Upscaling transmissivity in the near-well region

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Abstract Upscaling transmissivity near the wellbore is expected to be useful for well performance prediction. This article ascertains the requirement of upscaling and presents a scheme to upscale the transmissivity in the near-well region for 2-D steady flow, which extends the approach of Gómez-Hernández & Journel (1994) to the near-well region with radial flow. Several synthetic fields with different stochastic models are chosen to check the efficiency of this scheme. Both flow and transport simulations are carried out in finite confined heterogeneous aquifers to evaluate the results. It is shown that the proposed method improves the ability in predicting well discharge or recharge and solute transport at the coarse scale.

Keywords radial flow; stochastic model; transmissivity; upscaling

INTRODUCTION

Stochastic modelling of reservoir parameters with the aid of geostatistical techniques can provide effective high-resolution images of a reservoir at the measurement scale. Limitation in computing resources forces these models to be upscaled at the modelling scale. Several upscaling approaches have been developed to coarsen detailed aquifer or reservoir models into those at an appropriate scale for numerical simulations (Wen & Gómez-Hernández, 1996b; Renard & de Marsily, 1997). Many of them are quite efficient in upscaling transmissivity under uniform flow conditions in porous media where local piezometric head or pressure values normally vary slowly.

In the immediate vicinity of a well, however, these existing upscaling approaches may not be applicable due to the fact that the flow pattern is no longer uniform but convergent around a pumping well or divergent near an injection well. The pressure gradient typically increases close to the well and becomes highly sensitive to the spatial variation of transmissivity (Desbarats, 1992) and especially to the difference between the global mean $\ln T$ and the value at the wellbore (Axness & Carrera, 1999). Moreover, the concentration distribution and the breakthrough curve of injected conservative tracers will differ from those of the uniform flow. An effective upscaling scheme should capture this character of the pressure gradient around the well and honour the statistical structure of $\ln T$. Basically, two problems need to be addressed: (a) Can a coarse grid account for the flow geometry in the near-well region? and (b) Do block transmissivity values adequately honour heterogeneities of the aquifer or reservoir? This article presents a scheme to upscale the transmissivity in the near-well region for two-dimensional (2-D) steady flow.

METHOD

An upscaling procedure typically consists of three steps: (a) using geostatistical techniques, a series of fine, detailed equiprobable models of transmissivity are generated, each representative of the geology and hydrology of the area; (b) a coarse grid is designed to capture the main characteristics of flow and transport at the larger scale; and (c) an equivalent value, either scalar or vectorial, calculated from the fine model of scalar transmissivity is assigned to the coarse model.

The Thiem solution to the 2-D steady flow in a homogeneous porous medium with prescribed heads at the well radius and at an exterior circular boundary can be written as:

$$h(r) = h_w + \frac{h_e - h_w}{\ln(r_e / r_w)} \ln(r) \tag{1}$$

where r represents the normalized radius $r = r'/r_w$; r_w is the wellbore radius; r' is the radius from the well axis; and h_e and h_w are the heads at the outer and inner radii, respectively. Although this solution is only applicable for homogeneous media, it may be shown that gridding with respect to $\ln(r)$ other than r minimizes the error in the hydraulic head (Axness *et al.*, 2004). The coarse grid in terms of log-scale, therefore, will have more advantage than the normal scale. The former is expected to be able to capture the main features of gradient variations better than the latter, even for highly heterogeneous media.

We extend the concept of skin (Gómez-Hernández & Journel, 1994) to the computation of equivalent transmissivity for a coarse grid. Two sets of boundary conditions are considered for each coarse block: one is in the x direction and the other in the y direction. We obtain the corresponding T_{xx} and T_{yy} by solving the local flow problems at the fine scale. One of the crucial problems is the configuration of boundary conditions for each coarse rectangular block. We approximate the head value for each side by calculating it at the middle point of each edge by the Thiem solution.

The procedure for calculating the equivalent transmissivity is as follows: (1) define the smallest rectangular block that includes the non-rectangular target block; (2) solve the flow problem with boundary conditions described as above; (3) evaluate the average flow rate Q_V and the average head gradient Δh_V over the non-rectangular target block both in x and y directions; and (4) compute the equivalent transmissivity by:

$$T_{V,xx} = -Q_{V,xx} \big/ \Delta h_{V,xx} \tag{2}$$

$$T_{V,yy} = -Q_{V,yy} \big/ \Delta h_{V,yy} \tag{3}$$

NUMERICAL SIMULATION

We chose two types of parameters for the assessment of upscaled results: one is the water injection (or, similarly, the yield for a pumping well) in to the wellbore, and the other is the travel time of conservative tracers. The former can be achieved by calculating the flow from the wellbore after solving the flow equations. The latter can be accomplished by the random walk particle tracking method.

The flow problem is solved by tailoring the five-point block-centred finite-difference simulator to the radial flow case. The transmissivity at the interface between cells are computed using harmonic averages of the adjacent blocks. We model the radial flow to a well by specifying the fixed heads at the wellbore and at the exterior circular boundary, i.e. $h_w = 10$ and $h_e = 0$ (unit-free), respectively. The transport equation solver adopts a particle tracking scheme (Wen & Gómez-Hernández, 1996a). Two-thousand particles released from the wellbore are followed until they arrive at the control circle which is set 100 units away from the wellbore.

RESULTS

Four types of technique are assessed in this section: (a) the proposed scheme as previously stated, (b) traditional geometric average from the fine scale in the framework of a non-uniform grid (coined as GM), (c) non-upscaled method (noted as NP), and (d) the geometric average from the fine scale in the framework of uniform grid (referred to as UG). GM assigns the upscaled element transmissivity to be the geometric mean of support cells within the coarse elements. This is the simplest and traditional method for upscaling uniform flow. We present the results here with two aims: one is to check its efficiency under the radial flow conditions, and the other is to compare its results with the proposed approach. NP simply assigns the transmissivity in the upscaled element to be the point value at the upscaled element centroid or to be the transmissivity of the support scale element closest to the centroid when the field is generated at a fine support scale. This is the usual case when detailed reservoir models are not available, for example, in the region less than several correlation lengths away from the wellbore. The last technique, UG, is done in the same way as the second but only distinctly with the grid intervals uniform.

Multi-Gaussian transmissivity fields with isotropic structures

One hundred log-transmissivity fields are generated by GCOSIM3D (Gómez-Hernández & Journel, 1993), each of them with correlation lengths equal to 10 in both the x and y directions, i.e. $\lambda_x = \lambda_y = 10$, with variances set as $\sigma^2_{\ln T} = 2$. The transmissivity fields are scalar, that is, $T_{xx} = T_{yy}$, and $E[\ln T] = 0$. The flow rates of the wellbore are computed at the fine scale and named as reference values. Then the fine scale fields with 801×801 grids are upscaled to those with 12×10 coarse grids in the same way as described above. The fluxes over the wellbore are computed on the coarse scale for four different upscaling approaches, named as proposals GM, NP and UG, respectively.

Figure 1 shows the relationship between the wellbore reference fluxes and those determined using the four different approaches. Figure 1(a) displays one hundred wellbore discharges by the proposed upscaling approach compared to those of the reference fine scale. The average flux of one hundred realizations is 17.537 for the reference fields and 17.012 for the proposed method with a relative error of only 3%. It shows that the upscaled values are reasonable approximates. Note, moreover, that the x and y mean values are close to each other showing that this method is unbiased, that is, the upscaled Q tends to be close to the reference Q in the mean. Figure 1(b) shows the

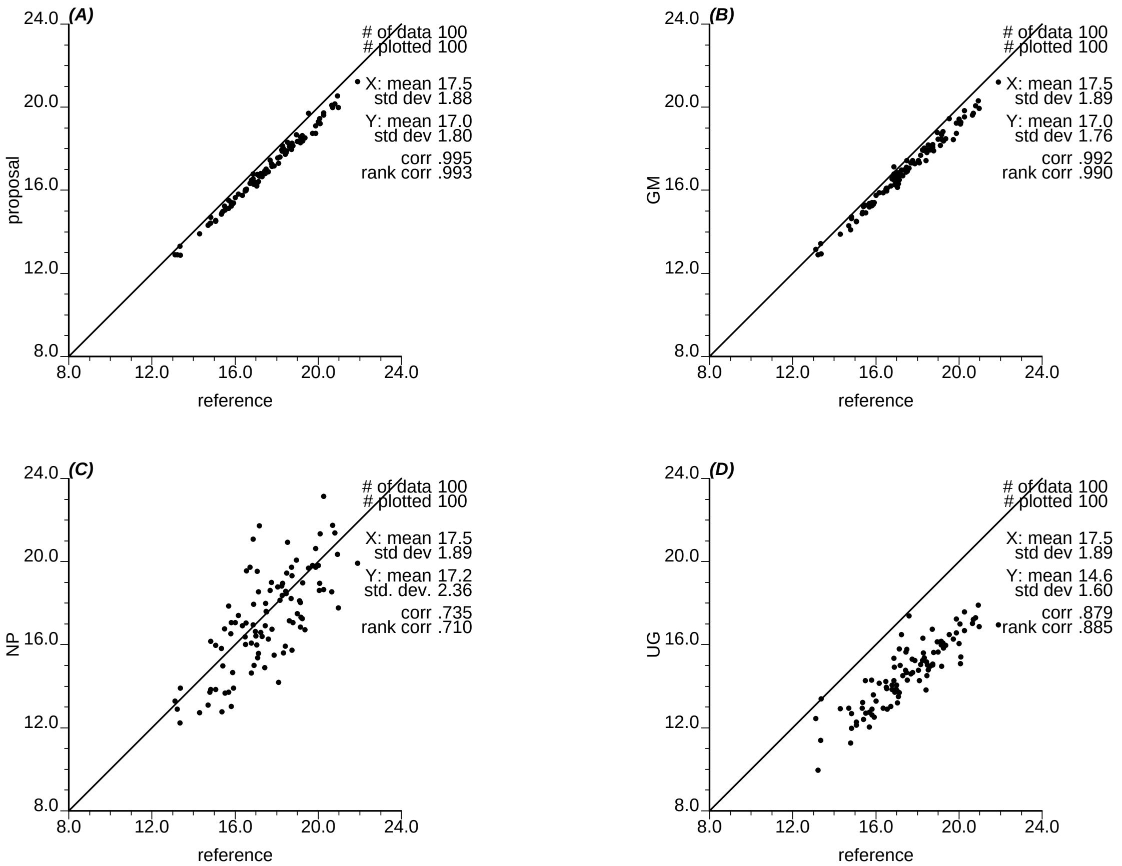

Fig. 1 Wellbore fluxes; correlations between the fine scale and the coarse scale of multi-Gaussian transmissivity fields with isotropic structures: (a) *Q* reference *vs* proposal, (b) *Q* reference *vs* GM, (c) *Q* reference *vs* NP, and (d) *Q* reference *vs* UG.

performance of the traditional geometric mean method. The correlation and rank correlation coefficient show a slight decrease relative to (a), but are still more than 99%. It seems that the geometric mean method is quite efficient and robust in upscaling radial flow in the near-well region for the scalar transmissivity fields with isotropic structures. Figure 1(c) plots the results of the NP method. The results are more variable with a correlation coefficient of only 73%. Obviously the reproduction ability is worse than the GM method. Figure 1(d) is the result of the UG method, i.e. that with a uniform interval grid. The wellbore fluxes calculated from the coarse grid severely deviate from those of the fine grid, and thus the method is heavily biased.

Non-Gaussian transmissivity fields with anisotropic structures

One hundred log-transmissivity fields are generated by ISIM3D (Gómez-Hernández & Srivastava, 1990), each with a correlation length equal to 10 in the *x* direction and 5 in the *y* direction, i.e. $\lambda_x = 10$ and $\lambda_y = 5$. The variance is set as $\sigma^2_{\ln Txx} = 2$. The transmissivity fields are constant vectorial, that is, $T_{xx} = T_{yy}/2$, and $E[\ln T] = 0$. The fluxes over the wellbore are computed for the fine scale fields and for the four different upscaled fields in the same manner as in the first scenario.

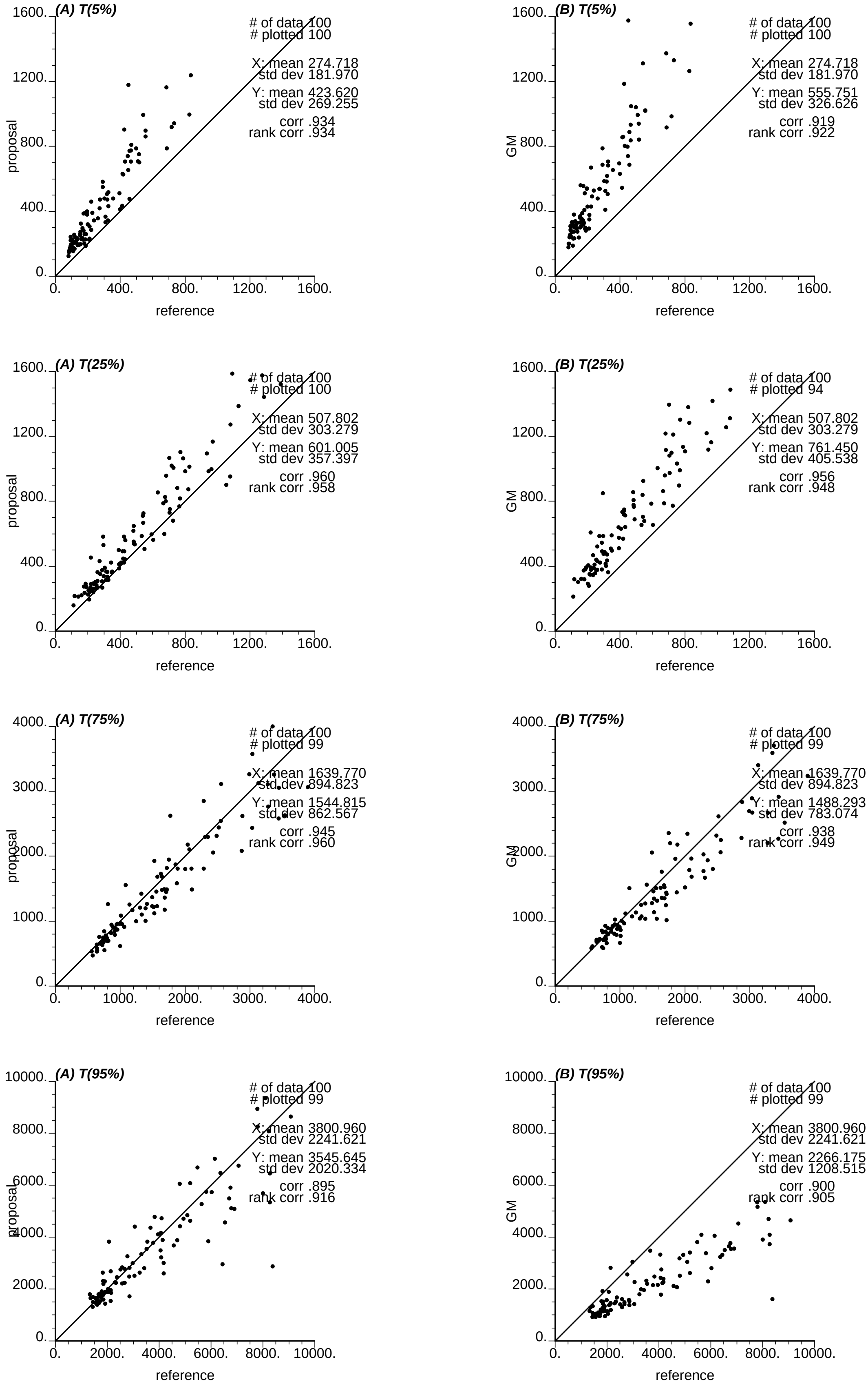

Fig. 2 Comparison of breakthrough curve reproduction of non-Gaussian transmissivity fields with anisotropic structures at four sample percentile points: (A) the left side cases represent the points at 5%, 25%, 75% and 95% of the proposed method, and (B) right side cases represent the points at 5%, 25%, 75% and 95% of the GM method.

Due to the difficulty of comparing the whole breakthrough curve for all realizations, we only sample four typical points of the breakthrough curve, e.g. 5%, 25%, 75% and 95%, which account for the early, middle and late arrival time. Figure 2 compares the breakthrough curve matching with reference fields between the proposed method and the GM method. As for the average breakthrough error, the proposed approach has a quite noticeable gain over the GM method. All four sample points of the former have values closer to the reference ones than those of the latter. The error reduction is 47% for the first point (5%), 63.3% for the second (25%), 37.3% for the third (75%), and 83.4% for the fourth (95%), respectively. The improvement in error using the proposed method is more apparent when anisotropic non-Gaussian models are used than the improvement when isotropic multi-Gaussian models are used.

CONCLUSIONS

Our comparison shows that upscaling may retain the main features of flow and transport in heterogeneous media. The proposed method improves the ability to predicting well discharge and solute transport error when a coarse grid is used for simulation. In other words, the proposed upscaling approach is efficient and robust both for flow and for transport simulations. Geometric mean upscaling is an alternative approach to upscaling in the near-well region, especially when the log-transmissivity is a multi-Gaussian field with an isotropic structure. A uniform grid fails to capture the flow and transport features in the near-well region.

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