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Integral pumping tests: average concentration and mass flow at the capture zone scale

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Abstract We are concerned with the application of the Integral Pumping Test (IPT) method for obtaining mass flow rates at the field scale. The IPT method, first proposed in Teutsch *et al.* (2000), is based on the interpretation of concentration-time data measured during pumping tests, increasing the observation scale to the size of the well capture zone. Here, we show how the novel non-recursive explicit solution for evaluation of single-well IPTs and the associated analytical framework developed in Bayer-Raich *et al.* (2004) can be used for the dimensioning of multiple-well IPTs.

Keywords analytical solution; field scale; integral approach; mass flow rate

INTRODUCTION

Research starting in the late 1980s has shown that under the right conditions, many organic contaminants in groundwater will naturally biodegrade sufficiently without expensive active remediation measures. The concept of allowing organic contaminants to naturally biodegrade, referred to as natural attenuation (NA), is increasingly used throughout North America and increasingly accepted in Europe. However, one pre-requisite for using NA as a remediation strategy is that one must be able to acquire field-scale observational evidence of decreasing contaminant mass fluxes down-gradient a source zone.

The Integral Pumping Test (IPT) method is a cost-effective alternative to conventional point measurements for estimation of mass flows, as shown in a number of field-scale applications (e.g. Bockelmann *et al.*, 2001; 2003; Bauer *et al.*, 2004; Jarsjö *et al.*, 2005). We present guidelines for systematic application of the IPT method using simple, new analytical expressions which enable dimensioning of total pumping time and distance between pumping wells in multiple-well IPTs.

GOVERNING EQUATIONS

We consider a two-dimensional contaminant concentration distribution $C_0(x,y)$ [M L^{-3}] which is developed for natural steady-state uniform flow conditions with specific discharge $q_{0y}(x,y)$ [L T^{-1}]. The mass flow rate M_{CP} [M T^{-1}] across a control plane defined by $y = 0$ (setting the y axis parallel to the mean flow direction) can be obtained by integrating the initial concentration $C_0(x,y)$ [M L^{-3}] as:

$$M_{CP} = \int_{\ell_{CP}} C_0(x,0)\, q_{0y}(x,0)\, b(x,0)\, \mathrm{d}x \tag{1}$$

where $b(x,y)$ [L] is the saturated thickness and $q_{0y}(x,y)$ [L T^{-1}] the y-component of the Darcy velocity.

During an integral pumping test, the concentration $C_w(t)$ at the extraction well located at $(x,y) = (0,0)$ is measured, typically during several days, while pumping at rate Q [L^3 T^{-1}]. In this situation, the time-dependent concentration $C_w(t)$ measured at the pumping well is related to the initial spatial concentration distribution $C_0(x,y)$ through the integral equation (Bayer-Raich *et al.*, 2004):

$$Q \int_0^t C_w(\tau)\, d\tau = n_e \int_{V_I(t)} C_0(x,y)\, \mathrm{d}V \tag{2}$$

where the left-hand side expresses the total contaminant mass pumped at the well up to time t while the right-hand side is the contaminant mass initially located within the well capture zone $V_I(t)$ [L^3], and n_e [–] is the effective porosity.

The general solution to equation (2), without restrictions on $C_0(x,y)$, is non-unique, i.e. an infinite number of possible realizations of $C_0(x,y)$ exist for a given $C_w(t)$. However, linking equations (1) and (2) and introducing some additional assumptions, it is possible to obtain relevant estimates of the total mass flow rate M_{CP} through estimates of the flow field $q_{0y}(x,y)$ and measurements of $C_w(t)$.

ANALYTICAL SOLUTIONS FOR SINGLE-WELL IPTs

For solving equations (1) and (2) we first consider homogeneous conditions and the assumption of constant concentration along the flow direction, i.e. initial concentration distribution being of the form $C_0(x,y) = C_0(x)$, with flow in the direction of the y axis. Under these conditions (homogeneous media) the geometry of the capture zone at pumping time t is given in Bear & Jacobs (1965) as:

$$t_D = -\ln\left(\cos x_D - \frac{y_D}{x_D}\sin x_D\right) - y_D \tag{3}$$

with $t_D = \dfrac{2\pi b q_0^2 t}{Q n_e}$, $x_D = \dfrac{2\pi b q_0 x}{Q n_e}$, $y_D = \dfrac{2\pi b q_0 y}{Q n_e}$

Solutions for equations (1) and (2) were provided in Bayer-Raich *et al.* (2004) for three different situations defined by the dimensionless duration of the IPT, t_D: (i) short dimensionless time $t_D < 1$ (i.e. radial flow); (ii) long dimensionless time $t_D > 2\pi$; and (iii) any dimensionless time t_D.

The dimensionless time t_D uniquely defines both the shape of the capture zone and the averaging function used to compute the mean concentration. Figure 1 shows the shape of the capture zone for a single well, in dimensionless coordinates for different t_D.

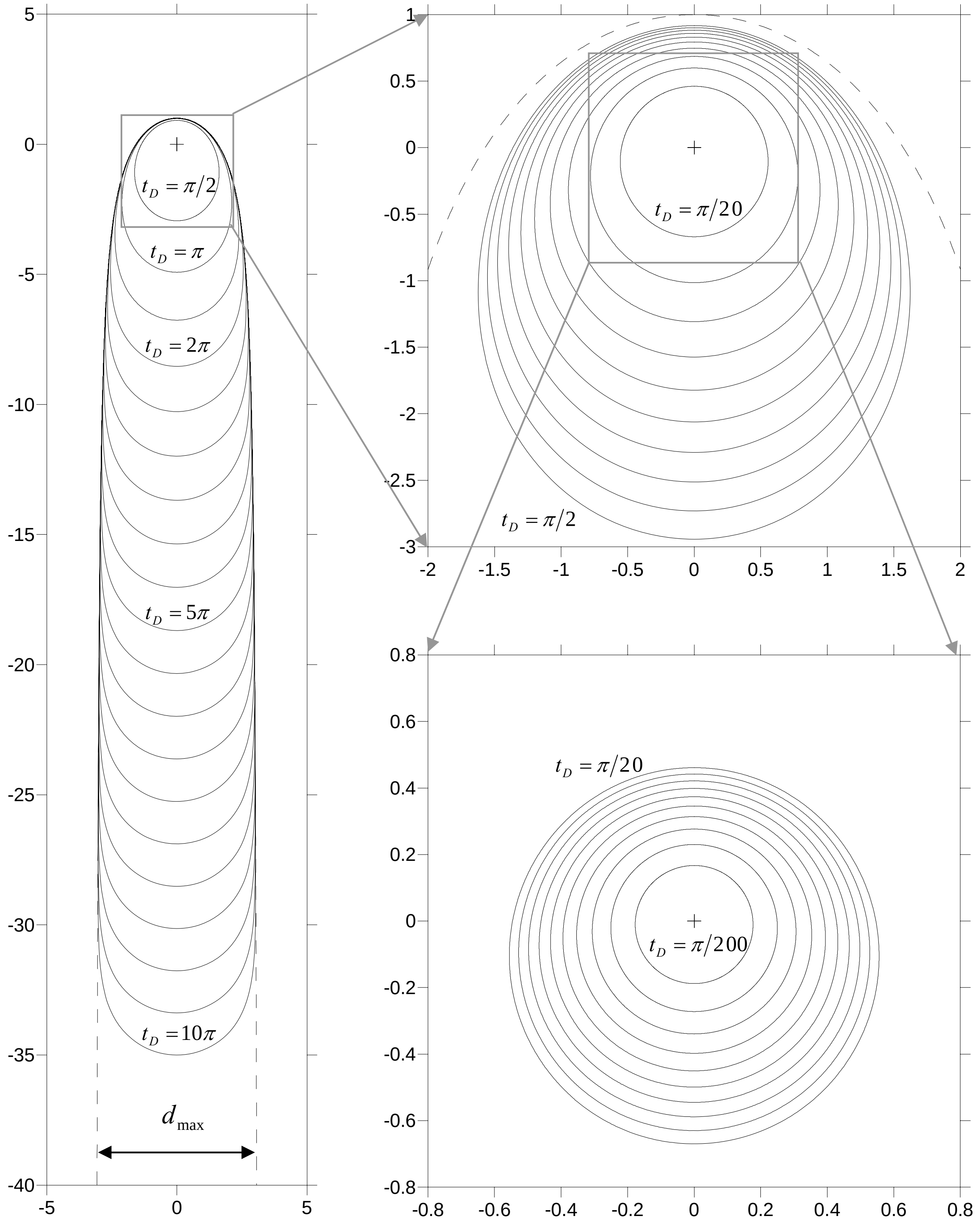

Fig. 1 Shape of the well capture zone in dimensionless coordinates (after Bear & Jacobs, 1965). Maximum width of the capture zone $d_{max} = Q/(bq_0)$. Natural flow q_0 upwards.

Analyses of more than 50 IPTs already conducted (Bayer-Raich *et al.*, 2004) showed that the solution (i) can be used in the majority of sites. In such conditions, $C_0(x,y) = C_0(x)$, defining $\overline{C}_0(x) = \left(\overline{C}_0(x) + \overline{C}_0(-x)\right)/2$ equation (2) can be simplified to:

$$C_w(t) = \frac{2}{\pi}\int_0^{r(t)} \frac{\overline{C_0}(x)}{\sqrt{r(t)^2 - x^2}}\,\mathrm{d}x \tag{4}$$

where $r(t) = \sqrt{Q\,t/(\pi\, b\, n_e)}$ (usually referred at the cylinder formula). In all IPT applications, we have used a recursive analytical solution for the average concentration $C_{av}(t)$ (Schwarz, 2002), which written in our notation is:

$$C_{av}(t) = \frac{1}{\sqrt{t}}\sum_{i=1}^{n} \overline{C_i}\left(\sqrt{t_i} - \sqrt{t_{i-1}}\right) \tag{5}$$

$$\text{with } \overline{C_i} = \frac{\frac{\pi}{2} C_w(t_i) - \sum_{j=1}^{i-1} \overline{C_j}\left[\arccos\sqrt{\frac{t_{j-1}}{t_i}} - \arccos\sqrt{\frac{t_j}{t_i}}\right]}{\arccos\sqrt{\frac{t_{i-1}}{t_i}}}$$

where n is the number of samples. Further, equation (4) expresses a Volterra Integral Equation of the first type (Bayer-Raich *et al.*, 2004). For this particular equation, a closed form solution can be obtained from Abel's Integral Transform (Abel, 1823), leading to a simpler (i.e. non-recursive) expression for evaluation of IPTs:

$$C_{av}(t) = \sum_{i=1}^{n} C_w(t_i)\left(\sqrt{1 - \frac{t_{i-1}}{t_n}} - \sqrt{1 - \frac{t_i}{t_n}}\right) \tag{6}$$

The advantage of this solution is that it is non-recursive, and therefore can be evaluated for instance, using a computer spread sheet.

DESIGN AND EVALUATION OF MULTIPLE-WELL IPTs

At many sites, the (potentially) contaminated area requires investigation along control planes on the order of hundreds of metres. In such cases, a single well is not capable of covering the entire cross-section downstream the (suspected) sources and therefore several wells are placed along a control plane perpendicular to the mean flow direction. In order to cover the entire cross-section, the capture zones of successive wells have to cover the entire region in between the wells.

If pumping is indefinitely long ($t_D \to \infty$) then the width of the well capture zone approaches $d_{max} = Q/(bq_0)$ while if pumping is short ($t_D \to 0$) then the capture zone becomes circular. In practice, the value of $t_D \approx 1$ gives an approximate limit for the range of validity of the radial flow approximation. On the other side, we set the limit $t_D \approx 2\pi$ as the maximum pumping time, since longer pumping does not significantly increase the capture zone width, but only leads to an elongation in the upstream direction (see Fig. 1).

If the IPTs are designed to fulfil the condition $t_D < 1$ then the new non-recursive equation (6) provides a simple tool for evaluation of the IPTs (i.e. the estimation of average concentration). Further, in such cases the capture zone shape can be approximated by the cylinder formula in order to estimate the distance between wells. Figure 2 gives the width of the capture zone as a function of the (dimensionless) time.

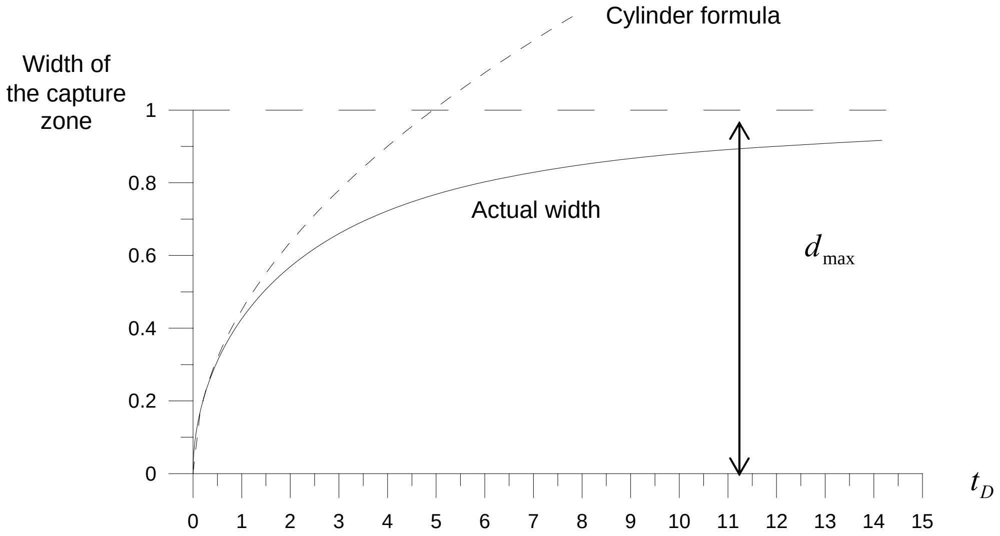

Fig. 2 Width of the capture zone as a function of the dimensionless time.

As shown in both Fig. 1 and Fig. 2, the cylinder formula $r(t) = \sqrt{Qt/(\pi b n_e)}$ provides a good estimation of the width of the capture zone for early times. The actual width is computed through equation (6) in Bayer-Raich *et al.* (2005). From Fig. 2 we see that the actual width of the capture zone is very close to the cylinder formula for tD < 1, after this pumping time the capture zone has reached 42.5% of the maximum width. For pumping durations equivalent to $t_D = 2\pi$ the capture zone has reached more than 80% of the capture zone width. At this point, increasing pumping duration by 100% (up to $t_D = 4\pi$) the width of the capture zone increases in 10%, i.e. reaching 90% of $d_{max} = Q/(bq_0)$. As pumping duration increases, the width of the capture zone approaches $d_{max} = Q/(bq_0)$ asymptotically. To avoid excessive deformation of the capture zone we establish the maximum pumping duration to $t_D \approx 2\pi$.

SUMMARY AND CONCLUSIONS

We have investigated how a novel simple analytical expression can be used for the planning and evaluation of Integral Pumping Tests. This solution is valid for pumping tests of duration $t < Q\, n_e \big/ \left(2\,\pi\, b\, q_0^{\,2}\right)$, where Q [L^3 T^{-1}], n_e [–], b [L] and q_0 [L T^{-1}] are pumping rate, effective porosity, aquifer thickness and natural (i.e. before pumping starts) Darcy velocity. This solution is based on the same assumptions as the previously existing one, but is considerably simpler since it is non-recursive. If the pumping time fulfils the condition $t < Q\, n_e \big/ \left(2\,\pi\, b\, q_0^{\,2}\right)$, the distance between wells can be designed to cover the entire control plane length by use of the cylinder formula $r = \sqrt{Q\,t \big/ \left(\pi\, b\, n_e\right)}$. However, in some cases, the simplified approach cannot be used, e.g. if Darcy velocities q_0 are high (e.g. more than 1 m day^{-1}) and the aquifer thickness is small (e. g. less than 2 m).

On the other side, if the pumping time is longer than $t > Q\, n_e \big/ \left(b\, q_0^{\ 2}\right)$, the capture zone extends towards the upstream direction without increasing significantly the width of the capture zone. IPTs of duration longer than $t > Q\, n_e \big/ \left(b\, q_0^{\ 2}\right)$ will provide information mostly related to the upstream aquifer region but not on the surroundings of the pumping well.

Parameters needed as input for these analyses include porosity n_e [-], aquifer thickness b [L] and natural Darcy velocities q_0 [L T^{-1}], as well as maximum pumping rates Q [L^3 T^{-1}] that can be extracted from wells. These four parameters are sufficient for effective dimensioning of the IPTs.

The solutions presented here are strictly valid only for confined conditions. In future work the range of validity of the above presented equations for unconfined conditions will be established.

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