Predictions in Ungauged Basins: Promise and Progress (Proceedings of symposium S7 held during the Seventh IAHS Scientific Assembly at Foz do Iguaçu, Brazil, April 2005). IAHS Publ. 303, 2006.

A multi-level and multi-scale structure of river network geomorphometry with potential implications towards basin hydrology

CHRISTOPHE CUDENNEC[1,2], JEAN-CHRISTOPHE POUGET[1], HOUDA BOUDHRAA[1,3], MOHAMED SLIMANI[3] & SLAH NASRI[4]

1 *UMR G-EAU, IRD, BP 434, 1004 Tunis, Tunisia*
christophe.cudennec@ird.fr

2 *UMR SAS, Agrocampus Rennes, 35042 F-Rennes Cedex, France*

3 *Génie rural, INAT, Mahrajène, 1082 Tunis, Tunisia*

4 *INRGREF, BP 10, 2080 Ariana, Tunisia*

Abstract The channelized water path distribution from any point in a basin to the outlet through the river network, and its hydraulic length L, are considered. Using the Strahler ordering scheme, each path is split into n i-order components whose lengths are l_i. The length L is studied as the sum $L = \sum_{i=1}^{n} l_i$, and subsets $L' = \sum_{i=1}^{m<n} l_i$ are considered. *pdf*(l_i) have a strong scaling property, except for the highest orders due to a hierarchical constraint. Consistently, *pdf*(L) is positively skewed, whereas successive *pdf*(L') display a more and more regular unimodal negatively-skewed shape. These evidences are exemplified with a set of Tunisian semiarid nested basins, showing regularity through levels, scales and heterogeneity, despite a weak global self-similarity.

Key words branched network; hydro-geomorphology; PUB; scaling; underlying pattern

INTRODUCTION

The river network is easily observed as a hydro-geomorphological structure, whatever the scale of the basin and the hydro-climatic context are, from various kinds of data sources and through diverse human-based and/or automatic procedures. Metric, organizational and scaling properties of a river network can thus be identified for any basin, independent of its degree of (un-)gauging. The identification of the major links between geomorphometry and hydrology may thus lead to a kind of robust "geomorphometric gauging", eventually to be adapted according to the hydrological stakes, scales and contexts.

The aim of this paper is to present: (a) the analysis of the river network organization through particular objects and variables—channelized paths down to the outlet, splitting through the Strahler levels, subsets of level cascade, and associated lengths and *pdf*s—which leads to the identification of a multi-level and multi-scale underlying structure of diverse, strongly self-similar river networks (see Cudennec *et al.*, 2004); and (b) its application to a particular set of nested basins whose self-similarity is weak because of the heterogeneity of drainage characteristics, in order to check its genericity.

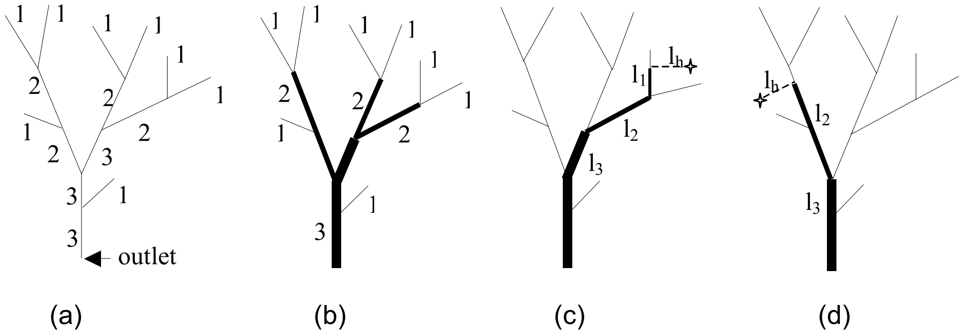

Fig. 1 Hypothetical basin of order $n = 3$. Strahler ordering: (a) Links. (b) Streams of order 1 ———, order 2 ▬▬ and order 3 ▬▬ . (c) and (d) Examples of paths run by a water drop from a point to the outlet. l_h ------ is the length of the path through the hillslope. l_1 ———, l_2 ▬▬ and l_3 ▬▬ are, respectively, the lengths of 1st, 2nd and 3rd orders components.

RIVER NETWORKS' GEOMORPHOMETRY: HIERARCHY-BASED AND GLOBAL APPROACHES

The branched topology and scaling

In a basin, the river network is made up of particular points: the unique outlet, upstream extremities and junctions. The part contained between two such successive particular points is a link. The Strahler (1952) ordering scheme permits the description of a link position in a river network (Fig. 1(a)). According to this ordering method, the stream is defined as the set of successive links of the same order (Fig. 1(b)). The spectrum of orders represented in the network can be seen as a cascade, in the frame of which scaling evidences have been identified.

The first two Horton laws give the number of streams N_i and the mean stream length $\overline{L_i}$ of Strahler order i (Horton, 1945; Schumm, 1956), on the basis of a bifurcation $R_B = N_i / N_{i+1}$ and a length $R_L = \overline{L_{i+1}} / \overline{L_i}$ ratios whose stabilities are statistically verified. These branching laws, among other pioneering evidence of scaling, were then embraced within the application of the fractal geometry to river networks, which led to the identification of generic self-affine and self-similar properties. From their synthesis of the topic, Rodriguez-Iturbe & Rinaldo (1997) reached the conclusion that "the search for invariance properties across scales as a basic hidden order in hydrologic phenomena is one of the main themes of hydrologic science". Our aim is to contribute to this search through the analysis of a basin-level geomorphometric function through the Strahler cascade.

Basin-level geomorphometric functions

From the hydrological point of view, one can focus on a function describing the basin organization in terms of flow paths until the outlet through the network. In this context, both the area function and the width function have been introduced (see Rinaldo &

Rodriguez-Iturbe, 1996). These functions are the *pdf*s, respectively, of the basin contributing area and of the number of links in the network, with respect to flow distance to the outlet. Any eventual regular pattern observable for these functions would be a useful network-level geomorphometric gauge, eventually linked with basin-level hydrological processes and features. But one can notice that, even if the width function proves to itself have scaling characteristics (see Rodriguez-Iturbe & Rinaldo, 1997), the relationship between the river network self-similarity and the shape—particularly the systematic skewness—of the area and width functions is not yet fully understood and assessed.

THE CONSIDERED GEOMORPHOMETRIC FRAMEWORK

Within the path followed by a water drop to reach the outlet, the hillslope path and the channelized path through the river network are considered separately. The length L of the channelized path is identified and called the hydraulic length. The *pdf* of L, i.e. the area–hydraulic length function of the basin, is slightly different from both the width function and the area–distance function, in a sense which opens two major interests and perspectives. It is indeed based on, and limited to, the river network seen: (a) as a fractal object within the two-dimensional (2-D) planar basin; and (b) as a structure determining both the drainage from hillslopes and the upstream–downstream transfer.

Following Cudennec *et al.* (2004), within the channelized path, the "ith order component" is defined as the part run through successive channels of the same order i (Fig. 1(c) and 1(d)). By definition, for any geographic point of the basin, the hydraulic length is the sum $L = \sum_{i=1}^{n} l_i$ where n is the basin order and l_i the length of the ith order component.

Furthermore, the network scaling is assumed to apply to the components. Thus, besides Horton's stream length ratio R_L, the component length ratio $r_l = \overline{l_{i+1}} / \overline{l_i}$ is defined, where $\overline{l_i}$ is the mean length of the ith order components. *pdf*(L) and *pdf*(l_i) are studied as structural functions, at the levels respectively of the whole network and of the Strahler levels. Moreover, in order to study subsets of the whole Strahler cascade, truncated hydraulic lengths $L' = \sum_{i=1}^{m<n} l_i$ are also considered for various values of m, as well as the corresponding functions *pdf*(L'). For a given basin, all these variables and functions are observable through a dedicated GIS-based analysis.

EVIDENCE OF A MULTI-LEVEL AND MULTI-SCALE STRUCTURE

A set of evidence of an underlying regularity has been shown for actual networks which respect a strong self-similarity (Cudennec *et al.*, 2004). Here we consider the Skhira basin in semiarid central Tunisia (Fig. 2) (see Cudennec *et al.*, 2005). The outlet coordinates are 35°44′15 N and 9°23′05 E in UTM system, the basin surface area S is 192 km^2 and the Strahler order n is 6. The river network was observed from 1:50 000

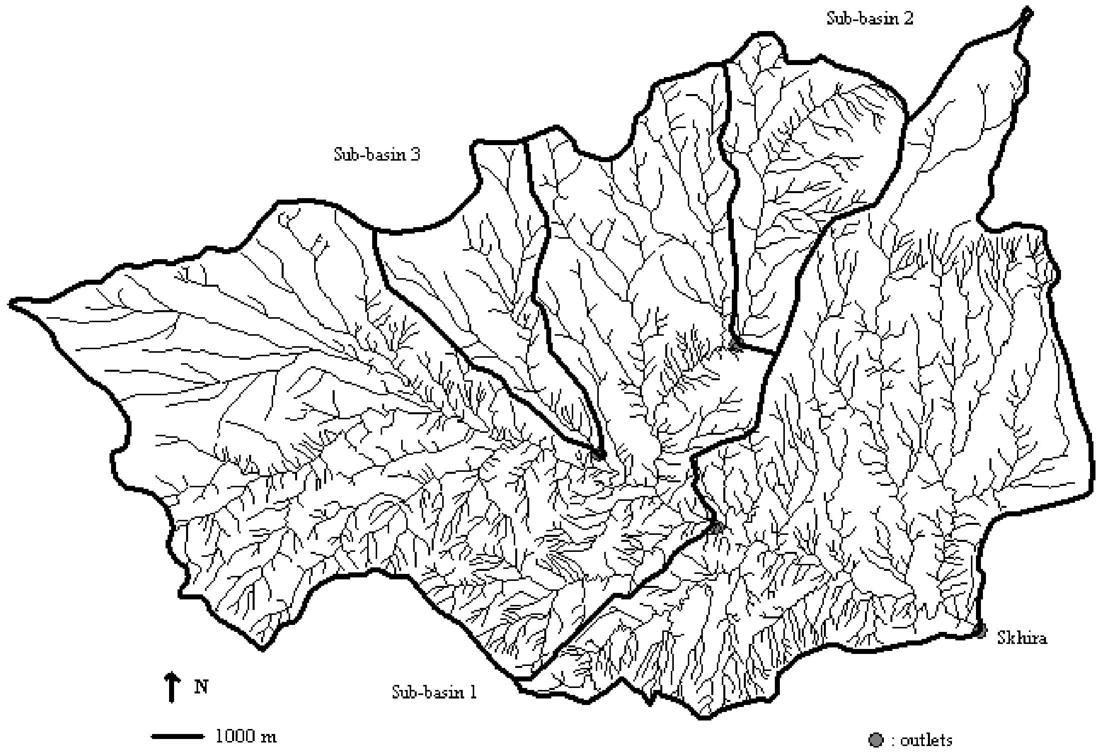

Fig. 2 River map of the Skhira basin (S = 192 km², n = 6) and of sub-basins 1, 2 and 3 (respectively S = 123 km², n = 6; S = 15 km², n = 5; S = 12 km², n = 3).

topographic maps. Three sub-basins are also considered in order to cross-study the influence of size, Strahler order and branching rules.

Basin-level observations

Table 1 presents the observed values of the Horton ratios R_B and R_L and of the newly defined component length ratio r_l. The global values are obtained by logarithmic regressions, whose R^2 coefficients are indicated. It appears that the Skhira basin respects a weak self-similarity: the quality of regressions is good for R_B for each basin, whereas it is very variable for R_L and r_l. The quality increases from the whole basin to sub-basin 1, over the same range of Strahler orders (1 to 6). Further than the size, this is related to the length of the stream of order n = 6. Furthermore, when focusing on smaller sub-basins, the quality either remains equivalent (sub-basin 3) or becomes very low (sub-basin 2). This variability of ratio values and stabilities is caused by the heterogeneity of geographic constraints and thus of the river network itself (see Fig. 2). It appears, nevertheless, that ratios of acceptable stability can emerge at certain aggregative levels and sizes (e.g. sub-basin 1) despite the heterogeneous quality of subsets. Moreover, it appears (Table 1) that the values of R_L and r_l, as well as the quality of the related regressions, behave similarly.

Figure 3 shows $pdf(L)$ of the four nested basins. Despite the big differences between the basins, the four functions are irregular and broadly positively skewed,

Table 1 Assessment of R_c, R_L and r_l ratios for the four basins.

		Order i – i+1					Global	R^2
		1–2	2–3	3–4	4–5	5–6		
Skhira basin	R_B	6.95	5.46	3.85	2.64	1.1	3.88	0.996
S = 192 km²	R_L	0.52	1.61	1.89	2.4	3.72	1.98	0.965
n = 6	r_l	0.72	1.6	1.9	1.76	3.6	1.88	0.88
sub-basin 1	R_B	6.49	5	3.47	2.3	1.1	3.47	0.993
S = 123 km²	R_L	0.49	1.45	1.63	2.39	2.82	1.79	0.98
n = 6	r_l	0.68	1.4	1.45	2.08	2.6	1.7	0.96
Sub-basin 2	R_B	4.34	2.94	1.61	0.7		2.73	0.97
S = 15 km²	R_L	1.13	3.19	0.25	10.46		1.46	0.43
n = 5	r_l	0.89	4.04	0.19	17.33		1.44	0.35
Sub-basin 3	R_B	5.14	7				6.19	0.997
S = 12 km²	R_L	1.75	5.77				2.82	0.9
n = 3	r_l	2.49	3.43				2.83	0.991

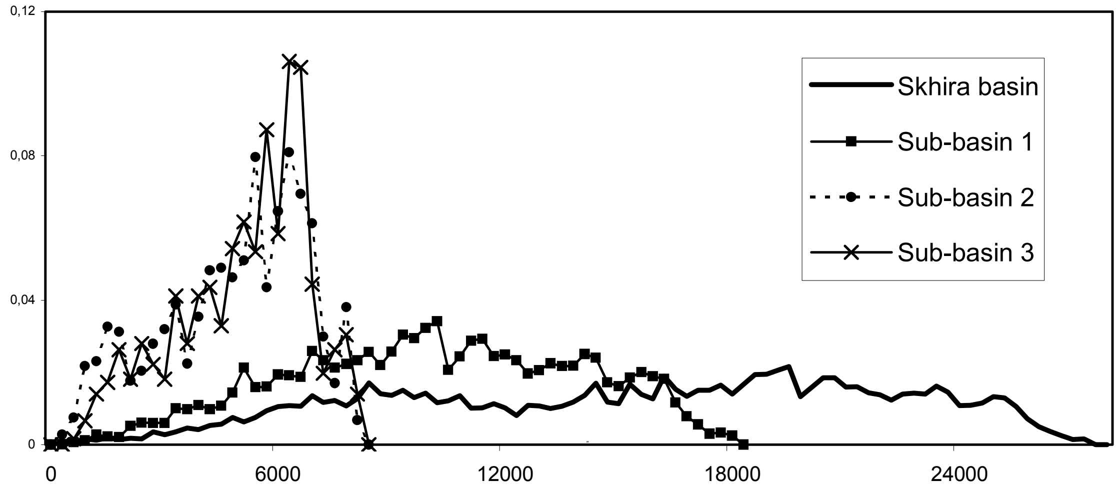

Fig. 3 *pdf* of hydraulic length L of the Skhira basin and the three sub-basins.

which is relevant with the generic skewness of width functions (see Rodriguez-Iturbe & Rinaldo, 1997). Moreover, it is verified that they are directly linked with the size of the basin and not with its Strahler order.

A multi-level underlying pattern

We observe how *pdf*(L) is structured and organized in terms of components. Figures 4(a), 5(a), 6(a) and 7(a) present the *pdf*s of the lengths of the components of the nested basins, reduced by r_l used as a scaling factor: l_i / r_l^{i-1}. The n-2 first curves are very close together, convex and rapidly decreasing, for each basin. This shows that the scaling of the components is verified in terms of distribution, in an even better way than in terms of geometric averages. The (n-1)th curve is more or less within the envelop of the others, whereas the nth one is very different. This shows a difference in the role and the representativeness of the highest order components, due to their particular hierarchical positions. Indeed the last order n is unavoidable for channelized paths from anywhere in the basin to the unique outlet (See Fig. 1(c) *vs* 1(d)). Thus $pdf(l_n)$ can necessarily not be decreasing. A similar—but statistically decreasing—effect may exist for the (n-1)th component, and maybe even further upstream, depending on the particular branching of the basin considered. Consistently, and stressing the use of r_l as a scaling factor, it appears that Figs 4(a), 5(a), 6(a) and 7(a) show: (a) the generic scaling of components under a "hierarchical constraint"; (b) a combined size and topological effect through both the shape and maximum abscissa of $pdf(l_n)$ (e.g. l_6 in Figs 4(a) and 5(a)) and the number of Strahler levels (e.g. 5 in Fig. 6(a) and 3 in Fig 7(a)).

(a)

(b)

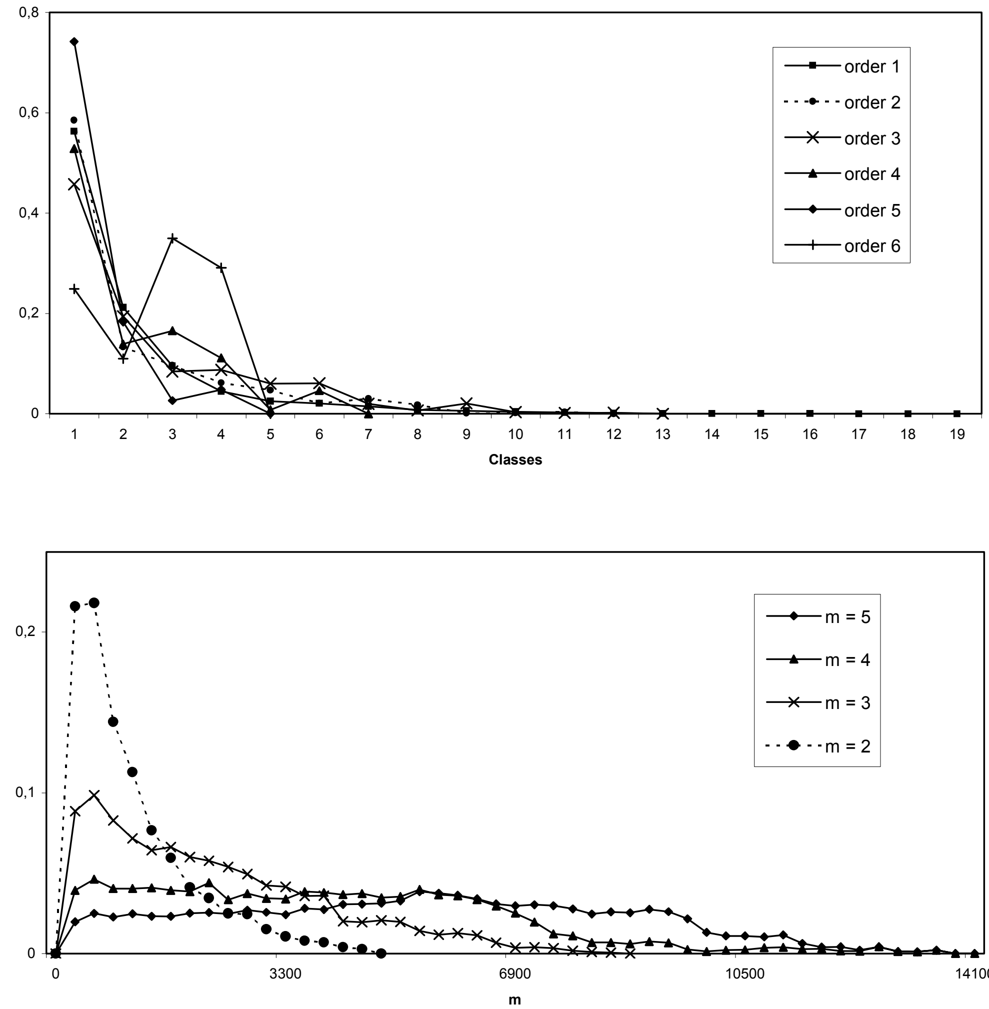

Fig. 4 The multi-level structure of the Skhira basin. (a) *pdf* of reduced lengths of components l_i / r_l^{i-1} (r_l = 1.88, classes of 200 m). (b) *pdf* of truncated hydraulic lengths $L' = \sum_{i=1}^{m<n} l_i$.

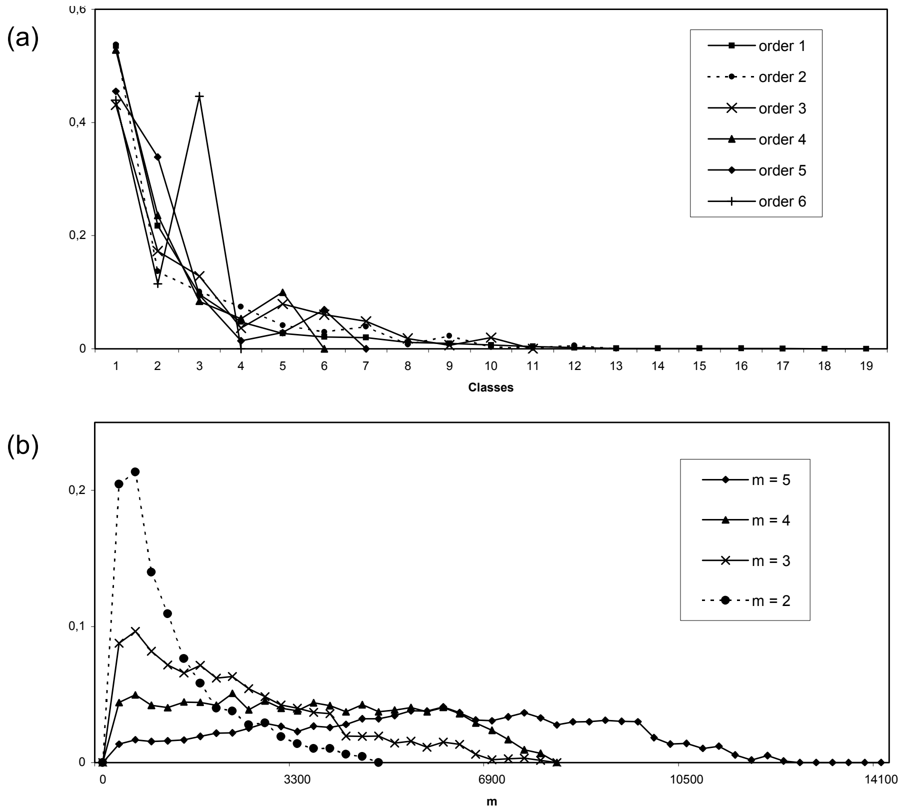

Fig. 5 The multi-level structure of the sub-basin 1. (a) *pdf* of reduced lengths of components ($r_l = 1.7$, classes of 200 m). (b) *pdf* of truncated hydraulic lengths L'.

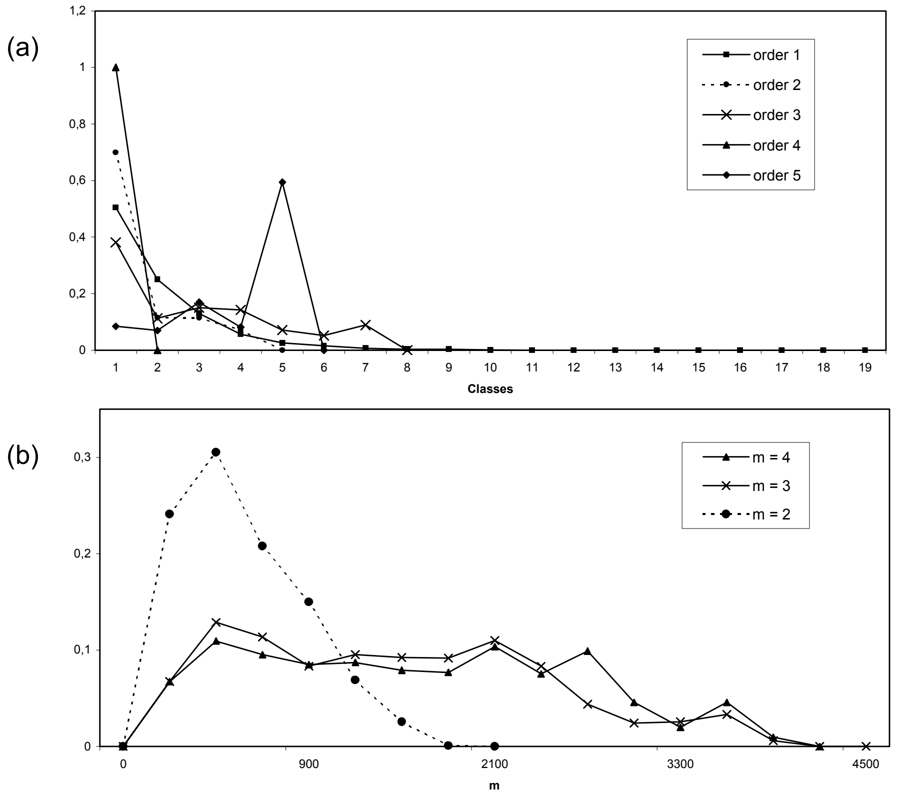

Fig. 6 The multi-level structure of the sub-basin 2. (a) *pdf* of reduced lengths of components ($r_l = 1.44$, classes of 200 m). (b) *pdf* of truncated hydraulic lengths L'.

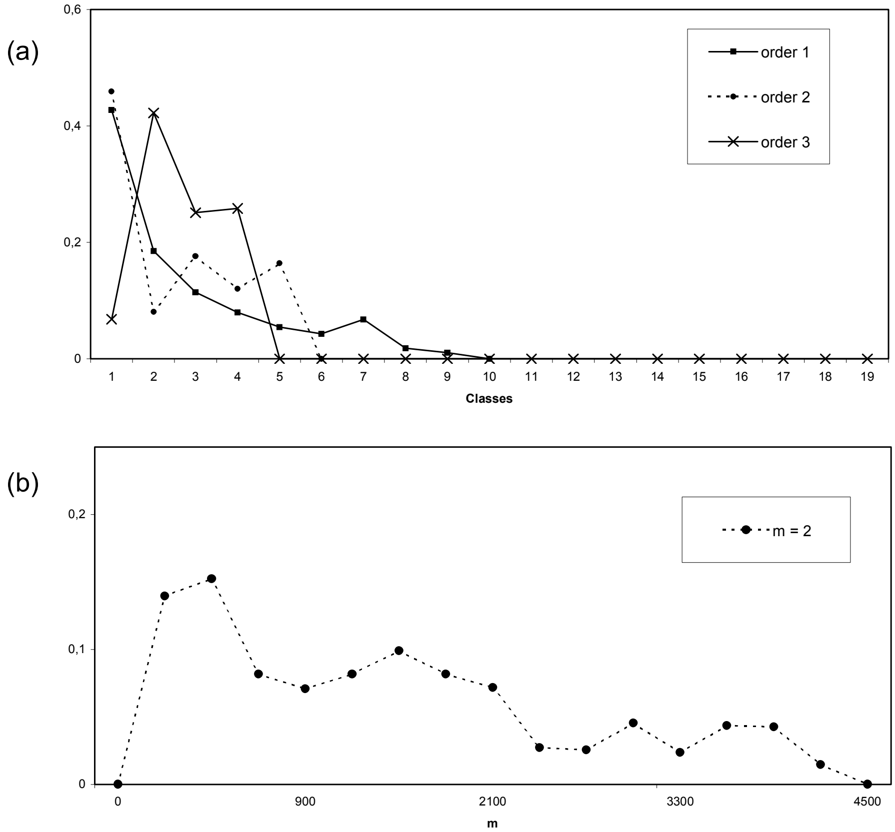

Fig. 7 The multi-level structure of the sub-basin 3. (a) *pdf* of reduced lengths of components (r_l = 2.83, classes of 200 m). (b) *pdf* of truncated hydraulic lengths L'.

This hierarchical constraint led us to consider the sub-system of the m ($m<n$) first Strahler levels, by defining the sum $L' = \sum_{i=1}^{m<n} l_i$, corresponding to the truncation of the highest orders components. Figures 4(b), 5(b), 6(b) and 7(b) show the *pdf*(L') of the four basins, for decreasing values of m. It appears that the skewness is obviously becoming negative and that a regular unimodal shape is appearing. This pattern comes from the combination of a subset of scaled—but also hierarchy-constrained—components, and is thus by nature underlying: the regular shape of *pdf*(L') for m = 3 of the whole basin and sub-basin 1 is very different from the shape of *pdf*(L) of sub-basin 3 whose Strahler order is n = 3.

CONCLUSION

We consider hydro-geomorphological objects and associated variables within the river network of a basin, based on the Strahler ordering scheme: the channelized path and its components, characterized by L and l_i. It appears that the *pdf*(l_i) follow convex and rapidly decreasing shapes and have a strong scaling property, except for the highest orders. This scaling is consistent with river networks fractality and its upper limit can be explained by the branched-out topology. Consistently the *pdf*s of the truncated hydraulic lengths L' display more and more regular unimodal and negatively skewed shapes, whereas $pdf(L)$ is irregular and positively skewed. It is shown here that these structural evidences can be identified even for a basin whose self-similarity is weak, due to geographic heterogeneities; it was deliberately chosen in order to face non-ideal contexts, in coherence with the PUB framework. The relationships between the scaled characteristics through the Strahler cascade, the regular underlying emerging patterns and the irregular geomorphometric function at the level of the whole basin, may be further understood and assessed in terms both of an analytical framework of deducible emergence, and of generality through scales and geographic heterogeneity. Moreover, the hydrological meaning of the variables and functions considered could open various modelling prospects, particularly in the framework of geomorphology-based transfer functions, within which new junctions between approaches based on branched topology and on geomorphometric functions could be identified (see Snell & Sivapalan, 1994; Robinson *et al.*, 1995; Rinaldo & Rodriguez-Iturbe, 1996).

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