*Calibration and Reliability in Groundwater Modelling: From Uncertainty to Decision Making*
(Proceedings of ModelCARE'2005, The Hague, The Netherlands, June 2005). IAHS Publ. 304, 2006.

# A simple procedure for the identification of aquifer parameters based on steady-state pumping tests

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**Abstract** Three-dimensional steady flow towards a fully penetrating well of radius $r_w$, in a confined aquifer, is studied by means of a finite-volume numerical scheme. The hydraulic conductivity $K$ is modelled as an axi-symmetric, stationary random space function mimicking hydraulic property variations at the local scale. We develop a new methodology for the identification of the geostatistical model of variability from a steady-state pumping test involving a few wells. A constant water discharge is extracted from each well in sequence while measuring the steady-state drawdown at the remaining wells. To show its potential, the new methodology is applied to the data set obtained from a synthetic example. We conclude that the new methodology may serve as a simple tool to assess the three-dimensional statistical structure of hydraulic conductivity at the local scale.

**Keywords** equivalent conductivity; heterogeneous aquifer; pumping test

## INTRODUCTION

Flow towards a pumping well has been the subject of several studies in recent years due to its importance in many applications of groundwater hydrology, such as groundwater supply, restoration of contaminated groundwater and identification of hydrogeological parameters, to mention a few. Most of these studies were conducted under the common assumption that the aquifer is homogeneous, and the result of such efforts is a large number of solutions for several hydrogeological and experimental conditions which are widely used in applications. However, the flow field is much more complex than predicted by the above solutions because the Earth's subsurface is inherently heterogeneous (see e.g. Rubin, 2003). Recently, a number of solutions were obtained assuming that hydraulic property variations can be modelled assuming the hydraulic log-conductivity $Y = \ln K$, where $K$ is the hydraulic conductivity, is a stationary Random Space Function (RSF) and adopting small perturbation expansion in the log-conductivity variance in order to obtain close-form solutions. We focus here on the three-dimensional local-scale flow towards an extracting well in a heterogeneous formation due to the importance that this problem assumes in aquifer characterization. The problem has been studied in the past mainly for weakly heterogeneous formations (e.g. Shvidler, 1966; Naff, 1991; Desbarats, 1994; Indelman *et al.*, 1996; Sanchez-Vila, 1997; Tartakovsky & Neuman, 1998; Fiori *et al.*, 1998; Hemker, 1999; Riva *et al.*, 2001; Guadagnini *et al.*, 2003; Indelman, 2003). The principal aim here is to explore whether the identification of the geostatistical model of variability is possible by using the head measurements in fully penetrating wells. We use the analytical solution for the equivalent conductivity obtained by Indelman *et al.* (1996) and a three-dimensional numerical scheme for solving the flow equation. A synthetic example showing the potential of the proposed methodology closes the paper.

## THE MATHEMATICAL FRAMEWORK

Interpretation of the drawdown caused by a pumping well by means of a flow model is the most widespread approach for aquifer characterization. The typical configuration for a pumping test requires a well extracting at constant discharge and an observation well at a known distance from the extraction well. Both the wells are assumed to fully penetrate the aquifer. In the standard procedure for aquifer characterization, the hydraulic conductivity is obtained by fitting a suitable analytical solution to the experimental data. The resulting $K$ value represents the hydraulic conductivity of a fictitious homogeneous aquifer producing at the monitoring well the same drawdown as the real aquifer. It is defined over a support volume whose size is much larger than the Darcy's scale. This has motivated Matheron (1967) to define the equivalent conductivity $K_{eq}$ as the hydraulic conductivity obtained by inverting the Thiem equation:

$$K_{eq} = \frac{Q \ln(r / r_w)}{2\pi D[< h(r) > - < h(r_w) >]} \tag{1}$$

where $r$ and $r_w$ are the distance from the extraction well and the well radius, respectively. Furthermore, $Q$ is the total discharge, $D$ is the aquifer depth and $<h(r)>$ the mean piezometric head at the distance $r$ from the extraction well. The resulting equivalent conductivity $K_{eq}$ is a global property characterizing the portion of the aquifer between the well and a cylinder of radius $r$ co-axial with the vertical axis of the extraction well.

Here we model the hydraulic log-conductivity $Y = \ln K$, as a normally distributed RSF with mean $<Y>$ and variance $\sigma_Y^2$ both constant, and the axisymmetric exponential two-point covariance function adopted by Gelhar & Axness (1983) to model hydraulic property variations at the local scale:

$$C_Y(\rho_x, \rho_y, \rho_z) = \sigma_Y^2 \exp[-\sqrt{(\rho_x^2 + \rho_y^2)\big/ I_{Yh}^2 + \rho_z^2 \big/ I_{Yv}^2}\,] \tag{2}$$

where $I_{Yh}$ and $I_{Yv}$ are the horizontal and vertical log-conductivity integral scales, respectively, and $\rho(\rho_x, \rho_y, \rho_z)$ is the two-point distance vector. A first-order approximation in $\sigma_Y = \sqrt{\sigma_Y^2}$ of the equivalent conductivity $K_{eq}$ for this model of spatial variability was obtained by Indelman *et al.* (1996) with the boundary condition of constant head at the extraction well which is modelled as a source line with flow rate proportional to the local conductivity:

$$K_{eq} = K_A[1 - \lambda(r, e)] + K_{efu}\lambda(r, e) \tag{3}$$

where $\lambda$ is an analytical function, bounded between 0 and 1, the expression for which is given in Indelman *et al.* (1996). Equation (3) is formally valid for $\sigma_Y^2 << 1$ and for $r_w/I_{Yh} << 1$. Equation (3) shows that $K_{eq}$ varies between the arithmetic mean $K_A$ for $r/I_{Yh}$

$\ll 1$ and $K_{efu}$ for $r/I_{Yh} \gg 1$, where $K_{efu}$ is the effective conductivity for uniform flow. Equation (3) has been obtained under the hypothesis that the thickness $D$ of the aquifer is much larger than $I_{Yv}$ such that ergodic conditions are obtained and $K_A$ converges to the expected value of the hydraulic conductivity: $K_A \cong \langle K \rangle = \exp[\langle Y \rangle + \sigma_Y^2 / 2]$.

## THE NUMERICAL MODEL

A fully penetrating well extracting a constant water discharge $Q$ is placed at the centre of a three-dimensional domain of sides $L_1 = L_2 = 41I_{Yh}$, in the horizontal plane and thickness $D = 60I_{Yv}$. A single realization of $Y$ was generated using HYDRO-GEN (Bellin & Rubin, 1996), and the flow equation was solved numerically using the seven-node finite volume (FV) scheme (Anderson & Wossner, 1992). The log-conductivity field was generated over a uniform grid with spacing $0.2I_{Yh}$ and $0.2I_{Yv}$ in the horizontal and vertical directions, respectively. The computational grid was uniform in the vertical direction with a spacing $0.2I_{Yv}$, and non-uniform in the horizontal plane with spacing increasing from $0.04I_{Yh}$ at the extraction well to $0.2I_{Yh}$ at a distance of $0.8I_{Yh}$ from it. For larger distances, the grid spacing was kept constant and equal to $0.2I_{Yh}$. Furthermore, the grid was designed to respect the condition that the ratio between the sides of two surrounding cells should not be larger than 1.5 (Anderson & Wossner, 1992, p.64). The numerical code was tested by comparison with the Thiem solution for the homogeneous case.

In the FV scheme the well is represented by blocks aligned along its axis. Under common operational conditions the water head $h_w$ does not change along the well column because the head loss inside the casing is negligible. We model this situation by assuming that the water flux at each cell of the well is proportional to the local conductivity (see Indelman *et al.*, 1996); the condition is applied in a discrete form subdividing $Q$ between the $N$ blocks of the well in proportion to the hydraulic conductivity of the block: $Q_j = QK_j / \overline{K}$, where $Q_j$ is the water discharge extracted from the block $j$, with hydraulic conductivity $K_j$, and $\overline{K} = \sum_{j=1}^{N} K_j / N$ is the vertically-averaged hydraulic conductivity at the well. After a best fit with the Thiem's curve, the equivalent well radius, $r_w$ (Anderson & Woessner, 1992) was found to equal $7.85 \times 10^{-3} I_{Yh}$.

## THE IDENTIFICATION PROBLEM

We are interested in a simple inverse procedure for the identification of the model of spatial variability, which here assumes the form of equation (2). Thus, our main objective is the characterization of the formation's heterogeneity at the local scale, in the terminology of Dagan (1989). Assuming second-order stationarity, the parameters that characterize the geostatistical model of variability (equation (2)) are: $K_G$, $\sigma_Y^2$, $I_{Yh}$, and $I_{Yv}$. We envision a simple inverse procedure which takes advantage of the fact that the vertically averaged head $\overline{h}(r)$, as measured in a fully penetrating monitoring well, depends on $K_{eq}$ from equation (1), and that in turn the resulting $K_{eq}$ depends on the geostatistical models of variability and the distance of the monitoring well from the extraction well, as described by equation (3). The procedure for obtaining $K_{eq}$ is the same used for the inference of $K$ in a homogeneous confined aquifer. Under the stationary and ergodic hypotheses, the mean head $\bar{h}$ is a deterministic quantity, and the response of the aquifer to pumping in terms of mean head is the same at all the locations with the same distance $r$ from the extraction well.

The inference proceeds as follows: first $\bar{h}$ is recorded at several distances from the extraction well, and $K_{eq}$ is computed by solving equation (1). Then the parameters $K_G$, $\sigma_Y^2$, $I_{Yh}$, and $I_{Yv}$ are obtained by best fitting equation (3) to the inferred $K_{eq}$ values.

To illustrate the inference procedure, we simulated synthetically a pumping test with $N_w$ wells, located at a relative distance $r_{ij}$ (i.e. the radial distance from well $i$ to well $j$). A constant water discharge is extracted from one of the wells and the head is measured at the remaining wells. The procedure is then repeated, changing the extraction well in such a way as to obtain $N_m = N_w\ (N_w - 1)/2$ independent measurements of $\bar{h}$.

For this illustration, we assume that the aquifer parameters are of the same order of magnitude as those of the Borden aquifer, (Sudicky, 1986), that is $K_G = 10^{-4}$ m s$^{-1}$, $\sigma_Y^2 = 0.5$, $I_{Yh} = 3$ m, $e = 0.1$. We assume three experimental setups with $N_w = 4, 5, 7$ wells as indicated in Fig. 1, for a total number of independent mean head measurements of $N_m = 6, 10, 21$. Each set is obtained from the previous one by eliminating the well labelled with the highest number (see Fig. 1). The well locations are generated randomly by selecting a set-up that produces a roughly uniform distribution of the distances $r_{ij}$ in an area of approximately $40 \times 60$ m$^2$. The mean head is calculated through our numerical solution, and is a function of the relative well distance $r_{ij}$. The

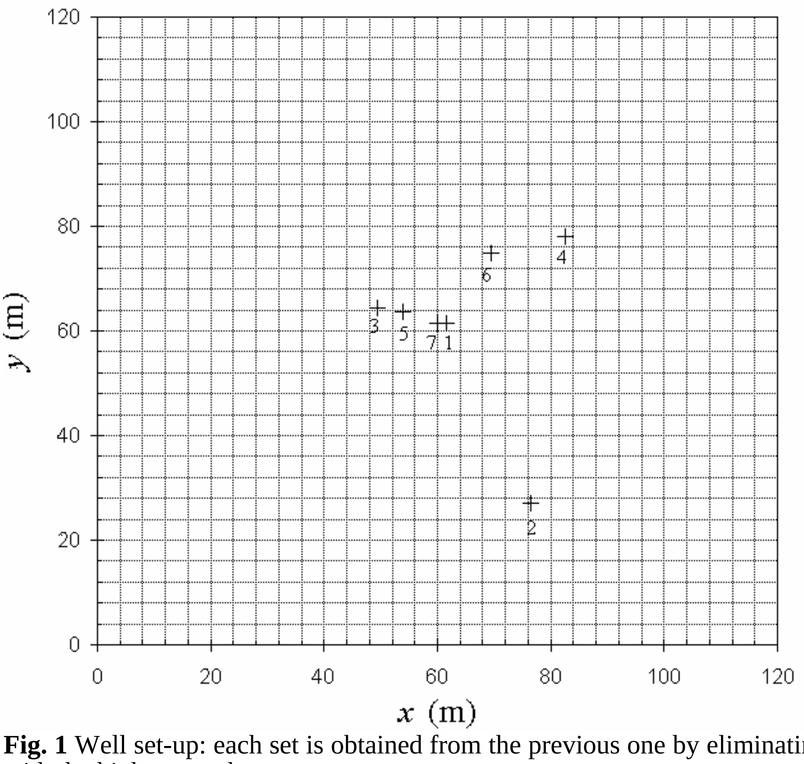


**Fig. 1** Well set-up: each set is obtained from the previous one by eliminating the wells with the highest number.

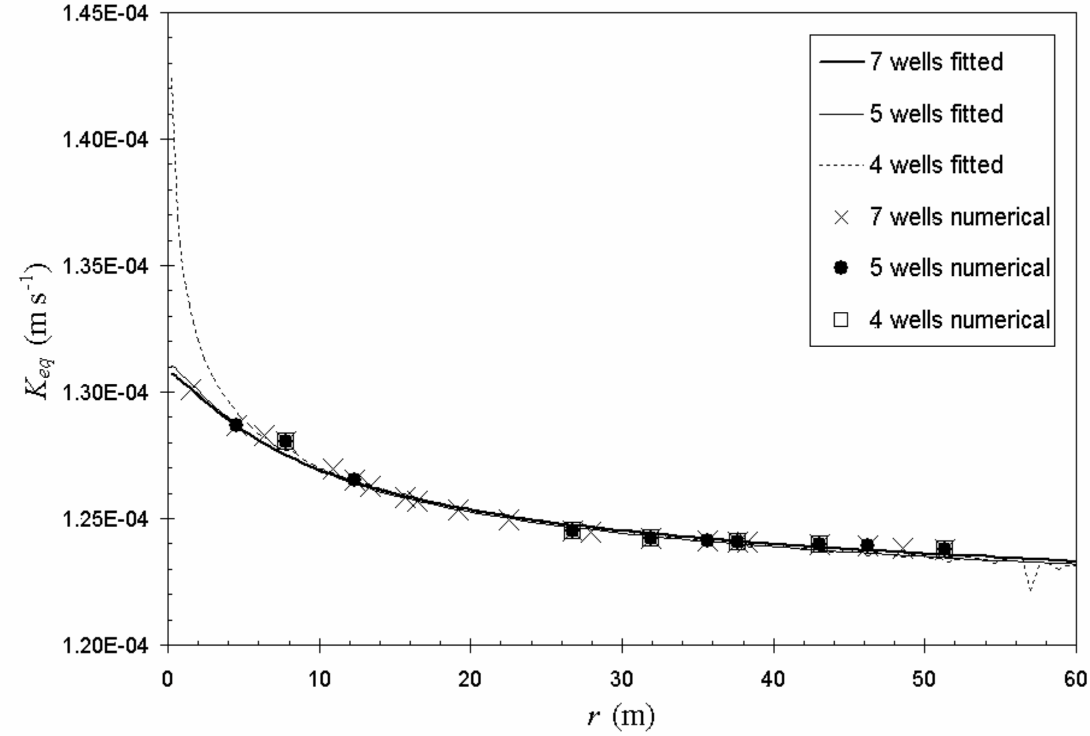


**Fig. 2** Fitting of the first order solution of $K_{eq}$ inferred from the pumping test.

first-order solution of $K_{eq}$ as given by equation (3) is then fitted to the values of $K_{eq}$ inferred from the pumping tests, and the result is shown in Fig. 2.

The parameters estimated through the fitting procedure for $N_w$ = 7 are as follows (the number in parenthesis is the standard error of the estimate): $K_G = 1.032 \times 10^{-4}$ ($0.156 \times 10^{-4}$) m s$^{-1}$, $\sigma_Y^2$ = 0.473 (0.307), $I_{Yh}$ = 4.059 (0.855) m, $e$ = 1.0 (1.86). With the exception of $I_{Yh}$ which is overestimated by about 35%, and $e$, the estimated parameters are close to the exact ones. The estimate of $e$ is error prone, and we also found similar problems after changing the other parameters. The principal reason of the bias in the inferred $e$ is the scarce sensitivity of $\lambda$ to $e$, which reflects negatively on the estimate yielding a relatively flat objective function. This is further highlighted by the large standard error of the estimate. The limited dependence of $\lambda$ on $e$ is also felt by the other parameters, and in particular $\sigma_Y^2$, and $I_{Yh}$, which are also characterized by relatively large standard errors; the estimate of $K_G$ is more robust.

Results for the other cases are as follows: for $N_w$ = 4, $K_G = 0.749 \times 10^{-4}$ ($8.533 \times 10^{-4}$) m s$^{-1}$, $\sigma_Y^2$ = 1.384 (23.792), $I_{Yh}$ = 0.162 (0.0006) m, $e$ = 0.440 (21.410); for $N_w$ = 5, $K_G = 1.025 \times 10^{-4}$ ($0.021 \times 10^{-4}$) m s$^{-1}$, $\sigma_Y^2$ = 0.493 (0.035), $I_{Yh}$ = 3.708 (3.324) m, $e$ = 1.0 (9.170). It is seen that the procedure roughly stabilizes after five pumping wells (10 head measurements available). The poor behaviour of the $N$ = 4 set-up is mainly due to the small number of independent measurements available for small/intermediate distances $r_{ij}$, which leads to an incorrect prediction of the horizontal integral scale. We note once again the large standard error of $e$, which confirms its unreliable estimate.

To further investigate the origin of the low sensitivity of $K_{eq}$ from $e$ we consider the following large distance ($r/I_{Yh} >> 1$) asymptotic limit of $\lambda$ (Indelman *et al.*, 1996): $\lambda \approx \ln(r/r_{Yh})/\ln(r/r_w)$. Substituting this expression into equation (3) we obtain: $K_{eq} = C_1 + C_2/\ln(r/r_w)$, with $C_1 = K_{efu}$ and $C_2 = \ln(I_{Yh}/r_w)(K_A - K_{efu})$ which is independent from $e$ and varies linearly with $1/\ln(r/r_w)$. Thus, the four parameters are embedded in the two coefficients $C_1$ and $C_2$, and two out of the four parameters have to be estimated independently. The use of the complete expression for $\lambda$ permits the solution of the mathematical problem, because of the dependence of $\lambda$ on $e$ (unfortunately a weak one), at the expense of an increased standard error and an uncertain and error-prone prediction of the anisotropy ratio.

Despite the above problems, the simple fitting procedure leads to reasonable estimates of the other three parameters, in particular $K_G$ and $\sigma_Y^2$, which are probably among the most important parameters characterizing natural aquifers. The order of magnitude of the horizontal integral scale $I_{Yh}$ is also well predicted, but the estimate here is more uncertain. In fact, the estimated $I_{Yh}$ is shown to depend on the density of wells at short distances $r_{ij}$. In fact, $I_{Yh}$ is inversely proportional to the slope of $(1 - \lambda)$ at $r \to 0$, and an insufficient number of measurements at small/intermediate distances $r_{ij}$ may lead to an uncertain estimate of $I_{Yh}$. Despite the above and other limitations, we consider our simple methodology as a promising tool for the characterization of hydraulic conductivity at the local scale.

## CONCLUSIONS

A simple inverse procedure for the identification of the geostatistical parameters that define spatial variability at the local scale, was presented. The method consists in the best fit of equation (3) by the use of "observed" values from a synthetic pumping test obtained from a large, three-dimensional numerical model. Different well set-ups have been used in the parameter estimation procedure. The main results can be listed as follows:

- for each well configuration, the estimate of the formation anisotropy ratio $e$ is error prone because of the scarce sensitivity of the function $\lambda$ (as it appears in equation (3)) to $e$, which reflects negatively on the estimate, yielding a relatively flat objective function;
- the low sensitivity of $\lambda$ on $e$ influences the other parameters like $\sigma_Y^2$ and $I_{Yh}$, which have also a large standard deviation;
- the estimate of the geometric mean of hydraulic conductivity $K_G$ is more robust;
- the procedure roughly stabilizes when using a number of wells equal or larger than 5;
- the estimates of the parameters $K_G$, $\sigma_Y^2$ and $I_{Yh}$ seem reliable and are of the same order as the model parameters; we emphasize that the latter three parameters are among the most important ones for natural aquifer characterization.

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