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Delineation of capture zones in transient groundwater flow systems

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Abstract Capture-zone analyses are widely used to facilitate protection of groundwater supplies. Even though frequently substantial, transients are commonly ignored in the capture-zone analyses assuming a steady-state flow. Furthermore, advection-only flow paths generally applied in capture-zone analyses might not provide an adequate representation of mean plume behaviour of potential contaminant transport, especially in transient conditions. Here we analyse the impact of the transients and dispersion in the groundwater flow and transport on the capture zone estimates for a series of synthetic cases. Conditions for performing transient advective–dispersive capture-zone analyses are defined. They depend predominantly on the magnitude of groundwater transport velocities at the spatial and temporal scales of interest.

Keywords capture zone; predictive uncertainty; transient analysis

INTRODUCTION

Capture zones are important for the efficient protection of groundwater resources produced by wells and springs. Typically, the capture zones are delineated using mathematical models. The models are based on simplifying assumptions for a representation of true hydrogeological systems. For example, the transients are commonly ignored in the flow and transport models assuming a steady-state flow. Actually, substantial transients might exist, for example, due to variability in pumping rates of water-supply wells (e.g. Reilly & Pollock, 1996; Festger & Walter, 2002). As a result, there might be a substantial bias in the steady-state capture-zone estimates. Furthermore, the groundwater transport might be represented by advection-only flow paths (e.g. Rock & Kupfersberger, 2002). This might not provide an acceptable representation of mean plume behaviour of potential transport. As a result, we might have an additional bias in the capture-zone estimates.

Here we analyse, both theoretically and numerically, the impact of the transients in the groundwater flow and transport on the capture zone estimates for a series of synthetic cases. We use an analytical approach to identify parameter groups controlling the solution, but we perform the actual computations numerically. Capture zone estimates derived by advective-only and advective–dispersive transport simulations are also compared. Conclusions are reached regarding the conditions at which transient advective–dispersive capture-zone analyses should be performed based on the properties of the transients (amplitude/frequency) and the medium (permeability/porosity/dispersivity).

METHODOLOGY

To delineate the transient capture zones, we couple the following partial differential equations describing transient groundwater flow and transport:

$$\nabla^2 h = \frac{1}{a}\frac{\partial h}{\partial t} + \frac{q}{k} \tag{1}$$

$$\nabla \cdot (\mathbf{D}\nabla c) - \mathbf{v}\cdot\nabla c = \phi\frac{\partial c}{\partial t} \tag{2}$$

$$\mathbf{D} = \frac{\alpha_L}{|\mathbf{v}|}\mathbf{v}\mathbf{v}^T + \frac{\alpha_L}{|\mathbf{v}|}adj\left[\mathbf{v}\mathbf{v}^T\right] = \frac{\alpha_L}{|\mathbf{v}|}\begin{bmatrix} v_1^2 & v_1 v_2 \\ v_2 v_1 & v_2^2 \end{bmatrix} + \frac{\alpha_T}{|\mathbf{v}|}\begin{bmatrix} v_2^2 & -v_2 v_1 \\ -v_1 v_2 & v_1^2 \end{bmatrix} \tag{3}$$

$$\mathbf{v} = k\nabla^2 h \tag{4}$$

where h is hydraulic head [L], k is hydraulic conductivity [L T^{-1}], a is hydraulic diffusivity [L^2 T^{-1}] ($a = k/S_S$, where S_S is specific storage [L^{-1}]), q is a source/sink function that can vary in space and time [L^2 T^{-1}], **D** is dispersion tensor [L^2 T^{-1}], α_L and α_T are the longitudinal and transverse dispersivities [L], **v** is the Darcy velocity vector [L T^{-1}], ϕ is porosity [–]. We assume that the flow is 2-D and confined, the media is uniform and isotropic, and there is no molecular diffusion. To solve equation (1), we use the Theis equation which defines the head drawdown s [L] at a radial distance r [L] from a pumping well at a given time t [T] since the pumping started:

$$s(r,t) = \frac{Q}{4\pi mk}\mathrm{Ei}\left(\frac{r^2}{4at}\right) \approx \frac{Q}{4\pi mk}\ln\left(\frac{2.25at}{r^2}\right) \tag{5}$$

where Q is well pumping rate [L^3 T^{-1}], m is aquifer thickness [L], **Ei** is the exponential integral function. When $r^2/4at > 0.1$, the exponential integral function can be approximated by a natural logarithm, and the flow regime is defined as a quasi-steady state. From equation (5), the radial hydraulic gradient is equal to:

$$\frac{\partial s}{\partial r} = -\frac{e^{\frac{r^2}{4at}}Q}{2\pi mkr} \approx -\frac{Q}{2\pi mkr} \tag{6}$$

When $r^2/4at > 0.1$, the exponential function is close to 1, and we obtain an approximate expression of the gradient that is time invariant. The pore (advective) velocity u [L T^{-1}] toward the well is:

$$u = -\frac{k}{\phi}\frac{\partial s}{\partial r} = \frac{e^{\frac{r^2}{4at}}Q}{2\pi mr\phi} \approx \frac{Q}{2\pi mr\phi} \tag{7}$$

Note that if $r^2/4at > 0.1$, u does not depend on k or a. Only if $r^2/4at < 0.1$ (large times/close to the well), u is not time invariant and depends on a, but not on k. In addition, u can be adjusted by a retardation factor to account for contaminant sorption.

If there are multiple wells pumping at stepwise changing rates, the composite drawdown can be computed using equation (5) and the principle of superposition:

$$s(x,y,t) = \sum_{i=1}^{N}\sum_{j=1}^{M}\frac{Q_{ij}-Q_{ij-1}}{4\pi mk}\,\mathrm{Ei}\left(\frac{r(x,y)_i^2}{4a(t-t_j)}\right) \approx \sum_{i=1}^{N}\sum_{j=1}^{M}\frac{Q_{ij}-Q_{ij-1}}{4\pi mk}\ln\left(\frac{2.25a(t-t_j)}{r(x,y)_i^2}\right) \tag{8}$$

where N is the number of pumping wells, r_i is the radial distance to pumping well i, M is the number of step changes of the pumping rates, t_j is the time elapsed since the pumping rate step change j, Q_{ij} is the pumping rate of well i during step j ($Q_{i0} = 0$). We limit our analysis to a simple case, where $N = 2$ and the pumping is cycled between the two wells at a rate Q ($Q_{11} = 0$, $Q_{12} = Q$, $Q_{13} = 0$, …; $Q_{21} = Q$, $Q_{22} = 0$, $Q_{23} = Q$, …). The time step t_C for pumping change is constant ($t_j = j\, t_C$).

To solve the transport equations (2)–(4), we use a Lagrangian (particle-tracking) technique (Robinson, 2002). The advective transport time from a given initial spatial location (A) to a pumping well (B) along a flow path can be defined as:

$$\tau_{adv} = \int_A^B \frac{\phi}{\mathbf{v}}dl = \int_A^B \frac{\phi}{k\nabla^2 h}dl \tag{9}$$

Note that due to transients, the flow path does not only depend on the initial location but also on the transient changes of the flow structure. The flow path between points A and B is not known *a priori*, but it can be determined using transient particle tracking. If the transport is also impacted by dispersion, the contaminant plume will disperse from the advective flow path. In our case, since we have flow toward pumping wells, the forefront of the plume will always experience higher velocities than the plume backside. As a result, the front portions of the plume will be moving and dispersing faster than the back portions. Therefore, we can expect that advective transport time τ_{adv} will be representative of neither the peak (τ_{peak}) nor the median (τ_{med}) breakthrough arrivals of the plume at the wells. Furthermore, the advective flow path will define a unique arrival to one of the wells for each initial location. In the advective–dispersive case, the plume may be distributed between the wells. Therefore, due to transients and dispersion, there will be overlaps in the capture zones associated with each of the wells.

Based on the above analytical expressions, we can derive the following dimensionless ratios that will help us analyse our transient capture zone estimates. The two-dimensional spatial coordinates can be scaled as x/d [–] and y/d [–], where d is the distance between the two pumping wells [L]. A dimensionless time can be defined as ta/d^2. The advective velocities are multiplied by a scaling factor $Q/md\phi$ [L T^{-1}]. We will compare this factor with the ratio between half the distance between the wells $d/2$ and the size of the pumping steps t_C ($d/(2t_C)$). Dimensionless longitudinal and transverse dispersivities are defined as α_L/d and α_T/d.

The mathematical problem described can be solved directly using the analytical expressions discussed above. For computational convenience, we solved the problem numerically using the finite-element simulator FEHM (Zyvoloski *et al.*, 1997). The 2-D model domain (Fig. 1) is defined to be large enough to minimize the boundary effects (about 20 times the distance between the wells, d). The computational grid is generated using LaGriT (Trease *et al.*, 1996). The grid is fine in the well vicinity (d/100) and the grid cells increase geometrically with the distance from the wells. The pumping periods t_C are discretized using 10 geometrically increasing time steps. The transport is simulated using the particle tracking capabilities of FEHM (Robinson,

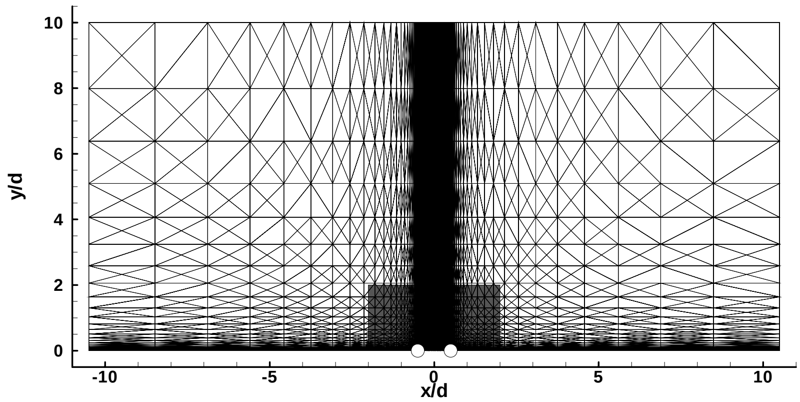

Fig. 1 Plainview of model domain, computational grid, pumping wells (white circles), and area for capture zone analysis (grey rectangle).

2002). The capture zones are delineated using instantaneous release at $t = 0$ of plumes at multiple initial locations defining a rectangle area around the wells (Fig. 1) The size of the rectangular area is $2d \times 1d$. In the advective-only case, we use 80 000 (400 × 200) regularly spaced initial locations. In the advective–dispersive case, 4000 (80 × 50) initial locations are used, and 1000 particles per release location are applied to characterize the plume distribution. The transient flow and transport are simulated for a series of pumping cycles until all the particles are captured. The capture-zone analyses are computationally very demanding. To achieve computational efficiency, we have used supercomputer clusters to parallelize the capture-zone delineation.

RESULTS

Capture-zone results using advective-only groundwater transport are presented in Fig. 2. If the two wells are pumping simultaneously at the same rate, their capture zones (white and grey regions) are separated by a straight line, Fig. 2(a). This capture-zone delineation is independent of any of the model parameters as well as of the type of flow system: steady-state or transient. When the well pumping is alternating as described above, the capture zones have more complicated structures, Fig. 2(b)–(f). If the pore (advective) velocities are very low (the factor $Q/(md\phi)$ is less than 10^{-7} m s^{-1}; Fig. 2(b)) the capture zone estimates are close to the steady-state estimate, Fig. 2(a). However, slightly faster pore velocities ($Q/(md\phi) > 5 \times 10^{-7}$ m s^{-1}) cause the level of interfingering between the capture zones to increase substantially, Fig. 2(d),(e). The velocity factor $Q/(md\phi)$ also impacts the number of fingers and the size of the fingers observed over our domain. The greater the value of $Q/(md\phi)$, the less and the thicker the fingers. Note that the cut-off value above which capture zone interfingering occurs compares well with the ratio between half the distance between the wells $d/2$ and the size of the pumping steps t_C, which in this case is $d/(2t_C) = 5.8 \times 10^{-7}$ m s^{-1}. Let us assume typical values for our model parameters $m = 100$ m, $d = 100$ m, $t_C = 1000$ days and $\phi = 0.1$. In this case, the pumping rates in Figs 2(b), 2(c), 2(d) and 2(e) are 1, 5, 10 and 20 L s^{-1}, respectively. Typical water-supply wells pump at rates higher than 10 L s^{-1}. Therefore, the interfingering of the advective-only capture zone due to transients will

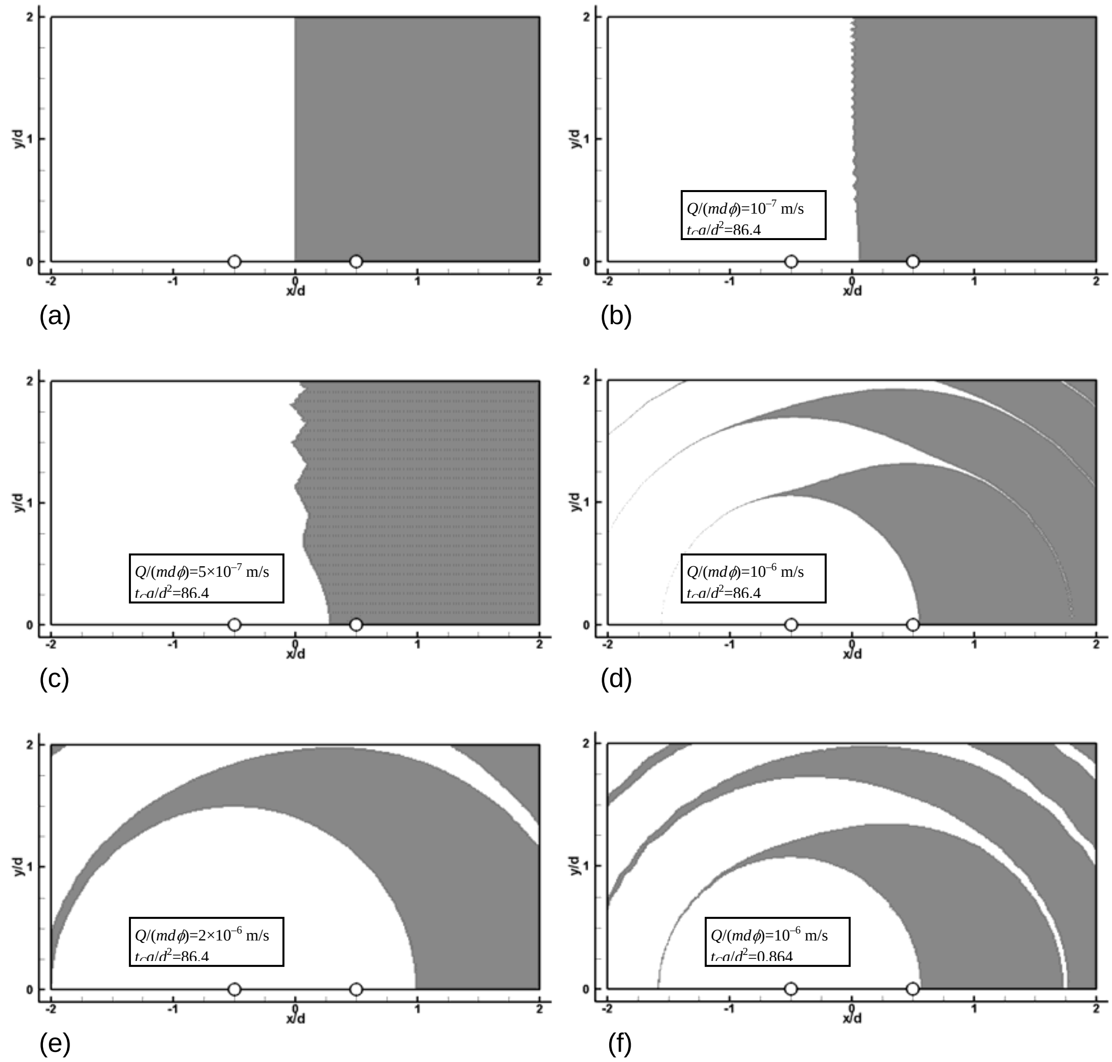

Fig. 2 Capture zones delineated using advective transient flow paths: (a) simultaneous pumping; (b)–(f) cycled pumping. The well locations are indicated by circles. The white and grey areas represent capture zones of the left and right well, respectively.

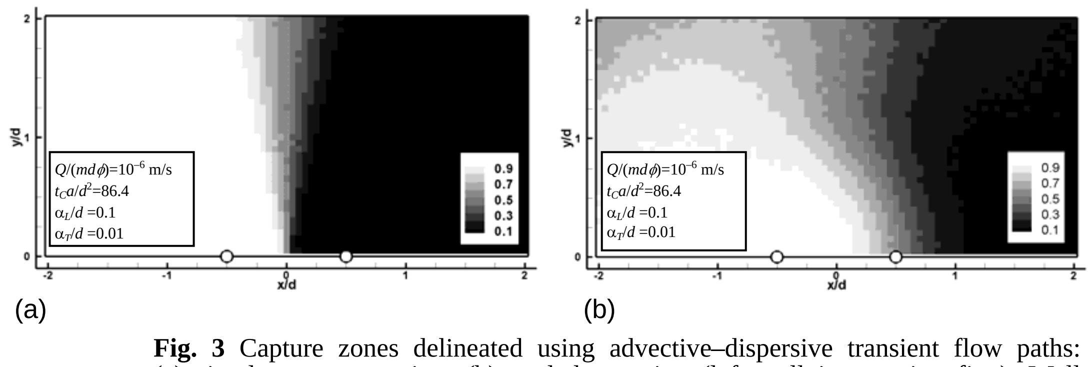

Fig. 3 Capture zones delineated using advective–dispersive transient flow paths: (a) simultaneous pumping; (b) cycled pumping (left well is pumping first). Well locations are indicated by circles. The grey scales define the ratio of the plume captured by the left well (for the right well, the ratio is 1 minus the ratio for the left well).

occur frequently. For our example, it can be suggested that whenever $Q/(md\phi) > d/(2t_C)$, the transients should be taken into account when estimating capture zones.

Other model parameters have a limited impact on the capture zone estimates. The zones shown in Fig. 2(b)–2(e) are obtained using $t_C\, a/d^2 = 86.4$. As a result, the capture zones in our domain are estimated for distances from the wells and simulation times for which $r^2/4at$ is predominantly less than 0.1 (a quasi-steady state regime). As it can be expected, further increase of the hydraulic diffusivity a ($t_C\, a/d^2$) does not change the capture zone estimates; therefore, our results are independent of a when $t_C\, a/d^2 > 86.4$. However, a decrease of $t_C\, a/d^2$ below 86.4 impacts the estimates by widening the fingers, Fig. 2(f); the capture zones of the right well (grey regions) in the left portion of the domain and the capture zones of the left well (white regions) in the right portion of the domain are thicker in Fig. 2(f) compared to Fig. 2(d). Still, the impact of hydraulic diffusivity on the capture zone estimates is minor compared to the impact of transport velocities. Also, note that our capture zone estimates are independent of hydraulic conductivity k which is assumed uniform and isotropic.

Capture-zone results using advective–dispersive groundwater transport are shown in Fig. 3. The grey scales define the ratio of the plume captured by the left wells (for the right well, the ratio is 1 minus the ratio for the left well). If the two wells are pumping simultaneously at the same rate, the dispersion will cause a slight smearing of the capture zone boundary (Fig. 3(a); compare with Fig. 2(a)). However, the cyclic pumping causes substantial smearing of the capture zones (Fig. 3(b); compare with Fig. 2(d)). Therefore, the interfingering of the advective-only capture zones due to transients causes a substantial additional dispersion of the tracked plumes.

FINDINGS AND CONCLUSIONS

Our results demonstrate the importance of transients and plume dispersion to capture zone analyses. A key parameter in that respect is the advective transport (pore) velocity. If for a given simulation time, the advective travel distances to the pumping wells are comparable to or less than half the distance between the wells, the transients have minimal effect on the delineated capture zones. If the advective travel distances are higher, we observe interfingering of the capture zones. The hydraulic diffusivity a impacts the rate (velocity) of propagation of the transients away from the pumping wells, but it has limited impact on the capture-zone estimates once a quasi-steady state flow regime is achieved. The greater the hydraulic diffusivity, the faster a quasi-state regime is obtained. The interfingering of the advective-only capture zone due to transients causes an additional plume dispersion when corresponding advective–dispersive capture-zone analyses are carried out.

Here we have performed a transient capture zone analysis using a set of release points distributed in space but associated with a single release time. Comprehensive transient capture zone analysis should also include a series of releases in time. Due to transients, capture zones associated with different release times will have different properties. This type of analysis was performed earlier by Vesselinov *et al.* (2003, 2004). They delineated transient capture zones for an existing multi-well water-supply system using multiple plume releases distributed in time and space. To address this more comprehensive case, analyses similar to that here presented will be performed in the future. Future work will also elaborate on the discrepancy between the advective capture travel times and the time of median and peak breakthrough arrivals at the wells when plume dispersion due to transients is considered.

For transient capture zone analyses, it is crucial to have accurate information about the transients. If the transients are unknown (e.g. future pumping rates) or very uncertain, stochastic analysis should be applied.

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