Modelling of regional-scale well-flow in heterogeneous aquifers: 2-D or not 2-D?

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Abstract Transmissivity of aquifers is spatially variable in the plane. Hoeksema & Kitanidis (1985) have analysed field data of tens of aquifers and found that the log-transmissivity, regarded as a normal stationary random function, is characterized by integral scales of the order of kilometres to tens of kilometres. We examine the impact of transmissivity heterogeneity upon steady flow toward a pumping well, for realistic values of the ratio between the integral scale and the well radius of 2000 or larger. Our main finding is that in the zone of influence of the well, of radius up to 2000 well radii, the aquifer practically behaves like a homogeneous one, with the constant transmissivity pertaining to the location of the well. Flow can be modelled, therefore, as one-dimensional (in the radial coordinate) and randomness is parametric. Thus uncertainty can be practically eliminated by conditioning the head on transmissivity measured at the well. We conclude that flow and transport in the zone adjacent to the well are impacted by the three-dimensional local variability of the hydraulic conductivity, but not by the large-scale transmissivity heterogeneity.

Keywords groundwater flow; groundwater hydrology; random media; scaling; steady-state; stochastic processes; transmissivity; wells

INTRODUCTION AND BACKGROUND

Natural formations (aquifers, reservoirs) display, as a rule, spatial variability of the hydraulic conductivity K, at various scales. By using the terminology of Dagan (1989), $K(\mathbf{x})$ at the local scale is defined as pertaining to a volume of the order of decimetres that surrounds the point of coordinate $\mathbf{x}$. It can be determined by laboratory measurements of samples or cores, or indirectly by profiling of boreholes using geophysical methods or flow meters. At this scale, K variation is of three-dimensional (3-D) nature and it is customary to model $\ln K$ as a normal random space function. The log-conductivity is generally of an anisotropic structure: the vertical integral scale I_v, of order O (10^{-1} m) is smaller than the horizontal one I_h = O (10^{0} m) (Rubin, 2003, Table 2.1). Since generally $I_v << D$ (the formation thickness), I_v can be determined from the histogram of measured K along a borehole that is fully penetrating the aquifer. The determination of I_h is more difficult, as it implies cross-correlation of different boreholes.

In contrast, at the regional scale, flow is modelled as 2-D due to the large ratio between the horizontal extent (tens of kilometres) and D. Then, the pertinent property is the transmissivity T. In numerical codes, block transmissivity can be viewed as the result of upscaling of the local K. Since natural flow at this scale generally varies very slowly in space and since numerical blocks are large compared to I_h, the transmissivity can be approximated by $K_{ef}D$, where K_{ef} is the effective conductivity for mean uniform flow. However, transmissivity is generally determined indirectly either by solving the inverse problem or by pumping tests. In the latter case, analysis of measurements of head at a distance $r >> I_h$ from the well and for a prolonged period may provide the value of $K_{ef}D$, and we assume this is the case.

Hoeksema & Kitanidis (1985) analysed pumping tests in tens of aquifers, each of which contained multiple wells. They found that $T(\mathbf{x})$ is spatially variable and $Y = \ln T$ can be modelled as normal. The range of values for the integral scale I identified by Hoeksema & Kitanidis (1985) are as follows: 1.4–45 km for aquifers made of consolidated materials and 2.6–39 km for unconsolidated materials. Hence the planar log-transmissivity integral scales are larger by at least three orders of magnitude than the local ones. Due to this wide separation, one may regard the heterogeneous aquifer as a superposition of two random structures: the quasi-stationary local ln$K$ (3-D) and the stationary log-transmissivity Y (2-D), where the latter represents a slowly varying trend of the first one.

The topic we are going to address here is the following: what is the impact of transmissivity spatial variation, characterized by the scales identified by Hoeksema & Kitanidis (1985), upon the head distribution around a pumping well? This problem has been investigated recently by Sánchez-Vila (1997), Sánchez-Vila *et al*. (1999), Riva *et al*. (2001), Neuman *et al.* (2004), and Copty & Findikakis (2004). The main difference between the present study and the previous ones is that we focus the analysis on formations characterized by the small, but realistic, values of the ratio between the well radius r_w and I, namely $r_w\, I^{-1} < 5 \times 10^{-4}$. We shall show that this restriction has a profound influence on the flow in the zone of influence of the well (say $r\, r_w^{-1} > 2000$): the aquifer practically behaves as a homogeneous one of transmissivity equal to its local value. Hence, modelling Y as a random 2-D space function has limited impact upon the zone of interest surrounding the well.

BASIC ASSUMPTIONS AND METHODOLOGY

We consider steady flow in a confined aquifer of unbounded extent. We assume that $Y = \ln T$ is stationary of mean $\langle Y \rangle = \ln T_G$, variance σ_Y^2 and two point covariance $C_Y(\mathbf{x}_1 - \mathbf{x}_2) = \sigma_Y^2 \rho(|\mathbf{x}_1 - \mathbf{x}_2|)$. Here ρ is the autocorrelation, characterized by the integral scale $I = \int_0^\infty \rho(\xi) d\xi$. A well of radius r_w is pumping steadily, with given discharge Q and head H_w. The head $H(r,\theta)$, with $x = r\cos\theta$, $y = r\sin\theta$, is random due to the randomness of Y.

Following Matheron (1967) we adopt the definition of the equivalent transmissivity $T = Q \ln(R\, r_w^{-1})\, [2\pi\, (H_R - H_w)]^{-1}$ of a domain surrounding the well $r_w < r < R$ with boundary conditions of constant heads $H(r_w) = H_w$, $H(R) = H_R$. Thus, T_{eq} is the transmissivity of a fictitious homogeneous aquifer that conveys the same discharge Q as the actual one, for the given head drop.

We slightly generalize the definition for any r and for an unbounded aquifer as:

$$T_{eq} = \frac{A}{\overline{H}(r)} \quad ; \quad A = \frac{Q \ln(r / r_w)}{2\pi} \tag{1}$$

where $\overline{H}(r) = (2\pi)^{-1} \int_0^{2\pi} H(r,\theta) d\theta$ is the average head over a circle surrounding the well, while for convenience $H_w = 0$. We are going to examine the impact of heterogeneity with the aid of T_{eq}, which is a parameter accessible by measurements of H (with $\overline{H} \cong H$).

The equivalent transmissivity T_{eq} is random since $\overline{H}$ is such. From considerations of dimensionality the moments of $T_{eq} T_G^{-1}$ are functions of σ_Y^2, $r\, r_w^{-1}$, and $r_w I^{-1}$. To determine $\langle T_{eq} \rangle$ and $\sigma_{T_{eq}}^2$ in an approximate manner, we adopt the common first-order approximation in σ_Y^2. First, we expand the head in a perturbation series $\overline{H} = H_0 + \overline{H}_1 + \overline{H}_2 + \ldots$ where $H_0 = \mathrm{O}(1)$, $\overline{H}_1 = \mathrm{O}(\sigma_Y)$, $\overline{H}_2 = \mathrm{O}(\sigma_Y^2)$. Then, from (1), $T_{eq} = (A\, H_0^{-1})[1 - (H_1 H_0^{-1}) + \ldots]$ and subsequently the expected value and the variance at leading order are found as:

$$\langle T_{eq} \rangle = T_{eq,0} = \frac{A}{H_0} \quad , \quad \sigma_{T_{eq}}^2 = \frac{A^2}{H_0^2} \frac{\sigma_{\overline{H}}^2}{H_0^2} \text{ with } \sigma_{\overline{H}}^2 = \langle \overline{H}_1^2 \rangle + 0(\sigma_Y^4) \tag{2}$$

with A defined in (1).

The general solution of $\sigma_{T_{eq}}^2 T_G^{-2} \sigma_Y^{-2} = f\left(r\, r_w^{-1}, r_w I^{-1}\right)$ is discussed in the next section. It is a simple matter, however, to derive it at the limit $r\, I^{-1} \to 0$, i.e. for a homogeneous aquifer of random T surrounding the well. This is a case of parametric uncertainty: in each realization the exact 1-D solution is given by $H = A\; T(0)^{-1}$. It follows that $T_{eq} = T(0)$, $\langle T_{eq} \rangle = T_G + 0(\sigma_Y^2)$, $\sigma_{eq}^2 = T_G^2 \sigma_Y^2 + 0(\sigma_Y^4)$ so that:

$$\frac{\sigma_{T_{eq}}^2}{T_G^2 \sigma_Y^2} = 1 \text{ for } r_w\, I^{-1} \to 0 \tag{3}$$

Our aim is to derive $\sigma_{T_{eq}}^2$ for small, but different from zero, $r_w\, I^{-1}$ in order to assess the impact of the spatial variability of T upon T_{eq}.

COMPUTATIONS, RESULTS AND DISCUSSION

The starting point for determining T_{eq} is the equation of well-flow:

$$\nabla(T \nabla H) = Q\, \delta(\mathbf{r}) \tag{4}$$

where $\delta(\mathbf{r})$ is the Dirac operator such that the well is replaced by a point-sink of strength Q at the origin $\mathbf{r} = 0$.

With $T = T_G \exp(Y') = T_G\,(1+Y' + \ldots)$, the expansion of equation (4) at zero and first-order in Y', the log-transmissivity fluctuation, yields:

$$\nabla^2 H_0 = \frac{Q}{T_G}\delta(\mathbf{r}) \qquad (r = r_w, H_0 = H_w = 0)$$
$$\nabla^2 H_1 = \nabla Y' \bullet \nabla H_0 \quad (r \to 0, H_1 \to 0) \tag{5}$$

The zero order solution is given by $H_0 = A\ T_G^{-1}$ which represents flow in a homogenous aquifer of transmissivity T_G. The solution of the random H in equation (5) can be written in terms of the Green function $G(\mathbf{r}-\mathbf{r}') = (2\pi)^{-1}\ln|\mathbf{r}-\mathbf{r}'|$ as follows:

$$H_1(\mathbf{r}) = \int \nabla Y'(\mathbf{r}') \bullet \nabla H_0(\mathbf{r}') G(\mathbf{r}-\mathbf{r}') d\mathbf{r}' \tag{6}$$

We have simplified the expression of $\sigma_{\overline{H}}^2$, based on (6), and reduced it to the computation of three quadratures (the details will be given elsewhere). The final results for the function $\sigma_{T_{eq}} T_G^{-1} \sigma_Y^{-1} = A\sigma_{\overline{H}} H_0^{-2}$ are given in Fig. 1. This figure, which encapsulates our main results, depicts the dependence of the ratio between the standard deviation of the equivalent transmissivity for $r_w\ I^{-1} > 0$ and the one corresponding to $r_w I^{-1} = 0$, as function of $r\, r_w^{-1}$.

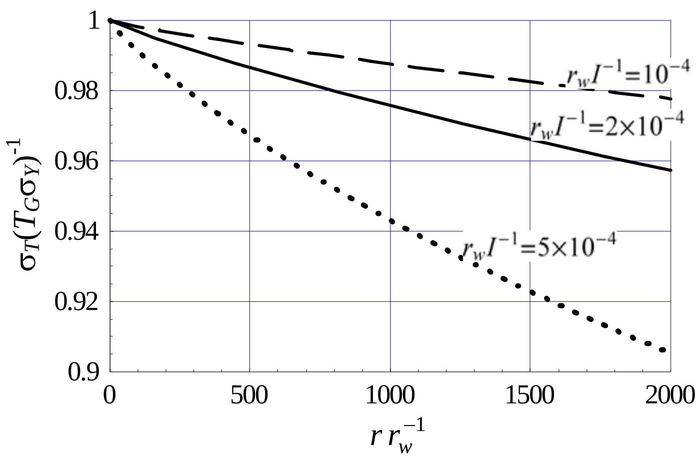

Fig. 1 Transmissivity standard deviation in the well influence zone for a few values of $r_w\, I^{-1}$.

Generally speaking, the head variance $\sigma_{\overline{H}}^2$ drops from its value $\sigma_Y^2 H_0^2$ for $r\, I^{-1} = 0$ to zero for $r\, I^{-1} >> 1$. Indeed, as $r\, I^{-1}$ increases, $\overline{H}(\mathbf{r})$ averages the flow over an expanding area relative to the heterogeneity scale. Eventually ergodicity is attained, with $\sigma_{\overline{H}}^2 \to 0$ and $\langle T_{eq} \rangle = T_{eq} = T_G$. However, for the aforementioned large field values of I, the latter limit is irrelevant since it is reached at a huge distance from the well, far beyond its radius of influence.

Figure 1 depicts the drop of $\sigma_{T_{eq}} T_G^{-1} \sigma_Y^{-1}$ within the radius of influence of the well, for $r\, r_w^{-1} < 2000$ (e.g. for $r_w = 0.25$ m, $r = 500$ m). It is seen that for the reasonable value of $r_w\, I^{-1} = 2 \times 10^{-4}$ (i.e. $I = 1.25$ km) $\sigma_{T_{eq}} T_G^{-1} \sigma_Y^{-1}$ stays very close to unity and even more so for larger *I*. Even in the unrealistic case of $r_w\ I^{-1} = 5 \times 10^{-4}$, the deviation is small.

We conclude that in the area surrounding the well and for typical values of the log-transmissivity integral scale the aquifer practically behaves as a homogeneous one of transmissivity $T(0)$ and the flow can be regarded as one-dimensional (in terms of the radial coordinate r). The randomness of T is of a parametric nature and is manifest in the uncertainty of prediction of H and T_{eq}, but not in spatial variability of the latter.

If the transmissivity is conditioned with the aid of that measured at the well, this large parametric uncertainty is eliminated and the flow becomes one-dimensional and deterministic (the topic of conditioning will be discussed elsewhere). Still, the local scale heterogeneity, of a three-dimensional nature, has a definite impact on flow and transport in the domain adjacent to the well. This effect is not captured by the transmissivity spatial variability (see Fiori *et al.* 1998; Indelman 2004; Lessoff & Indelman 2004).

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