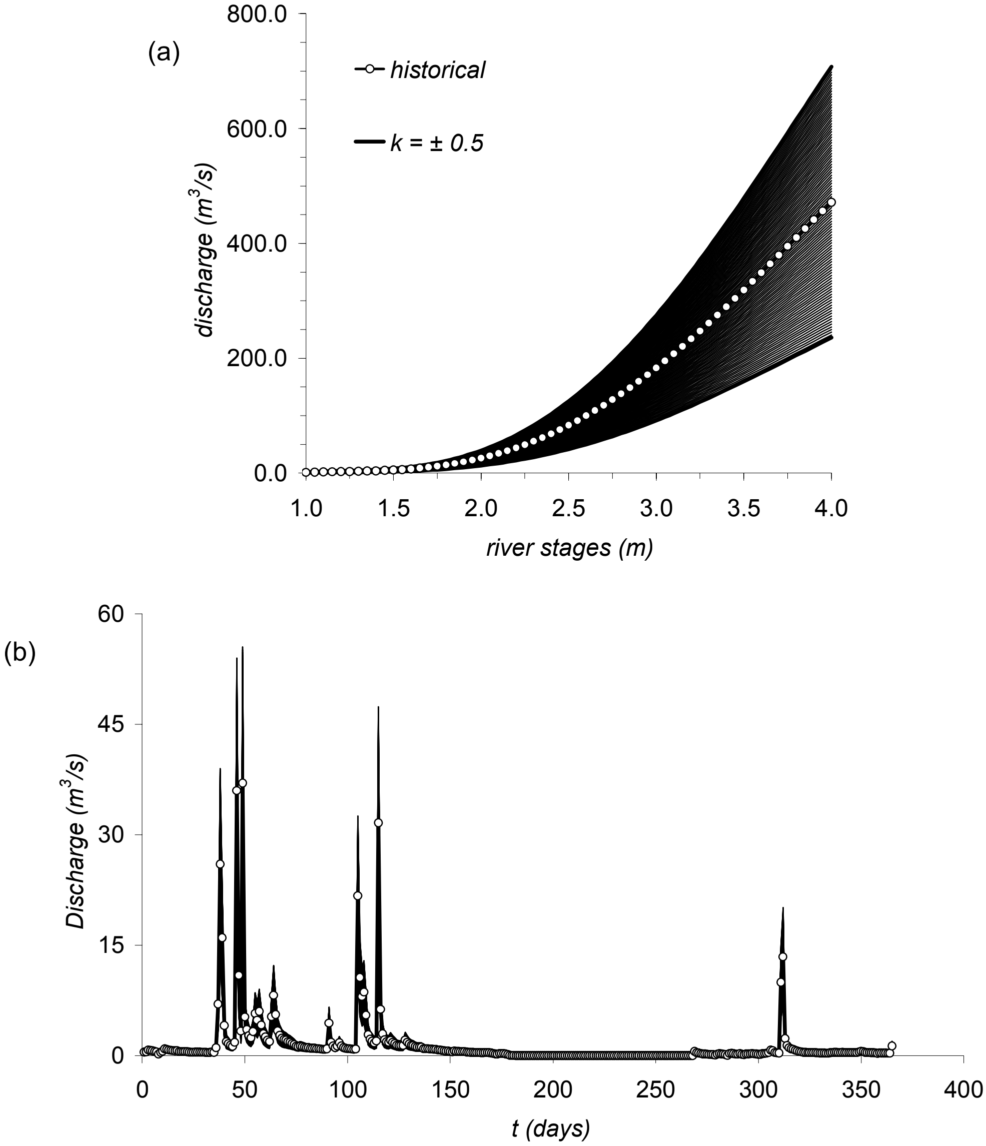

Fig. 3 (a) Example of generated rating curves. (b) Discharge series derived apart the generated rating curves compared with the historical values (white dots).

Again, 30 000 simulations were performed, for each parameter set and for each new rating curves generated. For each of these simulations a performance index has again been calculated in the form of the Nash and Sutcliffe Efficiency Criterion and the quantiles, 5 and 95%, have been calculated to represent the model uncertainty. For sake of simplicity, in Fig. 4(a) and (b) only the uncertainty bounds for the two groups of simulation corresponding to the Q_{bound} ($k = \pm 0.5$) were reported.

At first sight, the results reported in Fig. 4 seem to reveal how the uncertainty in the rating curve strongly affects the uncertainty in model predictions, but the comparison between the model predictions for all *k* values is not easy to interpret. To overcome this limitation, results of the simulation have been expressed using a global index defined as Normalized Mean Width of Bounds (MWB) and evaluated using the following expression:

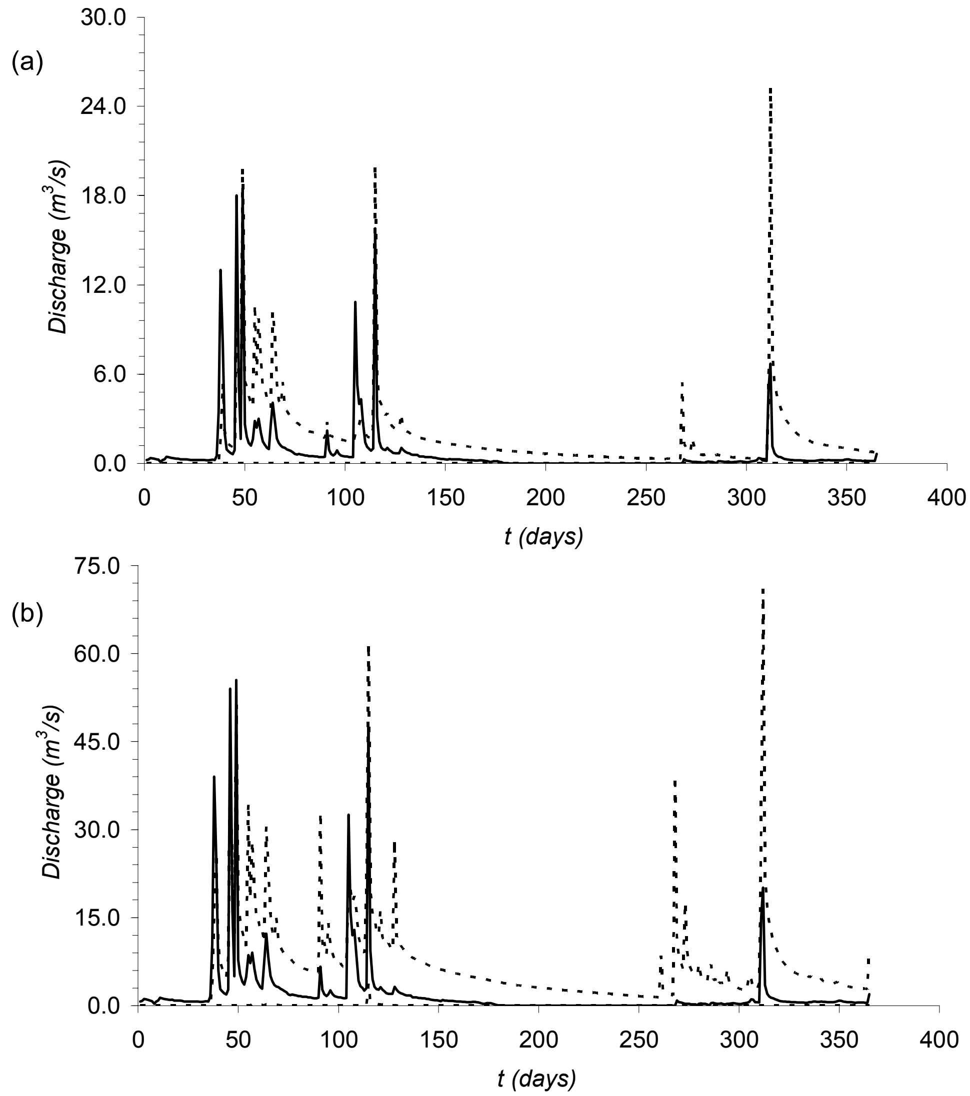

Fig. 4 (a) Uncertainty bounds for 5% and 95% quantiles (dashed lines) with Q_{bound} (k = –0.5) (solid line). (b) Uncertainty bounds for 5% and 95% quantiles (dashed lines) with Q_{bound} (k = 0.5) (solid line).

$$MWB_{norm,k} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{Q_{95,i} - Q_{5,i}}{Q_i}\right)_k \cdot \frac{1}{MWB_{norm,0}} \tag{8}$$

where $Q_{95,i}$ and $Q_{5,i}$ are the 95% and 5% quantiles, respectively, at ith time step for the model simulations performed using the rating curve defined by a certain value of k, Q_i is the discharge (“measured”) derived form the rating curve characterized by the same value of k, N is the length of the discharge series, (i.e. in this case $N = 365$). To allow a comparison between uncertainty bounds for different values of predicted variable, the index has been normalized with respect to the value of measured discharge. Figure 5 clearly shows the increase of uncertainty in respect to the historical condition ($k = 0$) for the majority of runs characterized by different values of k. Actually, the dashed horizontal line represents the value of MWB_{norm} for the historical rating curve and the majority of the black dots lies above this line. Further, this behaviour is more significant for values of *k* greater than *k* lower than 0.

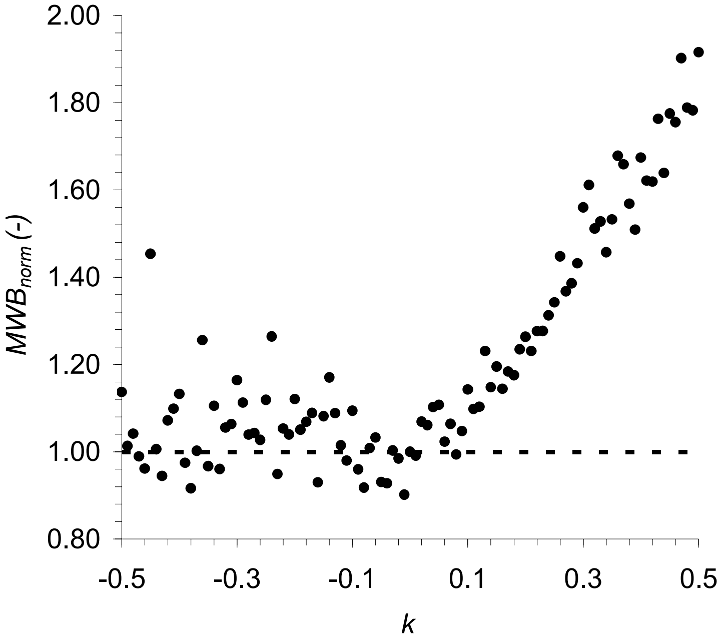

Fig. 5 Mean Width of Bound normalized *vs* *k* coefficient for all simulations.

CONCLUSIONS

A study on the influence of imperfect rating curve knowledge on the uncertainty of the response of a daily conceptual linear-nonlinear rainfall–runoff model was presented. The GLUE procedure was used to explore the influence of uncertainty of the rating curve in a classical rainfall–runoff model uncertainty analysis. The final results clearly show how, by taking into account the uncertainty of the rating curve in the calibration of the R-R model, a variation of uncertainty in the model predictions can be noted in comparison with the original situation. This behaviour seems to be more significant for overestimated rating curves than underestimated; this is to point out how the overestimation of the historical discharge has a great effect on the uncertainty of model predictions. Conversely, for underestimated rating curves the uncertainty seems to reduce or, even, to be lower then the uncertainty of original conditions. Probably, this latter result is due to the fact that underestimation of rating curves returns discharges with values very close to 0 and, hence, with a little uncertainty.

REFERENCES

Beven, K. J. & Binley, A. M. (1992) The future of distributed models - model calibration and uncertainty prediction. *Hydrol. Processes* **6**(3), 279–298.

Clarke, R. T., Mendiondo, E. M. & Brusa, L. C. (2000) Uncertainties in mean discharges from two large South American rivers due to rating curve variability. *Hydrol. Sci. J.* **45**(2), 221–236.

Kuczera, G. (1996) Correlated rating curve error in flood frequency inference. *Water Resour. Res.* **32**(7), 2119–2127.

Littlewood, I. G., Down, K., Parker, J. R. & Post, D. A. (1997) *The PC version of IHACRES for Catchment Scale Rainfall Streamflow Modelling: User Guide*. Institute of Hydrology, Wallingford, UK.

Montanari, A. (2004) Uncertainty assessment in rainfall–runoff modelling: a review. In: *Proc. Workshop: Statistical and Mathematical Methods for Hydrological Analysis*. Naples, Italy. *CNR-GNDCI* **2854**, 161–183.

Nash, J. E. & Sutcliffe, J. V. (1970) River flow forecasting through conceptual models. *J. Hydrol.* **10**, 282–290.

Ye, W., Bates, B. C., Viney, N. R., Sivapalan, M. & Jakeman, A. J. (1997) Performance of conceptual rainfall–runoff models in low-yielding ephemeral catchments. *Water Resour. Res.* **33**(1), 153–166.