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Uncertainty propagation in a sequential model for flood forecasting

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Abstract The aim of this paper is the estimation of uncertainty in an online data assimilation model applied to a sequential, multiple-step-ahead flood forecasting system. The main aim of the forecasting system under consideration is the derivation of real-time forecasts of the water levels with the maximum possible lead-time. This is achieved through a two-level, sequential data assimilation procedure. In order to extend the maximum lead-time, we incorporate the forecasts obtained from the earlier stages of the forecasting system, both rainfall-water level and water level routing processes. The updating of the gain of each of the subsystems introduces nonlinearity into the system performance. The Generalized Likelihood Uncertainty Estimation (GLUE) technique is used to estimate the uncertainty of model predictions in the decomposed online forecasting system.

Key words flood forecasting; Generalized Likelihood Uncertainty Estimation; Severn catchment; uncertainty propagation

INTRODUCTION

The estimation of the uncertainty of the output from an environmental model involves a difficult task in formulating the estimation problem. This includes the choice of the parameters of the model and its input variables, which mostly influence the uncertainty of the output and which are specified during the calibration stage of the model. Usually the choice is related to a statistical model of the residual error, determined during calibration, under the implicit assumption that the model is correct (for a discussion of some of the implications of this approach, see Beven, 2005).

One of the ways to check qualitatively the robustness of model predictions is by evaluation of a wide range of events. When the model uncertainty ranges (confidence limits) envelope most of the observations, the model uncertainty is well estimated. When many of the observed variables lie outside the specified confidence limits, this indicates that the parameter and input variability were assumed as too small or that the model structural error is too great. On the other hand, if the uncertainty ranges are too wide, the model loses its predictive power and is much less valuable to the user.

In hydrological modelling, it is difficult to specify a general prediction error structure. One of the possible solutions consists of the specification of behavioural parameter sets associated with an implicit error structure, assumed to be similar in prediction to that in calibration. This approach requires a subjective choice of criteria for the definition of a behavioural model (see Beven, 2005, for a formal approach based on a concept of effective observational error). Another approach consists of inflating the variance of predictions as a way of reflecting the reduced information content of the residual errors when the error structure is not clearly defined (Romanowicz & Beven, 2005). The uncertainty of input variables, i.e. rainfall and/or inflow, also influences the uncertainty of the predictions. However, proper accounting for this uncertainty requires introducing a statistical model for the input (Krzysztofowicz, 2002).

When a linear model is used to describe the process dynamics, the uncertainty propagation can be estimated analytically (Papoulis, 1965). The online adaptive sequential system for flood forecasting described in detail in Romanowicz *et al*. (2004) consists of interconnected one-dimensional (1-D) modules describing the dynamics of the rainfall–flow and flow routing processes in a catchment. It has linear dynamics, but it applies nonlinear gain for the updating of the predictions (Young, 2002), which introduces nonlinearity to the system. Two descriptions of this sequential system are possible. The first one consists of a Kalman filter formulation (Kalman, 1960) of the entire system and the second uses the decomposition of a system into sequentially connected modules, each of reduced dimensionality.

As the Kalman filter assumes that the input variables are deterministic, only the whole system formulation properly propagates uncertainty of the internal nodes. However, when the dimension of the system is large, the uncertainty analysis of sequentially connected modules may be more suitable. In this paper we present the estimation of uncertainty propagation in the modular (decomposed) system. The information about the input uncertainty for the middle stages of the forecasting system can be estimated from the sub-modules. We shall show here how this information may be used in the evaluation of prediction uncertainty of the online forecasting system for the River Severn, UK.

METHODOLOGY

The forecasting system applied in this study consists of a set of nonlinear static and linear dynamic Stochastic Transfer Function (STF) based modules, with sequentially updated nonlinear gain parameters, following the Kalman filter methodology introduced by Young (2002). The only assumption required for the application of Kalman filter to recursive data assimilation within the flood forecasting problem is the assumption of the linear process dynamics. This is ensured by the introduction of the nonlinear transformation of the rainfall into effective rainfall, thus assuming the process nonlinearity can be expressed by the input nonlinearity (Young, 2003). The initial application of this methodology to flow forecasting in the River Severn has been described in Romanowicz *et al*. (2004). The present case study considers a refined implementation based on the modelling and forecasting of water level (stage) measurements instead of flows. This approach avoids the errors introduced by conversion of levels to flow and directly yields the forecasts of water levels that are normally required for flood forecasting and warning. In addition, it enables gauging stations without well-defined rating curves to be used in data assimilation, without introduction of additional error. The error related to the rating curve varies between sites, reaching, for example, up to 30% of the maximum flow at Buildwas, River

Severn, used as a case study in this paper. Additionally, the rating curve often requires re-calibration after major flood events.

At each time step, recorded water level information is available at the Abermule, Meifod, Montford and Buildwas sites (Fig. 1). These levels can be used to update the gains in the transfer functions for each module of the forecasting system. It can be assumed that these measurements have small error and can be used directly in the Kalman filter algorithm. For forecasts at longer lead times, however, the forecasts at upstream sites, that form the inputs for the downstream routing transfer functions will be uncertain, and subject to the nonlinearities associated with the updating gains. Thus, an alternative methodology is required to cascade these uncertainties through the forecasting system. In this paper we apply the Generalised Likelihood Uncertainty Estimation (GLUE) technique (Beven & Binley, 1992) to estimate the uncertainty of online model predictions.

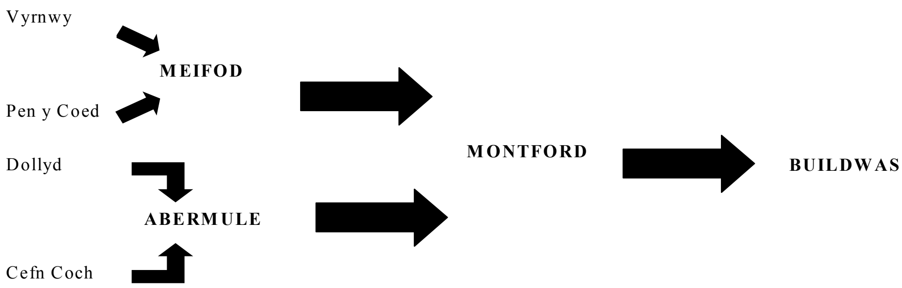

Fig. 1 Schematic presentation of the online forecasting system for the River Severn, UK, upstream of Buildwas.

GLUE is based on Monte Carlo realizations of the model parameters and (possibly) input variables; each realisation is associated with a likelihood weight to account for both prediction and parameter/structure related errors (Romanowicz & Beven, 2005). The STF parameter distributions and covariance estimates are assumed known from calibration of each transfer function. These parameters can be sampled directly from the identified Gaussian covariance structure. The nonlinear gain and error transformation associated with each STF involves additional hyper-parameters. In calibration, these are optimised, but without a direct estimate of their uncertainties. In the GLUE methodology, to propagate the joint effect of the parameter and hyper-parameter uncertainties down through the system, each realisation is associated with a likelihood weight. These weights are also determined in calibration using the following form of likelihood measure $f(\theta_i \mid \mathbf{z}, \mathbf{D})$ for the ith parameter set θ_i; $i = 1, ..., n$:

$$f(\theta_i \mid \mathbf{z}, \mathbf{D}) = \exp\left(-\sum_{t=1}^{T} \left(\mathbf{y}_{t,sim}(\theta_i, \mathbf{D}) - \mathbf{z}_t\right)^2 \right) / \sigma^2 \qquad (1)$$

Where $\mathbf{z}_t = \{z_{t,1}, ..., z_{t,m}\}$ denotes the vector of observations at time t, $\mathbf{D}$ is the input and $\mathbf{y}_{t,sim}(\theta_i, \mathbf{D})$ is a vector of simulated water levels.

The variance σ^2 is a scaling parameter reflecting our lack of knowledge of the true information content of the residuals in constraining the uncertainty in the model predictions. One possible form for this scaling is to take the sum of the variances of the errors between observed and simulated flows over all behavioural model for each time step as an estimate, such that:

$$\sigma^2 = \sum_{t=1}^{T} \operatorname{var}\left(\mathbf{y}_{t,sim}(\boldsymbol{\theta}) - \mathbf{z}_t\right) \tag{2}$$

This will increase the dispersion of the resulting posterior parameter distribution (relative to a formal likelihood measure) to account for the predictive model uncertainty without making additional assumptions about the model error structure. The multiple realisations are then used to forecast likelihood weighted cumulative distributions of predictions for lead times beyond the time for which measured inputs are available.

THE CASE STUDY: SEQUENTIAL MODEL FOR THE RIVER SEVERN REACH FROM ABERMULE TO BUILDWAS

As the case study we use the 120 km long reach of the River Severn, UK, between Abermule and Buildwas. The sequential updating model, described in detail in Romanowicz *et al*. (2004), consists of four STF sub-models as shown in Fig. 1. Here, the RIV (Refined Instrumental Variable) algorithm from the CAPTAIN (http://www.es.lancs.ac.uk/cres/captain) toolbox for Matlab™ and the associated Data Based Mechanistic (DBM) statistical modelling concepts are used to identify the order of the STF models and to estimate the associated parameters (Young, 2002). Using the DBM approach ensured that the identified models efficiently reflect the information content of the calibration data, so that the possibility of over-parameterization and associated poor identifiability is avoided.

The first two sub-models are rainfall–stage models. One model estimates the stage variable at Abermule, on the River Severn. The other is derived for the stage at Meifod, on the River Vyrnwy, a major Severn tributary that joins the Severn at Crew Green, above Montford. These models use the rainfall measurements from four sites in the Upper Severn: Cefn Coch, Dollyd, Pen y Coed and Vyrnwy. Both these models have 5 h delay, which means that they can provide forecasts 5 h ahead. Both models also apply nonlinear transformation of rainfall into effective rainfall (Young, 2002) to account for the changing antecedent conditions and varying storage capacity of the catchment.

The next sub-model is used to derive online forecasts of water levels at Montford, which are based on the water levels at Abermule and Meifod. This model is first order and has an 11 h delay. Together with the 5-h ahead forecast from the rainfall–water level models, we get 16-h ahead forecasts at Montford. The last sub-model gives water level forecasts at Buildwas. All the models apply both error transformation and online gain and variance updating to account for the changes of system response over time and heteroscedastic errors. These are controlled by a number of hyper-parameters which are determined during model calibration (Young, 2002; Romanowicz *et al*., 2004). An online parameter updating procedure is built into the system using real-time recursive estimation, Young, (1984). The STF models were developed using the autumn 1998 flood events (calibration stage) and autumn 2000 flood events were used for the validation of the models.

UNCERTAINTY ESTIMATION OF THE FORECASTS OF THE SEQUENTIAL MODEL

The parameters of the sequential model consist of 14 STF model parameters, estimated from the data during the derivation of the STF model structures for each sub-model and 15 hyper-parameters, covariance estimates of the state space KF form of the STF models and the covariance estimates of the random walk (RW) models (Papoulis, 1965) applied to the recursive updating of the gain and heteroscedastic variance of each STF model. It was found that the hyper-parameters, which only influence the statistical properties of the data assimilation and not the model dynamics, have little effect on the quality of the forecasts when the model was run with random samples of the hyper-parameters drawn from ranges of values (e.g. Fig. 2, for the Abermule rainfall–stage STF model). Here the hyper-parameters nvr1 and nvr2 are the covariance estimates of the state space form of STF Abermule model; q1 and q2 are the variances used in gain and variance updating. Allowing the STF model parameter values to vary, within the ranges determined by the KF covariance matrix determined during calibration, has a much greater effect on the model outcomes.

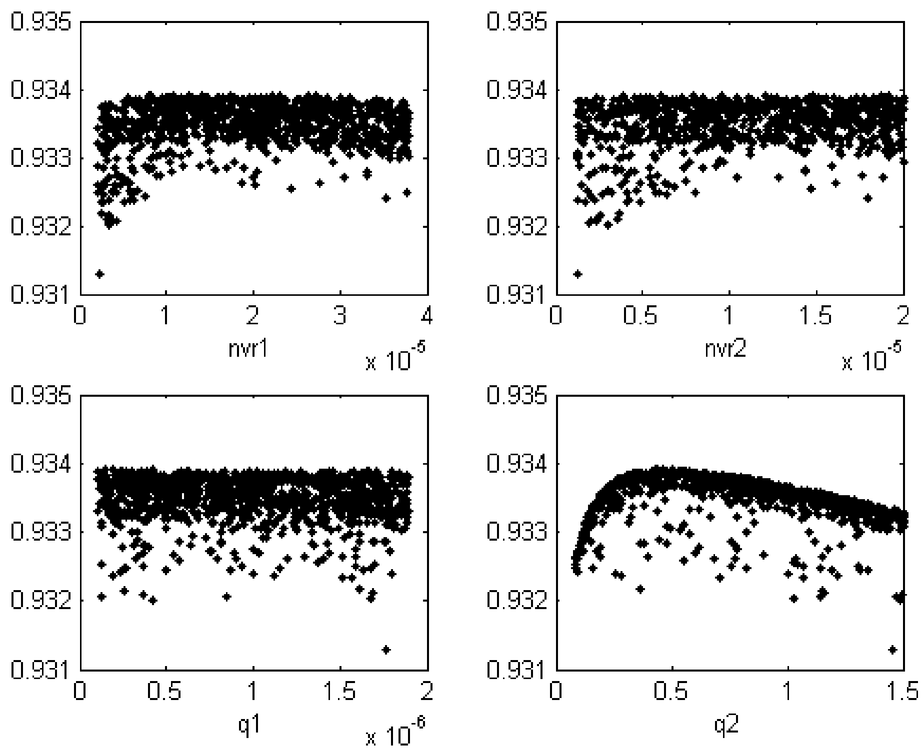

Fig. 2 Variation of R_T^2 criteria for 1000 simulations with hyper-parameters changing uniformly; nvr1 and nvr2 are the covariance estimates of the state space form of the STF Abermule model; q1 and q2 are the variances used in gain and variance updating.

In a further experiment 5000 Monte Carlo runs were performed with STF parameters and hyper-parameters varied. Likelihood weights for each realisation were derived using equations (1)–(2) for the 1998 calibration period and used to derive quantiles for the forecasts. Figure 3 shows the median estimates of 5 h ahead forecasts for Abermule during the Autumn 1998 flood, obtained from the 5000 MC simulations. The darkest shaded areas denote 0.95 confidence bands for the median. It is worth noting that these confidence bands are very narrow, as the estimated STF model structure obtained using the RIV procedure from the Captain toolbox is well identified (Romanowicz & Young, 2003). Also the estimate of the variance of the predictions, derived using the same weights (equations (1)–(2)), is shown in a lighter shaded tone in Fig. 3. The online updated variance for the predictions enables the variance of the predictions together with 0.95 confidence bands to be estimated (lightest shading). We overlaid the resulting 0.95 confidence bands for the predictions with forecast median and forecast variance to compare the resulting uncertainty.

Similar results were obtained for Meifod. The predicted stages at Abermule and Meifod then provide the upstream inputs for flood routing to Montford and Buildwas. Initially, the models for Montford and Buildwas were run with varying model structure and hyper-parameters and with input uncertainties not accounted for. Figure 4 shows the resulting model forecasts for Buildwas for the validation period in November 2000, obtained from 5000 simulations of the sequential model without input uncertainty.

In order to analyse the propagation of the uncertainties of the model input through the system, the Montford model was run 10 000 times for a 16-h ahead forecast, to allow for uncertainty in the input water levels and transfer function parameter values. The 10 000 runs were generated by choosing 1000 parameter sets, each of which was driven with 10 input sequences randomly selected from 1000 MC 5-h ahead forecasts

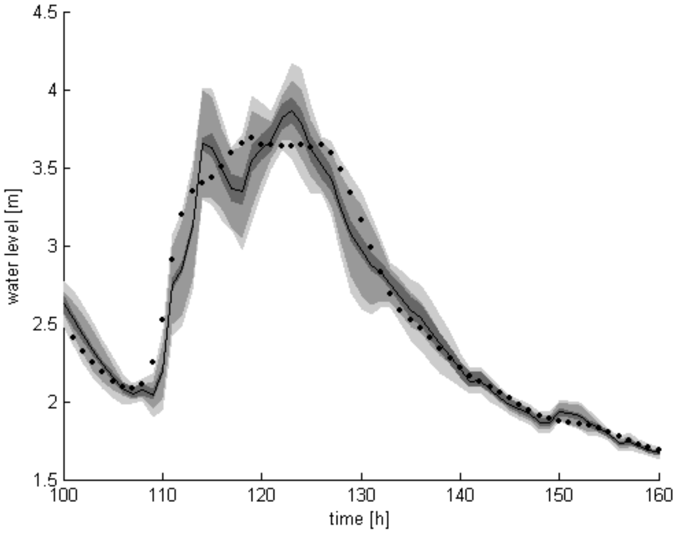

Fig. 3 Validation stage of rainfall-water level model for Abermule, River Severn, with updating of gain and variance, October 2000: the solid line denotes the 5-h-ahead forecast median obtained from MC simulations; the dotted line denotes the observations; the lightest shaded area denotes 0.95 confidence bands based on the estimated 0.95 variance quantile added to the median forecast; darker shading denotes estimated 0.95 confidence bands based on the median standard deviation added to the median forecast; darkest shading denotes 0.95 bands for the forecast median.

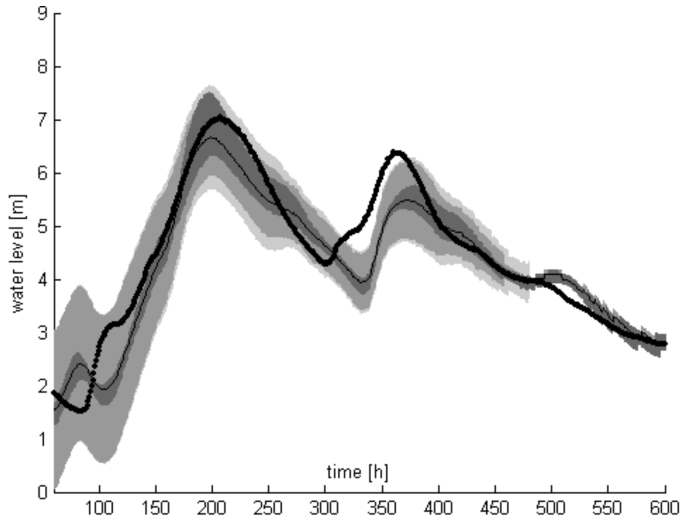

Fig. 4 Validation stage of rainfall-water level model for Buildwas, River Severn, with updating of gain and variance, 1 November 2000: the solid line denotes the 32-h-ahead forecast median obtained from MC simulations; the dotted line denotes the observations; the lightest shaded area denotes 0.95 confidence bands based on the estimated 0.95 variance quantile added to the median forecast; darker shading denotes estimated 0.95 confidence bands based on the median standard deviation added to the median forecast; darkest shade denotes 0.95 bands for the forecast median.

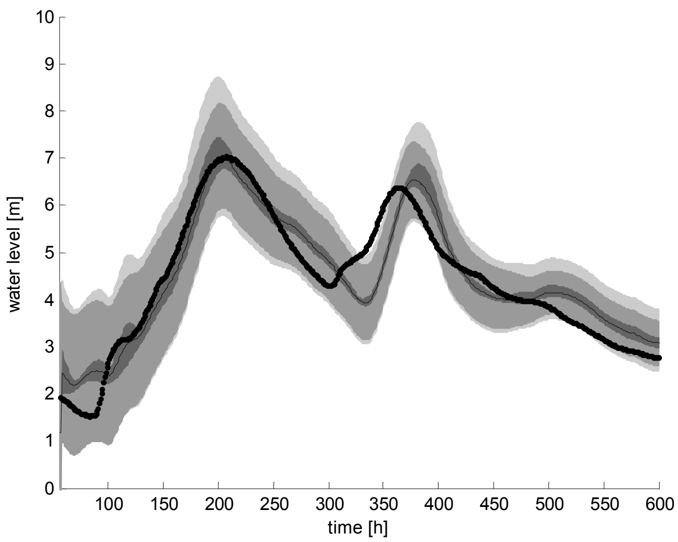

Fig. 5 Illustration of uncertainty propagation in the rainfall–water level model for Buildwas, River Severn, the solid line denotes the 32-h-ahead forecast median obtained from MC simulations; the dotted line denotes the observations; the lightest shaded area denotes 0.95 confidence bands based on the estimated 0.95 variance quantile added to the median forecast; darker shading denotes estimated 0.95 confidence bands based on the median standard deviation added to the median forecast; darkest shade denotes 0.95 bands for the forecast median.

at Abermule and Meifod. Finally, the Buildwas model has been run for a lead time of 32-h ahead with 10 000 realizations of the joint variation of the Montford 16 h ahead forecasts and Montford-Buildwas parameters and hyper-parameters (again using 1000 parameters sets, each of which is driven by 10 input sequences, randomly selected from those available at Montford). The resulting confidence bands for forecast water levels at Buildwas are shown in Fig. 5. This result shows forecast uncertainties that are larger than before, mainly due to the influence of the input variability, derived from the estimates of the Montford forecast variance.

CONCLUSIONS

In most forecasting problems for real catchments, maximization of forecast lead times requires the propagation of uncertainty through a cascade of rainfall–runoff and flow routing components. Given the nonlinear gain components within the uncertainty cascade, confidence bands for the forecasts were assessed using MC simulation and the GLUE methodology. The explicit conditioning of the results on input uncertainty gave much wider confidence bands, with variability related to the variability of the estimated variance of the forecast at Montford, due to the choice of the quantiles related to the estimated forecast uncertainty. This work indicates the importance of the model structure uncertainty and it will be followed by the comparison with the propagation of uncertainty in the full model of the sequential system.

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