Predictions in Ungauged Basins: Promise and Progress (Proceedings of symposium S7 held during the Seventh IAHS Scientific Assembly at Foz do Iguaçu, Brazil, April 2005). IAHS Publ. 303, 2006.

Physically-based model of heat and water transfer in frozen soil and its parameterization by basic soil data

ALEXANDER GELFAN

Water Problems Institute, Russian Academy of Sciences, 119991, Gubkin 3, Moscow, Russia

hydrowpi@aqua.laser.ru

Abstract A physically-based model of the hydrothermal regime of frozen soil has been developed. This model describes the process of coupled heat and moisture transfer through homogeneous, unsaturated frozen soil and accounts for the influence of phase changes on water flow. The process is formulated as one-dimensional partial differential equations, which are solved by an implicit, finite difference scheme. The model parameters are formulated through the water retention, hydraulic conductivity and freezing point depression curve of frozen soil, which, in turn, have been determined using the basic soil characteristics available from soil surveys. The model has been tested on the basis of both laboratory and field measurements. The results obtained have demonstrated an ability of the model to predict, without intensive calibration procedure, snowmelt runoff losses for infiltration.

Key words frozen soil process; parameterization; physically-based model; ungauged basin

INTRODUCTION

In cold regions, spring melting of seasonal snow cover contributes significantly to annual runoff. Snowmelt is also the main source of highest floods in the large plain rivers of these regions; for example, during 1985–2004, >60% of the registered disastrous floods in Russia were originated by snowmelt. Most of the river basins in cold regions should be classified as ungauged or at best poorly gauged, where runoff prediction by a hydrological model becomes more complicated because of the limited possibility for model calibration. This generates a need to invoke in the model structure more prior knowledge on the main features of flood generation processes. The prior knowledge, which contains both theoretical information (general physical laws) and experimental information obtained from gauged basins, can compensate, to some extent, for the unavailable data in the ungauged basin of interest. The perception of this need results in the development of a physically-based hydrological model, recognized as one of the “methods appropriate for ungauged basins” (PUB Science and Implementation Plan. Final Version, 2003, p. 11. From the viewpoint of PUB, the physically-based structure of the model looks attractive because the parameters of such a model have clear physical meanings and may, in principle, be related to commonly available characteristics of a river basin, such as topography, soils, vegetation, etc. Combined with and resulting from the physical background of the model, this feature makes it possible to minimize the quantity of hydrological data required for the model calibration (Gelfan, 2005).

Infiltration of water from melting snow into frozen soil is one of the main factors (in parallel with snowmelt) affecting the magnitude of snowmelt floods. Infiltration capacity depends on changes in soil condition; namely, changes of heat and water content of the frozen soil during pre-melt and melt periods. The respective measurements are not carried out for the vast majority of the river basins and improvement of snowmelt flood prediction is, consequently, intimately associated with developing a model for simulation of the frozen soil hydrothermal processes (soil freezing, thawing, infiltration, etc.), by using available meteorological data and measured soil properties. However, the existing models, which have been developed for predicting the snowmelt runoff at real basins, either ignore these processes or describe them by some conceptual sub-models requiring adequate records of hydrological observations for calibration (examples of one or the other of these models are given in US Army Corps of Engineers, 1998).

The main objectives of this study were to develop: (a) a physically-based model considered as a workable tool for simulating frozen soil processes in ungauged basins; (b) a technique for estimating the model hydrothermal parameters by commonly available soil data; and (c) to demonstrate (on the basis of the laboratory and field measurements) the ability of the model combined with the technique for adequately describing the principal features of the frozen soil processes without an intensive procedure of calibration.

A few sophisticated, physically-based models, which allow one to simulate combined heat and water vertical flow in frozen soil, were developed during the last decades. Harlan (1973) was probably the first who suggested such a model and applied it for simulation of water re-distribution within a hypothetical freezing soil. Different aspects of using Harlan's model for simulation of the hydrothermal regime of frozen soil were analysed and a variety of the model modifications were suggested by a number of authors. Motovilov (1977) improved the heat flow equation used by Harlan (1973) considering the dependence of phase changes on both temperature and total (ice + liquid water) water content and applied the model specifically to infiltration into frozen soil. Later, the model of Motovilov (1977) was incorporated, as a component, into the model of snowmelt runoff generation and used for prediction of runoff losses over the real watershed (Kuchment *et al*., 1986). Zaretcky & Lavrov (1986) developed a model used for establishing critical hydrothermal conditions resulting in formation of impermeable layers in frozen soil during the infiltration process. Jansson & Halldin (1980) presented the SOIL model, which was extensively used in soil heat extraction simulations (Lundin, 1985) and to study the influence of soil frost on drainage flow in an agricultural field (Jansson & Lundin, 1991). A sophisticated model considering the effect of vapour phase on infiltration into frozen soil was reported by Zhao *et al.* (1997). However, as far as we know, applications of the model were restricted to the hypothetical examples.

Most of the models listed above are similar in the main equations but differ in approximating the hydraulic properties (hydraulic conductivity and water retention curve) of frozen soil. In the model of Harlan (1973), the influence of ice on these properties was assumed to be identical to the influence of soil particles. In other words, the ice phase is considered as part of the soil matrix. A number of studies showed that overestimation of the frost-induced redistribution is a common problem of this assumption. It was suggested that the impedance parameter reflecting an increased tortuosity of the water flow under the freezing conditions should be used to improve approximation of the hydraulic conductivity in Harlan's model (Guymon *et al.*, 1980; Jame & Norum, 1980). Note that the first formulation of the impedance parameter was suggested by Kulik (1969). Later, his formulation was used by Motovilov (1977) and, with minor changes, by Zaretcky & Lavrov (1986). Lundin (1990) introduced the concept of impedance into the SOIL model. The concept of two water-conducting water flow domains, which takes into account the difference in the hydraulic conductivity through smallest and largest pores, was proposed and used for the SOIL model by Stähli *et al*. (1999).

The water retention curve of a frozen soil is assumed to be the same as one of an unfrozen soil in most of the aforementioned models. However, according to the experimental data reported by Kulik (1969), the distinction exists between the capillary potential in unfrozen and frozen soil under the equal liquid water content. This distinction is caused by the unequal specific surface of soil and ice particles. Motovilov (1977) was the first who accounted for this effect and showed its importance for the infiltration process.

Physically-based models require detailed information on soil properties, which are usually not available from the basic data of soil surveys; this is among the reasons why the detailed models have a limited application in prediction of snowmelt runoff in real watersheds. Determining hydrothermal characteristics of frozen soil from the available measured properties of an unfrozen soil should be, consequently, the focus of attention when developing such a model for the ungauged basins.

In this paper, a physically-based model of the hydrothermal regime of frozen soil will be described first, then the techniques for determining hydraulic parameters from the commonly available soil properties will be presented, and, finally, examples of reproducing the frozen soil processes either without, or almost without, the model's calibration will be shown.

MODEL DESCRIPTION

Water and heat transfer in frozen soil can be described by the following equations (e.g. Motovilov, 1977):

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial z}\left(K\frac{\partial \psi}{\partial z} - K\right) \tag{1}$$

$$c_{eff}\frac{\partial T}{\partial t} - L\rho_i\frac{\partial I}{\partial t} = \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) + \rho_w c_w\left(K\frac{\partial \psi}{\partial z} - K\right)\frac{\partial T}{\partial z} \tag{2}$$

where W is the total water content equals $W = \theta + \frac{\rho_i}{\rho_w}I$; θ and I are the volumetric content of liquid water and ice, respectively; ψ is the capillary potential; K is the hydraulic conductivity; T is the temperature; c_{eff} is the effective heat capacity of soil, calculated as weighted average of the heat capacities of soil matrix, water and ice, i.e. $c_{eff} = \rho_g c_g(1-P) + \rho_w c_w\theta + \rho_i c_i I$; ρ and c are the density and the specific heat capacity, respectively (indexes w, i and g refer to water, ice and soil matrix, respectively); P is the volumetric porosity of soil; L is the latent heat of fusion of ice; λ is the thermal conductivity.

Let us convert equations (1) and (2) in relation to W, I and T.

Assuming the liquid water content depends only on the temperature (i.e. θ does not depend on W under the fixed negative value of T), one can write:

$$\frac{\partial I}{\partial t} = \left(\frac{\partial I}{\partial T}\right)_W \frac{\partial T}{\partial t} + \left(\frac{\partial I}{\partial W}\right)_T \frac{\partial W}{\partial t} = -\frac{\rho_w}{\rho_i}\left(\frac{\partial \theta}{\partial T}\right)_W \frac{\partial T}{\partial t} + \frac{\rho_w}{\rho_i}\frac{\partial W}{\partial t} \tag{3}$$

The diffusion term $K\frac{\partial \psi}{\partial z}$ of water flow equals (Zaretcky & Lavrov, 1987):

$$K\frac{\partial \psi}{\partial z} = D\frac{\partial \theta}{\partial z} + D_I\frac{\partial I}{\partial z} \tag{4}$$

where $D = K\left(\frac{\partial \psi}{\partial \theta}\right)_I$ and $D_I = K\left(\frac{\partial \psi}{\partial I}\right)_\theta$

Substituting equations (3) and (4) in equations (1) and (2) gives:

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z} + D_I\frac{\partial I}{\partial z} - K\right) \tag{5}$$

$$c_T\frac{\partial T}{\partial t} - \rho_w L\frac{\partial W}{\partial t} = \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) + \rho_w c_w\left(D\frac{\partial \theta}{\partial z} + D_I\frac{\partial I}{\partial z} - K\right)\frac{\partial T}{\partial z} \tag{6}$$

where $c_T = c_{eff} + \rho_w L\frac{\partial \theta}{\partial T}$

Rearrangement of equations (1) and (2) suggested by Motovilov (1977) to (5) and (6) has been made in order to avoid some difficulties with numerical solution of the equations. In particular, numerical solution of the water flow equation in the form of equation (1) is associated with calculation of the specific moisture capacity, $c_\psi = \left(\frac{\partial \theta}{\partial \psi}\right)_I$ grossly changing by a small change of water content. In the diffusion form (5) of the water flow equation, we need to calculate derivatives of the coefficients of diffusion D and D_I, which vary in a much smaller range than c_ψ under the same change of the water content.

Equations (5), (6) were numerically integrated by an implicit, four-point finite difference scheme; the corresponding difference equations were solved by the double-sweep method (Samarsky, 1983). The following iteration algorithm was adopted. The rough estimates of W and I profiles at the current computational time-step are calculated first from equation (5) using the values of T assigned from the previous time-step. Calculated W and I are substituted in equation (6) and the rough estimates of T at the current time-step are found. Then the estimates of T are substituted in equation (5) again, and the next, more exact, estimations of W and I are defined at the same time-step. Iterations are stopped for the current time-step, when changes of the desired variables became negligible. In addition to the described "external" iterations between equations (5) and (6), "internal" iterations are also used for each equation separately to refine the parameters K, D, D_I, c_T and λ calculated as the functions (see next section) of W, I and T at each time-step. The numerical algorithm used combining the "external" iterations with the "internal" ones, gives more stable solutions if the boundary conditions are changed abruptly (e.g. sharply rising outflow from snowpack). Also, in distinction from the algorithm suggested by Motovilov (1977), we have changed the method of averaging of the hydraulic parameters at the boundary between a frozen compartment and an unfrozen one. Instead of the arithmetic averaging, which leads to overestimation of the predicted upward flow (Lundin, 1990; Motovilov, personal communication), the geometric averaging was applied to reduce predicted upward flow to a more realistic value. According to our numerical experiments, the effect of such averaging becomes especially visible when movement of a freezing front is retarded. In general, the improvements presented of the numerical algorithm allow one to use the difference scheme with larger time-space steps that may be important in the case of inadequate initial and/or boundary data.

Determining hydrothermal parameters of soil

In this section, we consider determining the parameters K, D, D_I, c_T and λ of equations (5) and (6).

The water retention ($\psi_u = \psi_u(\theta)$) and the hydraulic conductivity ($K_u = K_u(\theta)$) of unfrozen soil were calculated by the following formulas (van Genuchten, 1980):

$$\psi_u = -\frac{\left(S_u^{-1/m} - 1\right)^{1/n}}{\alpha} \tag{7}$$

$$K_u = K_0 S_u^{0.5}\left[1 - \left(1 - S_u^{1/m}\right)^m\right]^2 \tag{8}$$

where $S_u = \dfrac{\theta - \theta_r}{\theta_s - \theta_r}$ is the relative saturation; θ_s and θ_r are the saturated and residual water contents respectively; K_0 is the saturated hydraulic conductivity; $\alpha > 0$ is the parameter, which is related to the inverse of the air entry pressure; $n > 1$ is the parameter, which is a measure of the pore-size distribution; $m = 1 - \dfrac{1}{n}$; index u refers to an unfrozen soil.

Taking into account formulas (7) and (8), the coefficient of diffusion of an unfrozen soil equals:

$$D_u = K_u \frac{\partial \psi}{\partial \theta} = K_0 \frac{1-m}{\alpha m(\theta_s - \theta_r)} S_u^{0.5-1/m}\left[\left(1 - S_u^{1/m}\right)^{-m} + \left(1 - S_u^{1/m}\right)^m - 2\right] \tag{9}$$

To obtain water retention of frozen soil ($\psi = \psi(\theta, I)$), we invoked the relationship obtained by Motovilov (1977). He showed that for equally saturated unfrozen and frozen soil one can write:

$$\psi(\theta, I) = \psi_u(\theta_u)\frac{\theta_u}{\theta}(1 + \varphi I)^2 \tag{10}$$

where $\psi_u(\theta_u)$ is expressed (in our case) by equation (7); θ_u is the water content of an unfrozen soil; $(1 + \varphi I)^2$ is the impedance factor presented by Kulik (1969); φ is a characterization of the ratio between sizes of ice crystals and soil particles (Kulik (1969) suggested an average value of 8 for φ).

If the relative saturation of an unfrozen soil equals to one of frozen soil, i.e.:

$$S_u = \frac{\theta_u - \theta_r}{\theta_s - \theta_r} = S = \frac{\theta - \theta_r}{\theta_s - I - \theta_r}, \text{ then } \theta_u = \theta_r + \frac{(\theta - \theta_r)(\theta_s - \theta_r)}{\theta_s - I - \theta_r} \tag{11}$$

Substituting (11) in (10) gives water retention curve for frozen soil as:

$$\psi(\theta, I) = -\frac{\left(S^{-1/m} - 1\right)^{1/n}}{\alpha} \times \left[\frac{\theta_s - \theta_r}{\theta_s - I - \theta_r} + \frac{\theta_r}{\theta}\left(1 - \frac{\theta_s - \theta_r}{\theta_s - I - \theta_r}\right)\right](1 + 8I)^2 \tag{12}$$

In the following, we will assume $\theta_r = 0$ and $\theta_s = P$, i.e.:

$$\psi(\theta, I) = -\frac{\left(S^{-1/m} - 1\right)^{1/n}}{\alpha}\left(\frac{P}{P - I}\right)(1 + 8I)^2 \tag{13}$$

The hydraulic conductivity $K = K(\theta, I)$ of frozen soil was calculated from the corresponding value K_u (see formula (8)) of unfrozen soil using the same impedance factor as used in (13), i.e.:

$$K = K_u(S) \times (1 + 8I)^{-2} = K_0 S^{0.5}\left[\frac{1 - \left(1 - S^{1/m}\right)^m}{(1 + 8I)}\right]^2 \tag{14}$$

The hydraulic parameters $D = K\left(\frac{\partial \psi}{\partial \theta}\right)_I$ and $D_I = K\left(\frac{\partial \psi}{\partial I}\right)_\theta$ were calculated using obtained formulas (12) and (14).

According to a great body of the experimental data for different types of soil, under the decreasing negative temperature of soil the liquid water content decreases more or less steeply from the value of $\theta_0 = \theta(T = 0°\text{C})$ up to some critical value of $\theta_{\min} = \theta(T_{\min})$; then changes of θ can be neglected. For the most types of soil, $T_{\min}$ approximately equals –5°C (Globus, 1969; Lundin, 1990). Taking into account the experimental results, we described the freezing point depression curve as:

$$\begin{aligned} &\theta = \frac{5\theta_{\min}\theta_0}{5\theta_{\min} - T(\theta_0 - \theta_{\min})}, \ \theta_0 > \theta_{\min}, -5^\circ C \le T \le 0^\circ C \\ &\theta = \theta_{\min}, \qquad\qquad -5^\circ C > T \end{aligned} \tag{15}$$

From equation (15), one can obtain the value of c_T as:

$$\begin{aligned} &c_T = c_{eff} + \rho_w L \frac{\partial \theta}{\partial T} = c_{eff} + \rho_w L \frac{5\theta_{\min}\theta_0(\theta_0 - \theta_{\min})}{[5\theta_{\min} - T(\theta_0 - \theta_{\min})]^2}, \quad -5^\circ C \le T \le 0^\circ C \\ &c_T = c_{eff}, \qquad\qquad -5^\circ C > T \end{aligned} \tag{16}$$

The thermal conductivity λ was calculated as Kuchment *et al*. (1983):

$$\lambda = \lambda_u (1 + I) \tag{17}$$

where $\lambda_u = a_\lambda \lg \Theta + b_\lambda$ is the thermal conductivity of unfrozen soil; Θ is the moisture content of a soil, expressed as a percentage of the dry weight; a_λ and b_λ are the empirical coefficient.

The obtained equations (12), (14), (16), (17) contain seven coefficients: α, m, θ_0, θ_{min}, a_λ, b_λ, and K_0. Below we will describe determining all except one (K_0) of these coefficients through the basic soil characteristics, which are typically available from soil surveys.

Coefficients α and *m* of van Genuchten's model Let us define ψ_{FC} and ψ_{WP} as the capillary potentials under the field capacity, θ_{FC}, and wilting point, θ_{WP}, respectively. (θ_{FC} and θ_{WP} are the soil constants, which are typically measured at the Russian's agrometeorological stations). Then, according to (7) $\psi_{FC} = -\left(S_{FC}^{-1/m} - 1\right)^{1/n} / \alpha$ and $\psi_{WP} = -\left(S_{WP}^{-1/m} - 1\right)^{1/n} / \alpha$, where $S_{FC} = \theta_{FC} / P, S_{WP} = \theta_{WP} / P$. Assuming ψ_{FC} = –330 cm and ψ_{WP} = –15 000 cm, it follows that the coefficient m is calculated from:

$$\ln \frac{\psi_{FC}}{\psi_{WP}} = -3.817 = (1-m) \ln \frac{S_{FC}^{-1/m} - 1}{S_{WP}^{-1/m} - 1} \tag{18}$$

Then, after calculation of m, the coefficient α is found as:

$$\alpha = -\frac{\left(S_{FC}^{-1/m} - 1\right)^{1-m}}{\psi_{FC}} = \frac{\left(S_{FC}^{-1/m} - 1\right)^{1-m}}{330} \tag{19}$$

Coefficients θ₀ and θmin of formula (16) The liquid water contents $\theta_0 = \theta(T = 0°C)$ and $\theta_{min} = \theta(T = -5°C)$ were calculated by the following empirical relationships (Kalyuzhny *et al.*, 1988):

$$\theta_0 = 1.04\theta_{FC} - 0.06 \tag{20}$$

$$\theta_{min} = 0.94\theta_{WP} + 0.017 \tag{21}$$

Coefficients a_λ and b_λ for determining thermal conductivity (in J m^{-1} s^{-1} $°C^{-1}$) of unfrozen soil were calculated as (Koren, 1991):

$$a_\lambda = 1.5 \times 10^{-4} \rho_b - 0.09 \tag{22}$$

$$b_\lambda = -2.0 \times 10^{-5} \rho_b + 0.006 \tag{23}$$

where ρ_b is the bulk density of soil (in kg m^{-3}).

TESTING THE MODEL WITH THE LABORATORY AND FIELD OBSERVATIONS

Two tests of the performance of the developed model were carried out in this study. During the first one, we used laboratory measurements of the processes of soil freezing and infiltration into frozen soil, which were described by Zaretcky & Lavrov (1986). In the laboratory experiment of Zaretcky & Lavrov (1986), a 70-cm sample of a loamy chernozem was used. The available soil characteristics are the following: ρ_b = 1100 kg m^{-3}; $P = 0.560$; $K_0 = 1.7 \times 10^{-4}$ cm s^{-1}; $\theta_{FC} = 0.375$; $\theta_{WP} = 0.215$; $\theta_0 = 0.180$; as one can see from this list, all soil constants, needed for estimation of the model parameters, are available. Initial temperature of the soil sample was 6.0°C, initial water content smoothly increased from 0.42 at the upper layer to 0.44 at the lower layer. During the first 192 h of the experiment, the process of soil freezing was studied: temperature of the upper boundary of soil sample was decreased from 6 to –11.2°C; at the lower boundary, temperature was decreased from 6 to 2.5°C. Then, during the next 56 h, temperature of the upper boundary of soil was increased up to 0°C; beginning from the end of 248th hour, water was yielded on the soil surface during 64 h and infiltration was observed. We applied the developed model to simulate this laboratory experiment.

The measured water retention curve was compared with one obtained by van Genuchten's model with the parameters α = 0.049 cm^{-1}, m = 0.125 calculated by equations (18), (19) under the measured values of P, θ_{FC} and θ_{WP}. Comparison between the measured capillary potential of the soil sample and the calculated potential has shown perfect agreement.

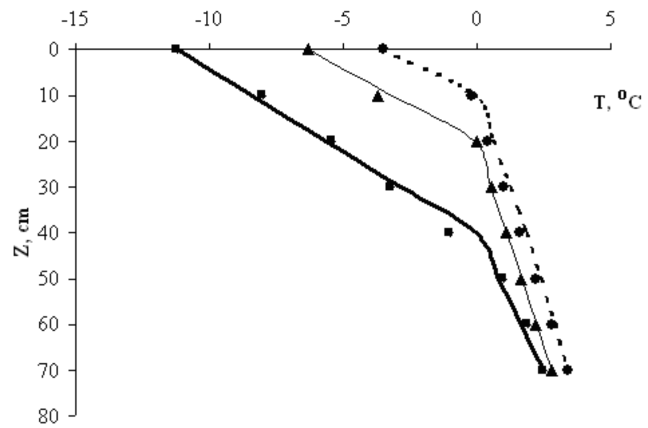

Fig. 1 Measured (points) and calculated (lines) temperature of freezing soil: points and dotted line, 72 h from the beginning of the experiment; triangles and thin line, 120 h; squares and bold line, 192 h.

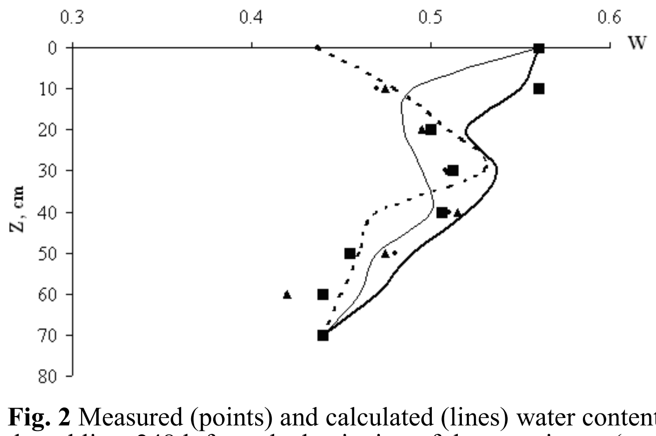

Fig. 2 Measured (points) and calculated (lines) water content of frozen soil: points and dotted line, 248 h from the beginning of the experiment (start of infiltration); triangles and thin line, 263 h; squares and bold line, 312 h.

In Figs 1 and 2, dynamics of the calculated T and W are compared with the respective measurements. One can see from Fig. 1 that the thermal regime of the freezing soil is well described by the model. Figure 2 demonstrates that the model satisfactorily simulated soil water re-distribution during both 248 h of freezing and the following 64 h of infiltration. Underestimation of W for the layers of soil deeper than 0.5 m is probably caused by errors in the lower boundary condition for equation (5); the constant value W = 0.44 was assigned because of lacking more detailed information. Note that the results presented in Figs 1 and 2 were obtained using the measured soil characteristics, without any calibration procedure.

The model was additionally tested using the field measurements at the Nizhnedevitckaya water balance station (51°31′N; 38°23′E), which is located in the central part of the forested-steppe zone, within the Don River basin. More than 70% of annual runoff takes place here during a few weeks of spring melt of snow cover. Snowmelt runoff losses for infiltration changes in a wide range from year to year because of the large variability of the condition of frozen soil.

The following physical properties of the typical soil (loamy chernozem) are available for the field site under consideration (Hanbook, 1975): ρ_b = 1120 kg m^{-3}; P = 0.562; θ_{FC} = 0.330; θ_{WP} = 0.208. Using these data, the constants α, m, θ_0, $\theta_{\min}$, a_λ, and b_λ were calculated by formulas (18)–(23). To determine saturated hydraulic conductivity K_0, we used the following calibration procedure based on measurements of W in the warm period from May to September of 1970–1973. For this period, 15 rainstorms were registered. Fifteen profiles of W measured before the beginning of each rainstorm were assigned as the initial condition for equation (5). Then changes of W caused by storm water infiltration were calculated for each case with the help of equation (5) (under D_I = 0 and I = 0). Fifteen calculated profiles of W were compared with the corresponding profiles measured after the ending of the storms. The value of $K_0 = 1.3 \times 10^{-3}$ cm s^{-1} was adjusted by minimizing the root mean square error averaged over 15 measured and calculated profiles. Note that the optimal values of K_0, which were estimated for separate profiles, varied in a wide range ($9.6 \times 10^{-5} - 4.0 \times 10^{-3}$ cm s^{-1}).

The soil hydrothermal regime was simulated for the period from 1 November to the end of spring melt season of 1970–1973. Initial profiles of T and W were taken from the available measurements. Using meteorological measurements for the simulation period, we reproduced snow processes by the model described in Gelfan *et al*. (2004). The snow model was successively tuned to the available measurements of snow depth that allowed us to believe that the calculated snow temperature and outflow from snowpack, which were utilized to assign the upper boundary conditions for equations (5) and (6), are reasonably accurate. Since there are no measurements of T and W in the winter seasons, the lower boundary conditions (assigned at the depth of 1 m) were prescribed as follows. Water flux at a depth of 1 m was assumed to be equal to soil hydraulic conductivity, heat flux was calculated from the assumption of linear temperature distribution between the depth of 1 m and the depth of 3 m where T was taken as constant, equals +5°C.

Measurements of T and W in spring seasons are scarce and not synchronous with each other, resulting in a limited possibility for the presentation and interpretation of the modelling results. To test the performance of the model, we used measurements of the infiltration-excess overland flow, which were carried out at a bounded rectangular

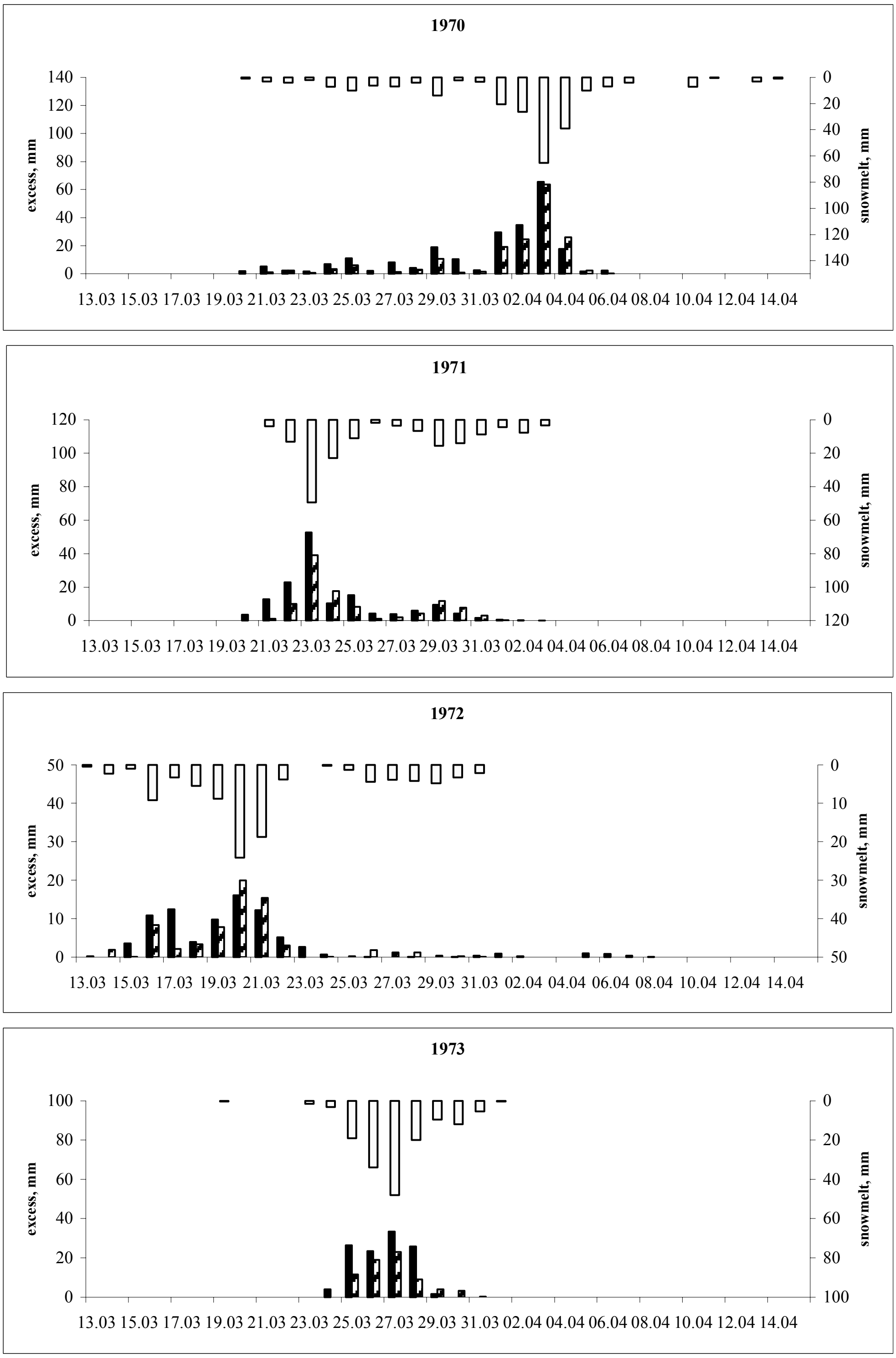

Fig. 3 Measured (black bars) and calculated (shaded bars) snowmelt runoff excess (white bars indicate calculated outflow from snowpack).

plot (20 × 5 m) representing a section of the watershed slope and located not far from the meteorological station. Daily excess measured for four spring melt periods was compared with the excess calculated as the difference in the modelled values of the outflow from snow and infiltration. It is seen from Fig. 3 that, in general, the results of the comparison are satisfactory. However, some underestimation of the calculated excess was obtained for each season.

CONCLUSIONS

The physically-based model of combined heat and moisture vertical flow in frozen soil has been developed. A specific procedure has been suggested for estimating the hydraulic conductivity, water retention, and freezing point depression curve of frozen soil through commonly available, measured characteristics (porosity, bulk density, field capacity and wilting point) of unfrozen soil and ice content. The model has been applied for simulation of frozen soil processes on the basis of laboratory and field observations. For the laboratory data, all parameters of the model have been estimated by measured soil constants and satisfactory results of the simulations have been reached. For the field data, only one parameter, namely, saturated hydraulic conductivity has been adjusted during the calibration procedure against measured soil water content in the summer season, whereas other parameters have been assessed using basic soil data. With a minimal calibration, the model has simulated the snowmelt flow excess from a small plot with a reasonable accuracy. This suggests that the developed model combined with the technique presented for parameter estimation may be a useful tool for modelling snowmelt infiltration losses at small ungauged basins in cold regions. However, experimentation with additional model applications is needed to either validate or disprove the suggestion.

Acknowledgement The present work was carried out as a part of the research project supported by the Russian Foundation for Basic Research.

REFERENCES

Gelfan, A. N. (2005) Prediction of runoff in poorly gauged basins using a physically-based model. In: *Prediction in Ungauged Basins: Approaches for Canada's Cold Regions* (ed. by C. Spence, J. Pomeroy & A. Pietroniro) (Proc. Workshop. Yellowknife, Canada, March 2004), 101–118. CWRA Publishers, Canada.

Gelfan, A. N., Pomeroy, J. W. & Kuchment, L. S. (2004) Modelling forest cover influences on snow accumulation, sublimation, and melt. *J. Hydrometeorology* **5**, 785–803.

Globus, A. M. (1969) *Experimental Hydrophysics of Soils*. Gidrometeoizdat. Leningrad, Russia. (in Russian).

Guymon, G. L., Hromadka, T. V. II & Berg, R. L. (1980) A one dimensional frost heave model based upon simulation of simultaneous heat and water flux. *Cold Reg. Sci. Technol*. **3**, 253–262.

Handbook of the Hydrological Properties of Soils in the Central-Chernozem Regions (1975). Publ. Hydrometeorological Survey of the Central-Chernozem Regions, Kursk, Russia (in Russian).

Harlan, R. L. (1973) Analysis of coupled heat-fluid transport in partially frozen soil. *Water Resour. Res*. **9**, 1314–1323.

Jame, Y.-W. & Norum, D. I. (1980) Heat and mass transfer in a freezing unsaturated porous medium. *Water Resour. Res.* **16**, 811–819.

Jansson, P. -E. & Halldin, S. (1980) SOIL water and heat model. Technical description. *Swedish Coniferous Forest Proj., Tech. Rep. 27.*

Johnson, H. & Lundin, L. -C (1991) Surface runoff and soil water percolation as affected by snow and soil frost. *J. Hydrol*. **122**, 141–159.

Kalyuzhny, I. L., Pavlova, K. K. & Lavrov, S. A. (1988) *Hydrophysical Investigations of the Meliorated, Humid Lands*. Gidrometeoizdat. Leningrad, Russia. (in Russian)

Koren, V. I. (1991) *Mathematical Models in Runoff Forecasting*. Gidrometeoizdat. Leningrad, Russia (in Russian).

Kuchment, L. S., Demidov, V. N. & Motovilov, Yu G. (1983) *River Runoff Formation (Physically-Based Models)*. Nauka, Moscow, Russia (in Russian)

Kuchment, L. S, Demidov, V. N. & Motovilov, Yu G. (1986) A physically-based model of the formation of snowmelt and rainfall runoff. In: *Symposium on the Modelling Snowmelt-Induced Processes* (ed. by E. M. Morris) (Budapest, July 1986), 27–36. IAHS Publ. 155. IAHS Press, Wallingford, UK.

Kulik, V. Ya (1969) About dependence of hydraulic conductivity of soil on ice content. *Met. Hydrol.* **9**, 70–77 (in Russian).

Lundin, L.-C. (1985) Simulated physical effects of shallow soil heat extraction. *Cold Regions Sci. Technol*. **11**, 45–61.

Lundin, L.-C. (1990) Hydraulic properties in an operational model of frozen soil. *J. Hydrol.* **118**, 289–310.

Motovilov, Yu. G. (1977) Numerical modelling of the infiltration of water into frozen soils. *Sov. Meteorol. Hydrol*. **9**, 55–62

PUB Science and Implementation Plan. Final Version. (2003), prepared by M. Sivapalan & J. Shaake. IAHS Bureau, Sapporo, Japan. July 2003 (available online http://www.cig.ensmp.fr/~iahs/)

Samarsky, A. A. (1983) *Theory of Finite-Difference Schemes*. Nauka, Moscow, Russia (in Russian).

Stähli, M., Jansson, P. -E. & Lundin, L. -C. (1999) Soil moisture redistribution and infiltration in frozen sandy soil. *Water Resour. Res.* **35**, 95–103.

van Genuchten, M. Th. (1980) A closed form equation for predicting the hydraulic conductivity of unsaturated. Soils. *Soil Sci. Soc. Am. J.* **44**, 892–898.

US Army Corps of Engineers (1998) *Runoff from Snowmelt. Engineer Manual No.1110-2-1406*. US Army Corps of Engineers, Washington, DC, USA.

Zaretcky, Yu. A. & Lavrov, S. A. (1986) Mathematical modelling the processes of soil freezing, thawing and infiltration. *Trans. State Hydrol. Inst*. **308**, 37–49 (in Russian).

Zhao, L., Gray, D. M. & Male, D. H. (1997). Numerical analysis of simultaneous heat and mass transfer during infiltration into frozen ground. *J. Hydrol*. **200**, 345–363.