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A tomographic approach to identify hydraulic conductivity of a phreatic aquifer by the "differential system method"

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Abstract The hydraulic conductivity of a phreatic aquifer, under the Dupuit assumption, can be determined by solving an inverse problem. If two stationary sets of data (heads and source terms) corresponding to different flow situations are available, the inverse problem can be solved with a simple modification of the differential system method. Numerical tests show the effectiveness of this method as well as the excellent results that can be obtained by applying it to multiple sets of stationary data; in this case different couples of sets of data can be used to identify conductivities in different subdomains.

Keywords hydraulic conductivity; inverse problem; phreatic aquifer

INTRODUCTION

The differential system method (DSM) has been proposed to identify the phenomenological coefficients of physical systems which are modelled with the classical diffusion equation (Parravicini *et al.*, 1995). Numerical applications to the identification of transmissivity, *T*, and storativity of a confined aquifer have been presented (Giudici *et al.*, 1995; Vázquez González *et al.*, 1997). Discussions of the stability of the method and of its link with upscaling are given by Giudici *et al.* (1998) and Lunati *et al.* (2001).

When applied to stationary conditions, the DSM requires two independent sets of data, $\left\{\left(h^{(l)}, f^{(l)}\right), l = 1,2\right\}$, i.e. piezometric heads and source terms corresponding to two different flow situations. The data sets are independent if the hydraulic gradients are not parallel.

In the previous papers we showed how the DSM works for a confined aquifer. Here we extend it to the case of the steady flow in a phreatic aquifer, where the balance equations are nonlinear with respect to the piezometric head, and the physical parameter appearing in the equation is the hydraulic conductivity *K*. Moreover we modify the DSM to handle multiple sets of data, a procedure which cures the main difficulties of the method, namely the choice of the starting point and the apparent necessity of two sets of data that are independent all over the domain.

THEORY

If the Dupuit approximation is valid, the physical parameter to be determined is K. Two independent sets of data are necessary, so that the following differential system can be written:

$$\begin{aligned} &\frac{\partial}{\partial x}\left(K\left(h^{(1)}-d\right)\frac{\partial h^{(1)}}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\left(h^{(1)}-d\right)\frac{\partial h^{(1)}}{\partial y}\right)= f^{(1)} \\ &\frac{\partial}{\partial x}\left(K\left(h^{(2)}-d\right)\frac{\partial h^{(2)}}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\left(h^{(2)}-d\right)\frac{\partial h^{(2)}}{\partial y}\right)= f^{(2)} \end{aligned} \tag{1}$$

where d is the height of the bottom of the phreatic aquifer, which is supposed to be known, and the unknown function to be determined is K.

The independence condition is the following:

$$\det A \neq 0 \text{ , with } A \equiv \begin{pmatrix} \left(h^{(1)}-d\right)\frac{\partial h^{(1)}}{\partial x} & \left(h^{(1)}-d\right)\frac{\partial h^{(1)}}{\partial y} \\ \left(h^{(2)}-d\right)\frac{\partial h^{(2)}}{\partial x} & \left(h^{(2)}-d\right)\frac{\partial h^{(2)}}{\partial y} \end{pmatrix} \tag{2}$$

If the independence condition (2) is satisfied, then the differential system (1) can be written in the normal form:

$$gradK = -K\underline{a} + \underline{b} \tag{3}$$

If a value of conductivity, K_0, is known at a point of the domain, $\underline{x}_0$, the procedure for the integration of equation (3) described, e.g. by Giudici *et al.* (1995), can be applied to find a solution to (1). The procedure requires the integration along an arbitrary line, e.g. a polyline, which connects the arbitrary point $\underline{x}$ to $\underline{x}_0$; the error propagation can be limited if the polyline is chosen in such a way that the integral of $|\underline{a}|$ along the line is least.

THE USE OF MULTIPLE SETS OF DATA

One of the main difficulties for the application of the DSM is the choice of the starting point, $\underline{x}_0$. As a simple rule, the best choice is at a location where $|\underline{a}|$ is small, so that the error propagation during the first steps of the integration is small. Another difficulty for the application of the DSM is the collection of two sets of data such that the independence condition (2) is satisfied all over the domain. In fact different independent sets of data can be created by different pumping schedules, which usually modify the flow field in a limited region around the pumping wells.

In order to cure these problems, we modify the DSM assuming that more than two sets of data are available, i.e. $\left(h^{(m)}, f^{(m)}\right)$, with $m = 1,\ldots,M > 2$. One might apply least squares techniques to compute the vectors $\underline{a}$ and $\underline{b}$, thus using all the available data at the same time. This is not always satisfactory, though, as the sets of data might not

provide independent information in some regions of the aquifer. On the other hand, the selection of the "best" couple of data sets at each point among the $M(M-1)/2$ possible couples is very important and successful. The "best" couple is that which at the same time makes: (i) large enough the determinant in the independence condition (2) and small enough both (ii) the value of $|\underline{a}|$ and (iii) the condition number of the matrix A that appears in equation (2). "Best" couples can be identified in different subdomains of the aquifer, so that the vectors $\underline{a}$ and $\underline{b}$ can be computed and the DSM can be applied with the standard procedure.

NUMERICAL TESTS

Some numerical tests are presented to show the effectiveness of the method for the computation of hydraulic conductivity of a synthetic phreatic aquifer.

The test data have been obtained from forward finite-difference modelling for the synthetic aquifer whose main features are described below. The reference K field is shown in Fig. 1; the domain is discretized with 51×51 cells; the aquifer bottom is assumed to be at sea level ($d = 0$). We assign Dirichlet boundary conditions in the form $h(x, y) = 50 - 0.005x$. A constant source term representing aquifer recharge is assigned at each cell of the finite difference grid (10^{-11} m³ s^{-1}).

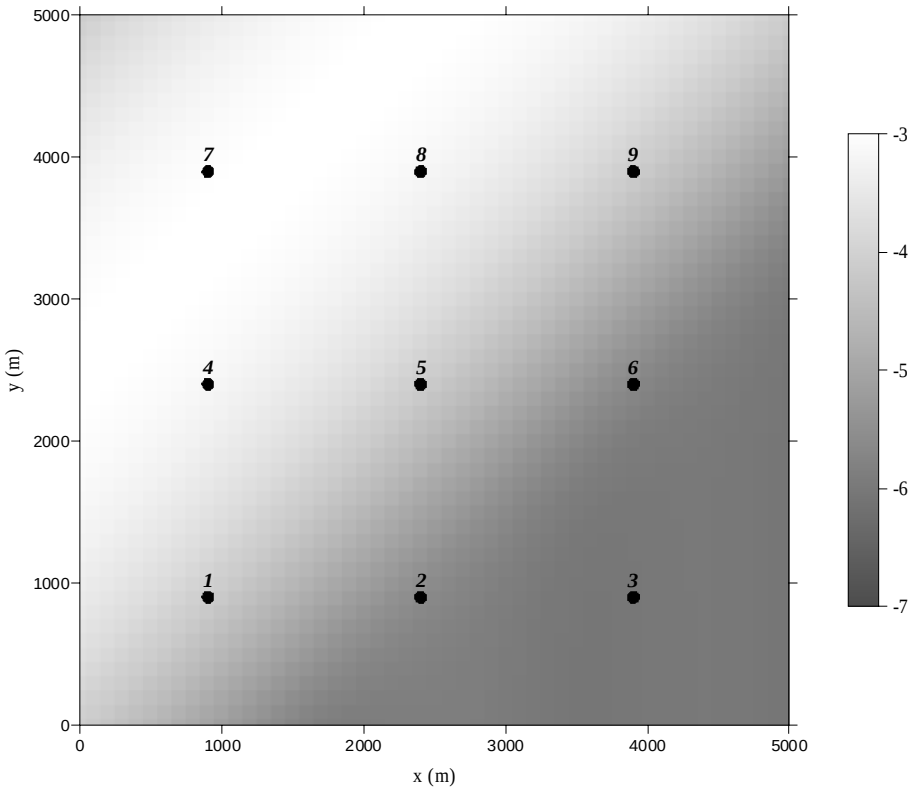

Fig. 1 Distribution of log$K$ and well positions for the test aquifer. The grey scale on the right corresponds to the decimal logarithm of hydraulic conductivity (in m s^{-1}).

Ten sets of data have been obtained by changing the source terms; in particular nine wells have been considered at the positions shown in Fig. 1. Nine sets of data have been obtained by solving the forward problem when only one well at a time is pumping; the tenth set of data is computed when no well is active. The well discharge rates have been chosen in such a way to produce a drawdown of about 5 m in the wells with respect to the undisturbed condition.

The *K* field identified with all the sets of data is shown in Fig. 2. The couple of sets of data to be used to integrate (3) along an internode segment, i.e. the segment connecting two adjacent nodes, has been chosen in such a way as to minimize the sum of $|\underline{a}| \cdot \mathrm{cond}(A)$ at the two nodes. The trend of the high *K* zone is clear, although the values of *K* at many cells cannot be computed because of numerical difficulties.

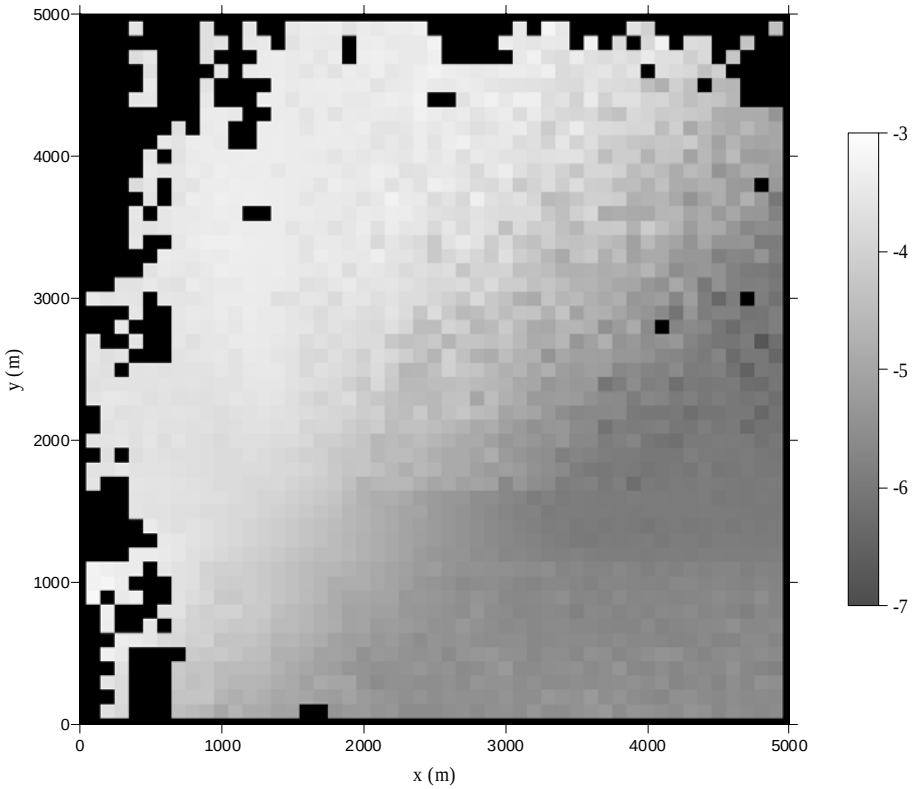

Fig. 2 Identified log*K* field. Black cells show the positions where *K* has not been computed due to numerical difficulties.

An improvement of the identified *K* field has been obtained by choosing the "best" couple at subdomains of 7×7 cells as the couple which minimizes the sum of $|\underline{a}| \cdot \mathrm{cond}(A)$ over all the nodes of the subdomain itself. The *K* field identified with this block-wise choice of the couples of sets of data is represented in Fig. 3 and shows a good agreement with the reference *K* field (Fig. 1).

The high *K* area is identified from these results much better than from those based on a point-wise choice of the couple of sets of data. Moreover there is no problem for the computation of *K*, but at the border. It is also to be stressed that the starting point

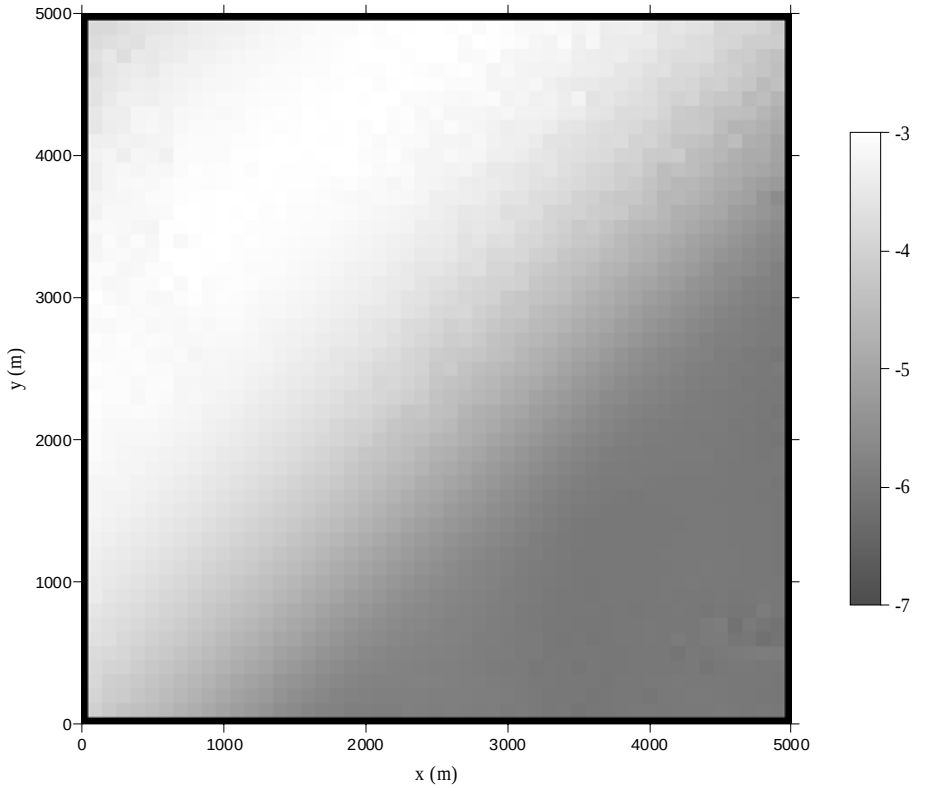

Fig. 3 Identified log*K* field with block-wise choice of the couple of data sets to be used to compute the vectors $\underline{a}$ and $\underline{b}$.

has been chosen at the coordinates (1400,1400) arbitrarily, i.e. without applying the criteria proposed in previous papers. This was a critical aspect of the DSM; in fact, if the starting point were chosen in an area with high values of $|\underline{a}|$, the identified *K* values would not be confident even at cells close to the starting point.

The following important remarks can be mentioned: (1) The couples of sets of data for which det*A* changes sign in the subdomain have been excluded, in order to guarantee the independence condition throughout the whole subdomain. This permits one to discard the couples involving the set of data corresponding to an active well in the subdomain; in fact the piezometric head has a minimum in correspondence of the well, a situation which causes several difficulties (Giudici *et al.*, 1995). (2) The worst set of data seems to be the one obtained for the undisturbed condition, which does not permit one to obtain such a “great independence” as two sets of data corresponding to pumping wells at different positions.

CONCLUSIONS

Numerical tests show that the innovative approach proposed in this paper is useful to reduce the dependence of the final solution on the starting point, which can be assigned almost everywhere without degrading the results of the DSM. This is important for the applications, because we use in a suitable manner the available data sets and because the choice of the starting point is usually constrained by the availability of the results

of field tests and can rarely be based on the analysis of the vector $\underline{a}$ proposed in previous papers.

The justification of the improvement of the results when the choice of the couple of data sets is performed block-wise seems to be related to the fact that $|\underline{a}|$ and cond(A) show great variations among the nodes and the use of more nodes to compute the criteria for the choice of the couple can filter the variability at high wave numbers.

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