Evaluation of the inverse modelling process for heterogeneous porous media through laboratory air injection tests

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Abstract In the modelling process, inversion techniques are generally used to estimate unknown parameters. However, we need to determine the model structure and other conditions before solving inverse problems, and there are many alternatives, such as the inversion method, the number of parameters, and the weightings in objective functions. The aim of this study is to discuss the process so that the alternatives can be assessed by evaluating their reliability. Multi-well transient flow tests, such as hydropulse testing, had been proposed for the treatment of heterogeneous media in subsurface flow modelling using data obtained through laboratory air injection tests for a dried heterogeneous sandstone plate. The numerical inversion method includes a quasi-Newton method and an adjoint-state method for pressure change rate matching that can deal with various types of objective functions. Porosity at the injection and observation points can be treated as different unknown parameters. Data quality and quantity are also important factors for the inverse problem. Several injection patterns were carried out to assess the observation pattern. The bootstrap re-sampling method was used to evaluate the reliability of the inversion results. We prefer this method because it is not necessary to assume a probability density function and because the method can estimate prediction errors. Using the estimated reliability indices, we can assess the best model, i.e. the adequate combination of injection pattern and model structure and objective function, etc.

Keywords air injection test; heterogeneous; inverse modelling; reliability index; sandstone

INTRODUCTION

Inverse analysis has been developed and used as a tool for groundwater modelling for years. However, there are still some unsolved problems, some of which are indicated below:

- To carry out inversion, an objective function and unknown parameters must be defined; however, there are too many alternatives to choose from.
- Whether the measured data is sufficient in quality and quantity for inversion is, for most of the complex real-world problems, unknown before calculation.
- Even when using the same data, results will vary for different inversion methods and different settings of the objective function and unknown parameters.
- Even if inversion is done successfully, the results will show some modelling errors in most cases.

Estimating the reliability of the inversion results is useful in order to deal with these problems. This is because some kind of reliability index is required to assess whether a result obtained by inversion is reliable or not. The purpose of this study then was to discuss the process so that the best alternatives could be assessed by evaluating the reliability of various inversion results. In studying the methodology of selecting the best method, a modified bootstrap re-sampling method proposed by Masumoto & Valle (2000) was used to estimate the reliability indices of a variety of inversion models for laboratory air-injection test data. A quasi-Newton method and an adjoint-state method were used for numerical inversions, by matching the pressure change rate (Masumoto *et al*., 1998). Porosity and permeability can be set as unknown parameters for flexible parameter setting. Several types of flow rate pattern in the air-injection test were carried out for comparison. Reliability indices were calculated for various measuring and inversion methods.

METHOD

Numerical inversion method

The governing equation for single phase flow in porous media is:

$$R \equiv \sum_{i=1}^{3} \frac{\partial}{\partial x_i}\left[\frac{K}{\mu B}\left\{\frac{\partial p}{\partial x_i} - \gamma_w \frac{\partial z}{\partial x_i}\right\}\right] - \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) + q = 0 \qquad (1)$$

where μ is viscosity (Pa s^{-1}), B the formation volume factor (dimensionless), K the permeability (m^2), p the pressure (Pa), ϕ the porosity (m^3 m^{-3}), γ_w, the specific weight of water (kg m^{-2} s^{-2}), q the injecting rate per unit volume (s^{-1}), z the depth (m), x_i the ith component of the spatial position vector (m) and t the time (s). The objective function with the change-rate matching term used here is:

$$J = J_1 + J_2 + A_{sm} \qquad (2)$$

where:

$$J_1 = \sum_{n=1}^{Nt}\sum_{m=1}^{Nm} W_1^{m,n}\left(p_{cal}^{m,n} - p_{obs}^{m,n}\right)^2$$

$$J_2 = \sum_{n=1}^{Nt}\sum_{m=1}^{Nm} W_2^{m,n}\left(\frac{p_{cal}^{m,n} - p_{cal}^{m,n-1}}{\Delta t^n} - \frac{p_{obs}^{m,n} - p_{obs}^{m,n-1}}{\Delta t^n}\right)^2$$

with n as time step and Nt the total number of time steps; m indicates the mth observation point, Nm the total number of observation points, W the weighting factor (dimensionless), p_{cal} calculated pressure (Pa), p_{obs} observed pressure (Pa), Δt the time step interval (s), and A_{sm}, the penalty function for the smoothing term, which consists of standard curvatures of distributed parameters (dimensionless). Inversion was solved by minimizing the objective function using a quasi-Newton method. The gradients of the objective function were calculated using the adjoint state method. The unknown parameter vector u is written as:

$$u = (K_1, K_2, \cdots K_{Nk}, \phi_1, \phi_2, \cdots \phi_{Np}) \qquad (3)$$

where subscript *Nk* is the total number of unknown permeabilities distributed on the intercell of the discrete model and *Np* the number of unknown porosities. Here, permeabilities at all intercells, and the porosities at observation points, are used as unknown parameters.

Reliability index

To create results with some deviation, the bootstrap re-sampling method (Efron, 1979), one of the Monte Carlo simulation methods, was used. This method can be applied to groundwater inverse problems with transient data (Masumoto & Valle, 2000). The procedure applied to our problems consists of five steps:

1. Repeat the procedure for $Nt \times Nm$ (the total number of data used for matching) times, randomly picking up the data which represent functions of time and well points. Replacement is allowed.
2. For each term shown in equation (2), the original weighting is multiplied by the chosen number. A new set of weightings will be made.
3. Calculate the inverse solution for the objective function with the set of weightings made in step 2.
4. Repeat steps 1–3 100 times to obtain 100 estimations.
5. Calculate the indices, such as the 90% confidence interval, with the 100 estimation sets.

The author prefers this method because the index can be estimated for a nonlinear problem without any information about the true model and we need not assume a probability function. Here, the 90% confidence intervals of *i*th parameter were calculated using equation (4); the reliability index R_k of the estimated permeability distribution is calculated using equation (5).

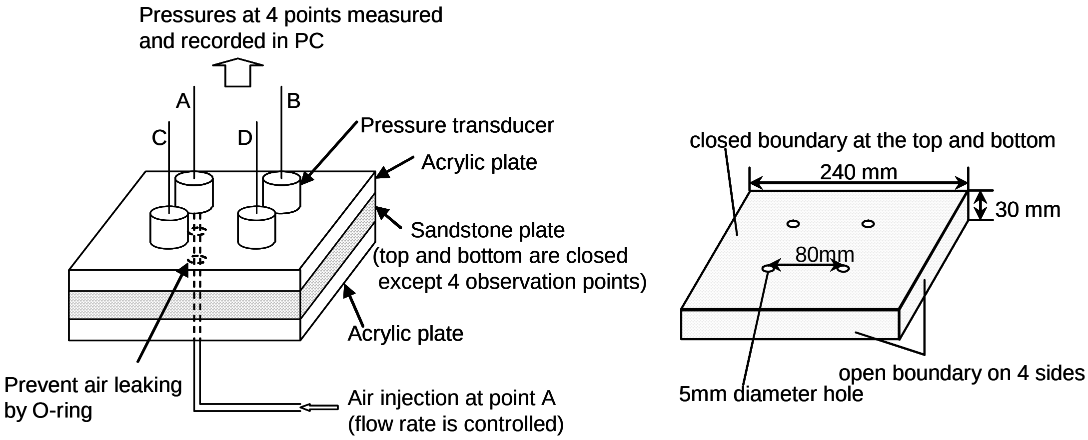
$$CI90K_i = K_i^6 - K_i^{95} \tag{4}$$

$$R_k = \left\{ \sum_{i=1}^{Nk} \left(CI90K_i \right)^2 \right\}^{-1} \tag{5}$$

Here, the subscript *i* indicates the *i*th component of the parameter, *Nk* is the total number of unknown permeabilities and K_i^6 (K_i^{95}) the 5th (95th) largest value of 100 inversion results of K_i. *CI*90*K* (in units m^2 or, as is used further here, milliDarcy) represents an estimation of the 90% confidence interval of *K*, and R_k, the inverse of squares of *CI*90*K* for permeabilities represents the reliability of an inversion result. In order to compare with the case of known porosity here, the reliability index R_k was calculated only through *CI*90*K* of permeabilities.

Laboratory test

The apparatus for the air injection test is shown diagrammatically in Fig. 1(a). Four observation holes were made in the sandstone plate that measured 24 × 24 × 3 cm



(a) Image of apparatus of air injection test (b) Sandstone plate

Fig. 1 Image of measurement system.

(Fig. 1(b)), and one pressure transducer was attached to each hole. The plate's surfaces with the largest area (upper part and bottom part of Fig. 1(b)) were made impermeable by epoxy adhesive. The other four sides remained open.

Air injection tests were carried out by repeating air injection and shut-off at an injection point. After an injection test at one hole was finished, the next injection test was started at another injection point. The process was performed four times. Thus, every hole was used as an injection point, and pressure performances were obtained at all holes. The injection pattern (injecting rate of air and timing of shut-off) was controlled for each case as shown in Table 1.

Table 1 Injection pattern for each measurement case.

	Timing of injection and shut-off	Air injection rate
Case1	[30 s injection & 570 s shut-off] × 2 cycles	1.8 $cm^3 s^{-1}$
Case2	[30 s injection & 270 s shut-off] × 2 cycles	1.8 $cm^3 s^{-1}$
Case3	[15 s injection & 285 s shut-off] × 2 cycles	3.4 $cm^3 s^{-1}$
Case4	[30 s injection & 1170 s shut-off] × 1 cycle	3.4 $cm^3 s^{-1}$

Numerical model

The permeability at each intercell of the discrete model shown in Fig. 2, and porosity at each observation point cell, were used as unknown parameters for numerical inversion of the heterogeneous permeability distribution of the sandstone used for the laboratory tests. Other conditions for numerical inversion are shown in Table 2.

The following two factors were studied for the four injection patterns to compare the effect of the difference of inversion model settings:

– whether porosities at observation point cells are known (K) or unknown (U);
– whether pressure change rate matching is used (C) or not (O).

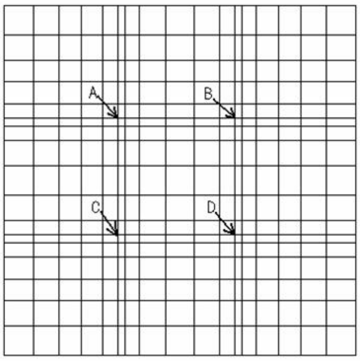


Fig. 2 Discrete model of the sandstone plate. (The size of the plate is 24 by 24 cm.)

Table 2 Main conditions for numerical inversion.

Time step interval	5 s
Porosity except for observation point cell	0.2 for all cells except for observation point cell
Weighting in objective function	$W_2/W_1 = 1.0 \times 10^{-6}$ or $W_2 = 0$
Convergence condition	Maximum number of iterations = 100

The indices – U, K, N and C – are added to each case number to show the inversion model setting, e.g. "Case1UO". Sixteen case studies were carried out in total. Each study requires 100 inversions, and so a total of 1600 inversions were carried out.

RESULTS AND DISCUSSION

Figure 3 shows reliability indices for 16 case studies. From Fig. 3, Case3UO is seen to be the most reliable model and Case4KC the least reliable one. According to the reliability indices, treating the porosities at observation points as unknown parameters seems to be effective. Although the number of unknown parameters is larger when observation point porosity is treated as unknown, the results appeared to be reliable. This means that the fixed porosity has a model error that was reduced by treating the porosity of observation points as unknown. Figure 3 shows that using pressure change rate matching is not effective except for Case4UC, which means that the effectiveness of a change-rate matching method depends on the combination of other settings of models and measurement pattern. Here, only one pattern of W_2, the values of J_2 weightings, was studied. Studying the effect of various weighting patterns in J_2 would seem to be reserved for the future. Figure 3 also shows the high flow rate pattern of air injection to be more effective for solving our problems. The reliability index R_k can be used to compare various data sets even if the time steps are different. Permeability distributions of these cases are shown in Fig. 4. Here the best result is observed to have no remarkable artefact, especially near the observation points and the boundary, while the worst one has some contrasts of permeability near the observation points and boundary, which is considered to reflect several model errors. The distributions of $CI90K$ (or the confidence interval of estimated permeability) of these two cases are

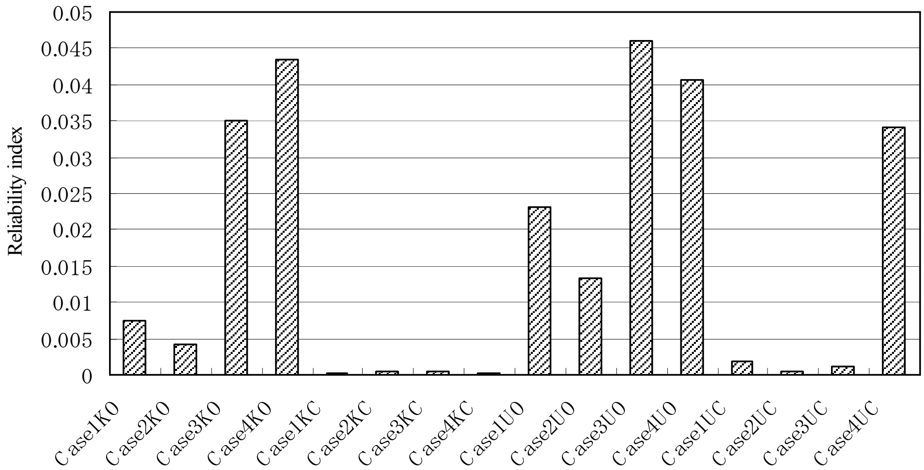
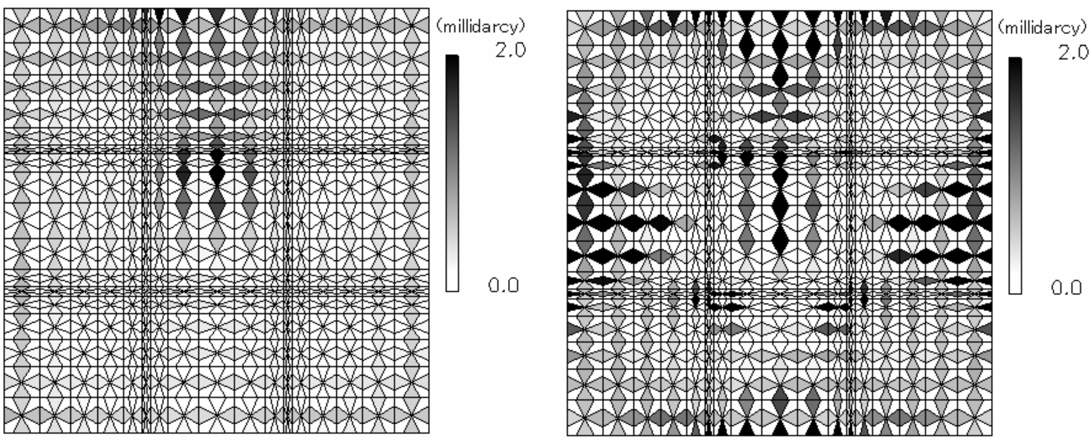
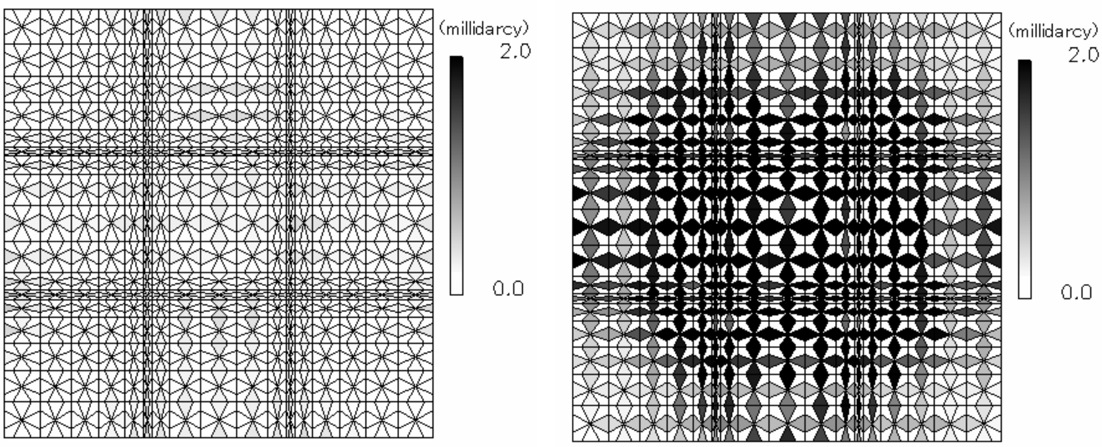


Fig. 3 Reliability indices obtained by 16 inversion models.



(a) Case3UO (the best case) (b) Case4KC (the worst case)

Fig. 4 Estimated permeability distributions of the best result and the worst result.



(a) Case3UO (the best case) (b) Case4KC (the worst case)

Fig. 5 Distribution of *CI*90*K* of Case3UO and Case4KC.

shown in Fig. 5. From Fig. 5, the *CI*90*K* at the centre of the plate is seen to be larger for Case4KC, and the permeability *CI*90*K* is considered to have large model errors.

CONCLUSION

Reliability indices for different measurements were compared so as to evaluate the process of inverse modelling for heterogeneous porous media using laboratory air injection test data of some different numerical models (different objective functions and different unknown parameter settings). These indices were calculated using a bootstrap re-sampling method. By employing this method, reliability indices, even for data for different time steps, could be evaluated. The results show that the combination of: (a) porosity treated as an unknown, and (b) a high flow rate of injected air is more effective. The pressure change-rate matching method, which uses an objective function with an added term, was, however, in most cases not effective, with the exception of Case4UC. Further study is required of the weighting patterns for the objective-function term regarding the pressure-change rate matching.

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