(IAHS Publ. 305, 2006).

Geomorphic flood research

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Abstract The paper discusses empirical and theoretical evidence on the structure of river basins, emphasizing the intertwined form and function of such complex adaptive systems. Geomorphic signatures on the structure of floods are discussed. It also deals with the emergence and the implications of common features in a variety of natural networks, chiefly the scaling signatures of chance and necessity in their evolution, appearing in somewhat disparate contexts from physics to biology. Hence the claim that a comparative study of networks relates to hydrological research and to the general theme (flood research) of the Kovacs Colloquium. Of some importance, arguably, is the claim that river basins and their floods constitute one of the most reliable and fascinating laboratories for the observation of how Nature works across a wide range of scales. Specific applications are discussed that deal with transport through fractal networks. In passing, a geomorphic/ stochastic perspective of large-scale flood studies, and how floods relate to pollution loads generated by the transport of matter at basin-scales, are considered.

Key words basin-scale transport; fractal networks; hydrological response; watershed theory

Recherche géomorphologique des inondations

Résumé Cet exposé présente les éléments expérimentaux et théoriques concernant la structure des basins hydrologiques en insistant sur la forme entrelacée et la fonction de tels systèmes complexes et adaptatifs. Des signatures géomorphologiques de la structure des inondations seront discutées. Cet exposé discute aussi de l'apparition et des implications des traits communs que l'on trouve dans une variété de réseaux naturels, particulièrement les signatures invariantes d'échelle du hasard et de la nécessité dans leur évolution, qui apparaissent dans des contextes quelque peu disparates allant de la physique à la biologie. D'où une affirmation qu'une étude comparative des réseaux est liée à la recherche hydrologique et au thème général de ce colloque Kovacs. Il est peut être d'une certaine importance que l'affirmation proposée dans cet exposé que les bassins hydrologiques et leurs inondations constituent un des laboratoires les plus fiables et les plus fascinants pour observer comment la Nature fonctionne à travers une large gamme d'échelles. Les applications spécifiques discutées dans cet exposé traitent du transport à travers un réseau fractal—au passage, je vais discuter une perspective géomorphologique/stochastique des études d'inondation à grandes échelles, et comment les inondations sont liées aux charges de pollution produites par le transport de matière à l'échelle du bassin.

Mots clefs transport à l'échelle du bassin; réseau fractal; réponse hydrologique; théorie du bassin versant

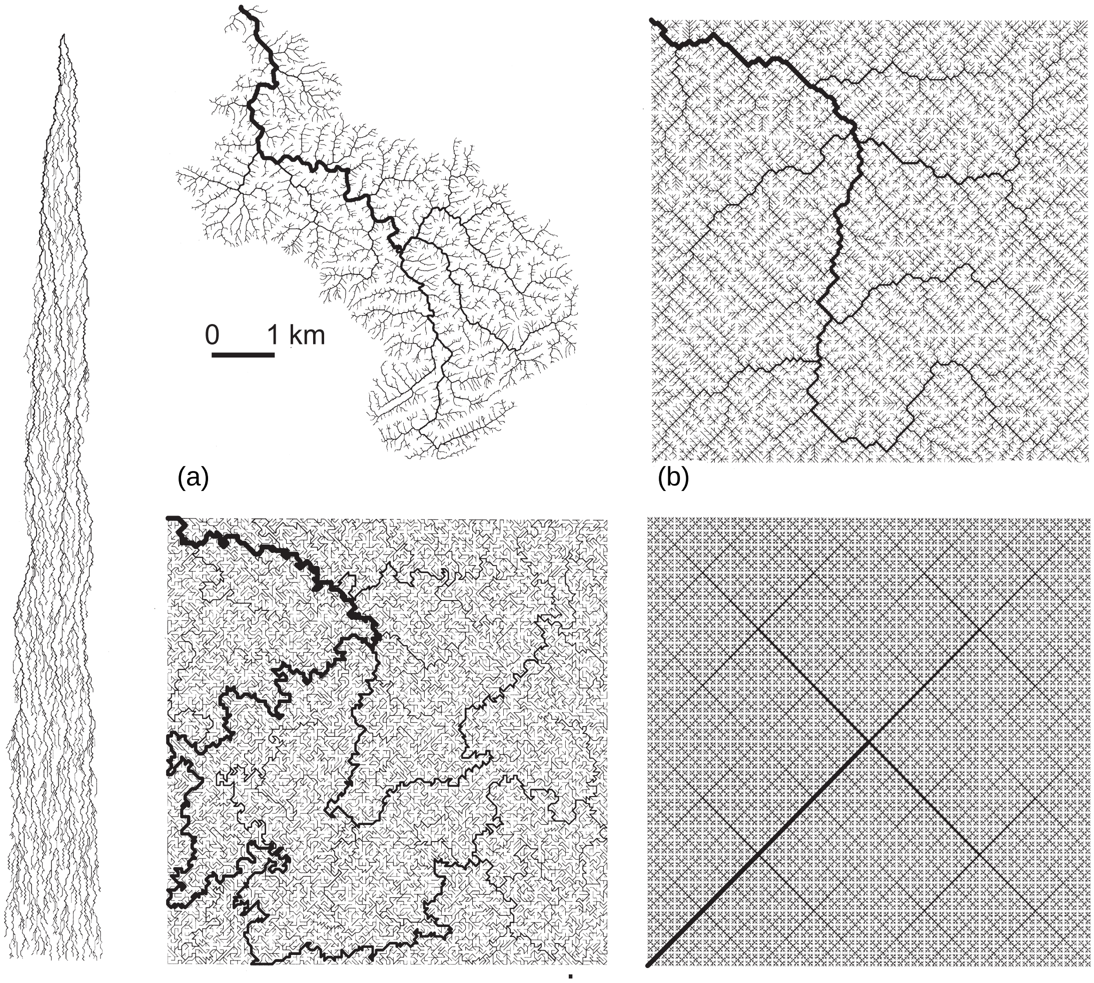
1. INTRODUCTION

The effective management of hydrological systems, including the general architecture of systems capable of mitigating all effects of floods and including measures aimed at improving the quality of receiving water bodies, can benefit from the use of reliable models describing hydrological fluxes and storage terms both in space and time. Note that I intentionally include in the general subject of flood studies the class of models describing transport processes at the scale of the basin. Several reasons support my choice. On one hand, in fact, most pollutant loads are carried by a few intense flood events rather than by chronic high concentrations at low stages of flow. This was not apparent earlier, as e.g. mitigation measures for nonpoint source pollutions were aimed at the treatment of low flows. On the other hand, transport through fractal networks is getting great attention from a variety of other fields where landscape heterogeneities strongly affect processes like the spreading of populations (Campos *et al.*, 2006) or diseases (Méndez *et al.*, 2004). Lastly, theoretical approaches suited to large-scale transport domains are still lagging behind, especially if solidly rooted in the stochastic framework that seems appropriate for large-scale applications (Rinaldo & Marani, 1987; Rinaldo *et al.*, 2005). The term geomorphic flood research is thus meant as an analogue of catchment theory.

Branching river networks are crucial in all this. They are striking examples of natural fractal patterns which self-organize, despite great diversities in forcing geological, lithological, vegetational, climatic and hydrological factors, into forms showing deep similarities of the parts and the whole across several orders of magnitude, and recurrent motifs everywhere (Rodriguez-Iturbe & Rinaldo, 1997). Interestingly, the drainage network in a river basin shows tree-like structures that provide efficient means of transportation for runoff and sediment and show clear evidence of fractal behaviour. Our observational capabilities are also noteworthy. Accurate data describing the fluvial landscape across scales (covering up to 5 orders of magnitude) are extracted from digital terrain maps remotely collected and objectively manipulated. Raw data consist of discretized elevation fields, say z_i, on a lattice. The drainage network is determined assigning to each site i a drainage direction through the steepest descent at i, i.e. along $\vec{\nabla} z_i$. Multiple flow directions in topographically convex sites, and their derived hydrological quantities, are also easily tackled. Many geomorphic features are then derived and analysed. To each pixel i (the unit area on the lattice) one can associate a variable that gives the number of pixels draining through i, i.e. following the flow directions. In the case of uniform rainfall injection, area provides a measure of the flow at point i allowing one to use these two quantities interchangeably. Drainage directions uniquely determine network lengths. Down-stream lengths L_i (i.e. from a site i to the outlet following the largest topographic gradient, i.e. steepest descent) can be computed easily to derive their distributions which clearly show the characters of finite-size scaling (Rodriguez-Iturbe & Rinaldo, 1997). Channelized patterns are now reliably extracted from $\{z_i\}$ fields through the exceedence of geomorphic thresholds, and have thus much improved our ability to describe objectively natural forms over several orders of magnitude. Large-scale observations have allowed thorough comparisons across scales defining fractal river basins. It is my belief that the detailed knowledge (automatically acquired and objectively manipulated) of the geometry and the topology of the flow paths within river basins is the key player of any current transport model—hence the central role of geomorphology, and the title of this paper.

General transport models would serve well, however, both research and applications within the same framework, given the timeliness of design criteria that include directly concepts of probability—much of what flood research is all about, I guess. Management objectives require, in fact, models capable of: (a) reproducing system functioning as described by observations; and (b) predicting system functioning under conditions and during events which have not been observed, possibly generating statistical ensembles of events. This must be possible without the burden of making unphysical or unrealistic assumptions, like, typically, statistical stationarity of the response of ever-changing watersheds. Thus one can hardly overestimate the importance of basin-scale models of transport for society at large. Towards that end, the formulation of transport by travel time distributions is well suited (Rodriguez-Iturbe & Valdes, 1979; Gupta *et al.*, 1980; Dagan, 1989; Rinaldo & Rodriguez-Iturbe, 1996). I shall briefly address here elements of this formulation in a framework somewhat broader and more comprehensive than that of the original approach—results from transport theories concoct a formulation that applies regardless of whether we deal with flow or with transport models at catchment scales (e.g. as in Rinaldo & Marani, 1987; Rinaldo *et al.* 1989, 1991; Destouni & Graham, 1995; Cvetkovic & Dagan, 1996; Rinaldo & Rodriguez-Iturbe, 1996; Simic & Destouni, 1999; Gupta & Cvetkovic, 2002; Botter & Rinaldo, 2003; Botter *et al.*, 2005). The aim is to use the large-scale collection and objective manipulation of geomorphic, hydrological or land-use data now available, which indeed allow the automated description of the features of the geometry and topology of the flow domains—hence the river network, principally.

Figure 1 illustrates a sample of the variety of network forms that are relevant to geomorphic flood studies. A real fluvial network (Fig. 1(a)) of the Dry Tug Fork River (California, USA) is suitably extracted from a digital terrain map (Rodriguez-Iturbe & Rinaldo, 1997). Notice its clear tree-like structure, usual in the runoff production zone of the river basin. Its morphological features (like aggregation and elongation) are typical of fluvial patterns and recurrent “modules” appear regardless of the scale of total contributing area, such that the parts and the whole are quite similar notwithstanding local signatures of geological controls, here marked by a fault line clearly visible across the landscape. Scheidegger’s directed network (Fig. 1(c)) is constructed by a stochastic rule (Scheidegger, 1991); with even probability, a walker chooses between right or left forward sites only. The model was devised with reference to drainage patterns of an intramontane trench and maps exactly into a model of random aggregation with injection or voter models (Takayasu *et al.*, 1991) and also describes the time activity of a self-organized critical (Abelian) avalanche (Bak, 1996). Peano’s network (Fig. 1(e)) is a deterministic fractal whose main topological and scaling features, some involving exact multifractals (Schertzer & Lovejoy, 1989), have been solved analytically (Marani *et al.*, 1991; Flammini & Colaiori, 1996; Colaiori *et al.*, 1997). The basic prefractal is cross seeded in a corner of the square domain that covers the cross and its ensuing iterations. All subsequent subdivisions cut in half each branch to reproduce the prefractal on four, equal subbasins. Here the process is shown



(c) (d) (e)

Fig. 1 Samples of trees where a unique pattern links any inner site to the outlet of the network. (a) A real river network, the Dry Tug Fork (California), suitably extracted from digital terrain maps; (b) a single-outlet optimal channel network (OCN) selected starting from an arbitrary initial condition by an algorithm accepting random changes only of lowering the total energy dissipation of the system as a whole – thus incapable of reaching the ground state and settling in a local minimum dynamically accessible; (c) Scheidegger's random construct built by choosing with even probability right or left in a directed walk; (d) a "hot" network where any arbitrary change randomly assigned to an evolving network is accepted provided it maintains a tree-like form; (e) Peano's deterministic construct (from Rinaldo *et al.*, 2006b).

at the 11th stage of iteration. Optimal channel networks (OCNs) (Fig. 1(b)) are described elsewhere (Rodriguez-Iturbe & Rinaldo, 1997). They hold fractal characteristics that are obtained through a specific selection process from which one obtains a rich structure of scaling optimal forms that are known to closely conform to the scaling of real networks, even in the case of unrealistic geometric boundaries. To design a very inefficient tree, we have constructed a non-directed structure constrained to be tree-like (Fig. 1(d)) by using a Metropolis algorithm (Rinaldo *et al.*, 2005). This basically corresponds to an algorithm that accepts any change attributed sequentially at random sites of an evolving spanning network—an existing link is disconnected at a random site and rewired randomly to another nearest neighbour provided the change maintains a tree-like structure. Hot networks (Fig. 1(d)), so-called because they correspond to high temperatures of a Metropolis scheme where every spanning tree seeded in the outlet is equally likely, are thus abstract forms meant to reproduce a rather undirected tree. From the comparison of transport features along these geomorphic (or geomorphic-like) structures, several issues have been highlighted.

This paper is organized as follows: An introductory section reviews the kinematics and the elements of general transport theory that allow us to blend flow and transport of matter for a single transport volume. The ensuing section uses the theoretical results obtained for a single transport volume to obtain a formulation valid for arbitrary sequences (in series or in parallel) of transport states, distinguishing the effective functioning of any geomorphic paths upon the fraction of input rainfall conveyed therein. A few examples aimed at clarifying a somewhat convoluted procedure are then shown, which are related to the naturally nested structure of control volumes within a catchment rather than to unnecessary complications of our models. A section on possible extensions of the theory then closes the paper.

2. FLOW AND TRANSPORT AT BASIN SCALES

Once net rainfall is suitably partitioned into surface and subsurface pathways—not a subject of this paper as any model would do—the flux of the water carrier within natural formations is seen as a conservative process where water particles move within the control volume towards the outlet without significant variations of their mass. Thus, let m_w be the (time-independent) water mass transported by a single particle injected at time t_0 = 0 in the initial position x_0. Each trajectory is defined by its Lagrangian coordinate $\mathbf{X}(t) = \mathbf{X}(t;\mathbf{x}_0,t_0) = \mathbf{x}_0 + \int_0^t \mathbf{v}(\mathbf{X}(\tau),\tau)d\tau$ where $\mathbf{v}(\mathbf{x},t)$ is the point value of the velocity vector. The spatial distribution of water concentration in the transport volume V as a result of the injection of a single particle is given by Taylor (1921):

$$c_w(\mathbf{x},t;\mathbf{x}_0,t_0) \propto m_w\delta(\mathbf{x}-\mathbf{X}(t)) \quad (1)$$

where δ is Dirac's delta distribution and, without loss of generality, we have assumed unit porosity within the whole control volume (i.e. $\int_V c_w d\mathbf{x} = m_w$). Equation (1) states that, in the one-particle one-realization case, volumetric water concentration (water mass per unit transport volume) is nonzero only at the site where the particle is instantaneously residing (i.e. at its trajectory). Thus uncertainty in the dynamical specification of the particle (i.e. the evolution in time and space of the trajectory $\mathbf{X}(t;\mathbf{x}_0,t_0)$ of the labelled, travelling "water particle") is reflected in the transport process. Owing to the heterogeneity which characterizes transport processes and environments at basin scale, the trajectory is seen as a random function. Let therefore $g(\mathbf{X})d\mathbf{X}$ be the probability that the particle is found within the infinitesimal volume $d\mathbf{X}$ located around the position $\mathbf{X}$ at time t (notice that the functional dependence $g(\mathbf{X})$ implies $g(\mathbf{x},t)$ in terms of cartesian coordinates because of the evolution of the trajectory with time). The ensemble average concentration $\langle c_w(\mathbf{x},t)\rangle$ is given by the relationship (Taylor, 1921; Dagan, 1989):

$$\langle c_w(\mathbf{x},t)\rangle = \int_{-\infty}^{\infty} m_w \delta(\mathbf{x}-\mathbf{X}) g(\mathbf{X}) d\mathbf{X} = m_w g(\mathbf{x},t) \tag{2}$$

The distribution $g(\mathbf{x},t)$ is usually called a displacement probability density function (pdf). Important models describing displacement distributions, *g*, or $\langle c_w \rangle$ (from equation (2) $g \propto \langle c_w \rangle$), notably the cases deriving from the Fokker-Planck's equation, are reported in the literature (see, for a summary relevant to hydrology, Rinaldo *et al.*, 1991; Rinaldo & Rodriguez-Iturbe, 1996).

The displacement pdf $g(x,t)$ is determined by the kinematics of the carrier flow, and defines the travel time distribution $f(t)$ of the water carrier within the control volume. The definition of travel time distribution implies the identification of a suitable control section. We thus assume that the time t at which a particle crosses such a control section is unique and, most importantly, that all particles injected in V ensuing from $\mathbf{x}_0 \in V$ must transit through it. The probability density of travel times is proportional to the instantaneous mass flux at the absorbing barrier of the control volume (Dagan, 1989). In fact, water mass in storage within the control volume $M_w(t)$ is expressed by:

$$M_w(t) = \int_V \langle c_w \rangle d\mathbf{x} = m_w \int_V g(\mathbf{x},t)\, d\mathbf{x} = m_w P(T \geq t) \tag{3}$$

where $P(T \geq t)$ is the probability that the residence time is larger than current time t. Thus, by continuity, one has $dM_w(t)/dt = I - Q$ (where I [MT^{-1}] is the mass water input and $Q_w(t)$ [MT^{-1}] is the mass flux at the outlet of V), and therefore, for an instantaneous water pulse (i.e. $I(t) = m_w \delta(t)$):

$$Q_w(t) = -m_w \frac{dP(T \geq t)}{dt} = m_w f(t) \quad \text{for} \quad t > 0 \tag{4}$$

where $f(t)$ is the probability density function (pdf) of travel times for the water carrier. In surface hydrology, when the input is a unit of net rainfall, such a pdf is usually termed the instantaneous unit hydrograph (IUH). The equivalence of travel time distributions with IUHs was at the origins of the theory of the geomorphic unit hydrograph (Rodriguez-Iturbe & Valdes, 1979), i.e. the GIUH.

In using the travel time formulation of transport in surface hydrology, two courses have been pursued: one course assumes the form of the pdf, and characterizes it by some parameters of clear physical meaning, like mean travel times. An example of this is the exponential pdfs used to describe travel times of water particles in the original approach by Rodriguez-Iturbe & Valdes (1979) to derive the geomorphic unit hydrograph. The second course exploits the equivalence of water fluxes and pdfs to deduce travel times from the equations of motion. Eulerian, Lagrangian or travel time approaches therefore may differ formally although they are derived from the same assumptions. A discussion on the relative balance of the merits of the above approaches can be found in Dagan (1989).

Let us turn to reactive transport of solutes carried by hydrological waters in the same framework. A given amount of solute (of mass m_s) is injected within the control volume through an instantaneous release of water, and is thus allowed to move within the transport volume driven by the hydrological carrier flow and to exchange mass with the surrounding environment. The "reactive" character of the transport is described by the (spatial and/or temporal) variability of the solute mass associated with the water particles moving within the control volume, that is, the function $m_s = m_s(\mathbf{X},t)$ which embeds physical, chemical or biological exchanges with immobile phases in some contact with the carrier flow. One-particle, one-realization concentration fields resulting from the injection of a single reactive particle are given by the following equation:

$$c_s(\mathbf{x},t;\mathbf{x}_0,t_0) \quad \propto \quad m_s(\mathbf{X},t;t_0)\delta(\mathbf{x}-\mathbf{X}(t)) \tag{5}$$

The reactive components involved define the instantaneous solute mass m_s attached to the moving particle without affecting the trajectory $\mathbf{X}$ of the particle itself, which is determined by the usual kinematic relationship. The mass transfer occurring between the carrier and immobile phases (e.g. chemical or physical sorption, ion exchange, precipitation) leads in general to variability for m both in time and space. We assume, however, that the injection area is much larger than any correlation scale of heterogeneous transport properties and/or that the temporal scales relevant for the undergoing advective processes are larger than the characteristic times for the reaction processes. This suggests (Rinaldo & Marani, 1987; Rinaldo *et al.*, 1989, 2005; Botter *et al.*, 2005) that the spatial gradients of mass exchange become negligible and that, therefore, the contact times drive mass transfer between phases (i.e. the well-mixed approximation). The injection of identical particles labelled by carrier and solute masses m_w, m_s at different initial locations $\mathbf{x}_0$ at time t_0 produces, at time $t > t_0$, the sampling of different trajectories $\mathbf{X}(t)$ but yields roughly the same temporal evolution of the mass of solute transported $m_s(t - t_0, t_0)$, which thus depends (for a given injection time t_0) solely on the time available for the reaction processes, $t - t_0$. The expected value of the volumetric concentration $\langle c_s(\mathbf{x},t)\rangle$ (solute mass for unit transport volume) is then given, from equation (3), by the relation (Rinaldo & Rodriguez-Iturbe, 1996):

$$\langle c_s(\mathbf{x},t;t_0)\rangle = m_s(t-t_0,t_0)g(\mathbf{x},t-t_0) \tag{6}$$

where the similarity of structure with respect to passive transport stems from the fact that m_s is unaffected by ensemble averaging. Thus we obtain a generalization of Taylor's theorem for reactive transport problems. The displacement distribution g defines the structure of the carrier residence time distribution within the control volume and thus epitomizes the complex chain of events determining the hydrological flow. The mass function $m_s(t - t_0, t_0)$ accounts for all sorption/desorption processes which determine the temporal variability of the solute mass transported by the moving water particles. The decoupling of the reaction component from the transport problem is quite expedient because the displacement and the travel time distributions may be directly employed.

The solute mass instantaneously stored in the water carrier within the transport volume V (as a result of a solute injection occurring at $t = t_0$) may be thus expressed by the use of equation (6) as:

$$M_s(t) = \int_V \langle c_s(\mathbf{x},t;t_0)\rangle d\mathbf{x} = m_s(t-t_{0,}t_0)P(T \geq t-t_0) \tag{7}$$

where $p(T \geq t)$ is the probability that the residence time is larger than the current time t. Thus, deriving equation (7) with respect to t, one has:

$$\frac{dM_s(t)}{dt} = -m_s(t-t_0,t_0)\,f(t-t_0) + \frac{dm_s}{dt}P(T \geq t-t_0) \tag{8}$$

where the last term of the above equation represents the rate of solute, say R [MT^{-1}], transferred from the immobile phase to the water carrier due to the active reaction processes. Since for $t > t_0$ by continuity one has $dM_s/dt = -Q_s + R$ (where Q_s [MT^{-1}] is the solute flux at the outlet of V), by comparison with equation (8) we obtain:

$$Q_s(t;t_0) = m_s(t-t_0,t_0)\,f(t-t_0) \quad \text{for} \quad t > t_0 \tag{9}$$

which expresses the solute flux at the outlet due to the injection within the control volume at $t = t_0$ of an instantaneous water pulse carrying a solute mass m_s, which is time dependent owing to mass exchange processes.

In what follows, it is assumed that the solutes transported by the carrier undergo sorption phenomena with other immobile phases in contact with the water flow (e.g. soil grains, bed sediment, dead-end zones). The mass transfer between the phases is therefore driven by the difference between the solute concentration sorbed in the immobile phase and the solute concentration, say C, characterizing the water particles moving along the control volume (solute mass for unit water volume) (Van Genuchten, 1981). The latter may be straightforwardly derived by use of equations (2) and (6) as:

$$C(t-t_0,t_0) = \rho\frac{\langle c_s(\mathbf{x},t;t_0)\rangle}{\langle c_w(\mathbf{x},t;t_0)\rangle} = \rho\frac{m_s(t-t_0,t_0)}{m_w} \tag{10}$$

where ρ is the (constant) water density [ML^{-3}]. Notice that in equation (10) the capital letter C is employed for the solute concentration of the water particles (solute mass per water volume), so as to highlight the difference with respect to the volumetric concentration of solute c_s (mass per unit transport volume). Notice that at a given time t, the water particles injected into the system at the same injection time t_0 are all marked by the same resident concentration $C(t - t_0,t_0)$, independently from their trajectory. This is, of course, an important assumption which nonetheless seems applicable to most cases where rainfall is the driving factor (Botter *et al.*, 2005).

Note that it is appropriate to state clearly the mathematical analogies that stem from the position $\tau = t - t_0$, where τ is the travel time of a single particle within the control volume after injection at time t_0, thereby the contact time between phases, and t is chronological time. Thus, one may express the solute concentration of the water carrier as a function of only two of the above timescales (e.g. $C = C(\tau,t_0)$, or $C = C(\tau,t)$, see below). Within the above framework, solute mass transported by the water carrier, m_s, is thus defined by the rate of change of the scalar property $C(t - t_0,t_0) = C(\tau,t)$ attached to the mobile phase. Incidentally, when the scalar is simply the density of the carrier, i.e. $C(t - t_0,t_0) = const = \rho$, the above derivation reduces to the description of flow rates. In the general case, instead, the temporal variability of the function C (which retains all sorption/desorption processes determining the temporal variability of the mass transported by the moving particles) is related to the active reaction processes between the phases. For the sake of simplicity, linear rate-limited kinetics are assumed to drive the temporal evolution of the concentration function $C(t - t_0, t_0)$ (Rinaldo & Marani, 1987):

$$\frac{\partial C(\tau,t)}{\partial \tau} = k\left(\frac{N(t)}{k_D} - C(\tau,t)\right) \tag{11}$$

where N [MM^{-1}] is the concentration in the immobile phase (properly transformed by k_D [L^3M^{-1}], the equivalent of a partition coefficient) and k [T^{-1}] is the overall rate coefficient of the reaction kinetics between mobile and immobile phases. Accordingly with the well-mixed assumption, the concentration in the immobile phase N is assumed to solely depend on time and not on the position **x**. The temporal evolution of the function $N(t)$ may be thus described on the basis of a global (rather than local) mass balance, applicable to each "state" which is physically meaningful to identify. This is not the case, for instance, in the other approaches known from the literature (Cvetkovic & Dagan, 1994).

An important indicator of the validity of the above assumptions comes from an application where the carrier flow is forced to be in steady state, which is a particular case of the above framework for constant input flow rates (Botter *et al.,* 2005). Consider a steady-state flow through a generic heterogeneous medium and assume that the underlying Eulerian velocity field is a stationary random vectorial function **v**(**x**). The ensemble mean of the local velocity **v** is assumed to be positive (such as a mean flow direction is determined) and—without loss of generality—aligned with one axis. Under the above assumptions, the transport domain may be thought of as a collection of independent and stationary streamlines, which are characterized by different residence times owing to the heterogeneity of the transport properties involved. Solute particles injected within the flow field, or released from the soil, are simultaneously advected by the carrier and affected by sorption–desorption processes with immobile phases in contact with the water flow. In this context, a noteworthy simplification of the transport problem may be achieved by projecting the transport equation along a single streamline and embedding all the heterogeneities of the transport properties within a single variable, the travel time τ (for appropriate details the reader is referred to the original paper by Cvetkovic & Dagan, 1994). If we assume that linear and reversible sorption processes occur between the mobile and the immobile phases, mass conservation yields:

$$\frac{\partial C(\tau,t)}{\partial t} + \frac{\partial C(\tau,t)}{\partial \tau} = R(\tau,t) = -k_1 C(\tau,t) + k_2 N(\tau,t) \tag{12}$$

and:

$$\frac{\partial N(\tau,t)}{\partial t} = k_1 C(\tau,t) - k_2 N(\tau,t) \tag{13}$$

where C [ML^{-3}] represents the solute concentration in the mobile phase, N [ML^{-3}] is the solute concentration in the immobile phase (mass of solute per unit fluid volume), R [$ML^{-3}T^{-1}$] is the sink/source term due to chemical and/or physical reactions and k_1,k_2 [T^{-1}] are the forward and backward reaction coefficients, respectively. It is worth mentioning that τ is the time needed for a particle injected in $\mathbf{x}_0$ at $t = 0$ (i.e. $\mathbf{X}(0) = \mathbf{x}_0$), with $\mathbf{X}(t) = (X(t),Y(t),Z(t))$ (as usual the trajectory of the particle) to reach a control plane, perpendicular to the mean flow direction, located at a distance x (measured along the mean flow direction) from the injection site (Cvetkovic & Dagan, 1994):

$$\tau(x) = \int_0^x \frac{d\xi}{u(\xi,\eta(\xi),\zeta(\xi))} \tag{14}$$

The quantities η and ζ in equation (14) are the transversal displacements of the considered particle, i.e. $\eta(x) = Y(\tau(x))$ and $\zeta(x) = Z(\tau(x))$ (for a complete treatment, only sketched here, see Cvetkovic & Dagan, 1994, 1996). It should be noted that equation (12) is actually fully three dimensional (3-D), since the Lagrangian variable τ retains the 3-D structure of the velocity field. Furthermore, in (12) we neglect pore-scale dispersion; in heterogeneous formations, in fact, pore-scale dispersion may only affect the local values of resident concentrations but bears a negligible overall effect on global quantities, such as mass fluxes and the spatial/temporal plume moments (Dagan, 1989), particularly in the case of reactive solutes (e.g. see the discussion in Botter *et al.*, 2005).

When considering basin scales, it has been shown that ensemble averaging over different injection points $\mathbf{x}_0$ embedding source areas larger than the scales characteristic of heterogeneous properties (thereby typically for particles injected by rainfall patterns) smooths out the dependence on the features of the single trajectory and that the above framework forced to steady state often gives negligible differences with respect to the full Lagrangian framework, and that in practice one has $N(t,\tau) \approx N(t)$ (Botter *et al.*, 2005). This leads to the simplified formulation provided by equation (11), where the spatial gradients of immobile concentration are neglected. Thus the solute mass flux [MT^{-1}] due to an instantaneous injection of a water flux $J(t) = m_w/\rho)\delta(t - t_0)$ [L^3T^{-1}] may be expressed by the use of equations (9) and (10) as:

$$Q_s(t,t_0) = \frac{m_w}{\rho} C(t - t_0, t_0) f(t - t_0) = J(t_0)\Delta t_0 C(t - t_0, t_0) f(t - t_0) \tag{15}$$

where $J(t_0)\Delta t_0 = m_w/\rho$ is the water volume injected in the system during the time interval Δt_0. Equation (15) states the equality between the mass response function (i.e. the solute release corresponding to a unit water input) and the product between the carrier transfer function f (i.e. the travel time distribution for the water flow) and its solute concentration C. Flow rates [L^3T^{-1}] (constant m_w) and mass fluxes [MT^{-1}] (variable m_s) generated by an arbitrary sequence of rainfall volumes $J(t)$ [L^3T^{-1}] (which we may treat as clean for $\tau = 0$, i.e. $C(0,t) \equiv 0$) are thus derived, for a single transport volume, from equations (4) and (15):

$$Q_w(t) = \int_0^t dt_0 J(t_0) f(t - t_0) \quad [L^3T^{-1}] \tag{16}$$

and:

$$Q_w(t) = \int_0^t dt_0 J(t_0) C(t - t_0, t_0) f(t - t_0) \quad [MT^{-1}] \tag{17}$$

respectively.

In the case of unsteady forcing one may need to distinguish resident concentrations, $C(t - t_0,t_0)$, from flux concentrations, say $C^F(t)$, at the outlet of single transport volumes (thereby only a function of current time t):

$$C^F(t) = \frac{Q_s(t)}{Q_w(t)} \tag{18}$$

$C^F(t)$ being the solute concentration at the outlet resulting from the simultaneous arrival of water particles which have experienced different travel times and have come into contact with different immobile phase concentrations. The distinction between resident and flux concentrations for non-steady advection is indeed well known (e.g. Rinaldo & Marani, 1987). Flux concentrations are needed, in particular, when considering serial transport volumes, as we shall see in the next Section.

3. OF GENERALIZED APPLICATIONS

In general, the determination of travel time distributions must be accomplished following an analysis of the detailed motion of water particles in space and time over a channel network. Indeed a complex catchment entails a nested structure of geomorphic states, quite different from one another, where hydrological transport occurs. Typically one thinks of hillslopes (where solute generation to hydrological runoff mostly occurs) and channel states (where usually routing occurs, though exchanges with hyporheic zones or riparian vegetation or biological decays may be significant, especially if travel times therein become large) (Rinaldo & Rodriguez-Iturbe, 1996). We thus need to define the collection Γ of all individual paths $\gamma\in\Gamma$ that a particle may follow up to the basin outlet. The collection of connected paths $\gamma = x_1,x_2,\ldots,x_\Omega$, (where we define Ω as the closure of the catchment) consists of the set of all feasible routes to the outlet, that is: $x_1 \rightarrow x_2, \rightarrow \ldots \rightarrow x_\Omega$.

A different notation emphasizes the geomorphic framework adopted. If $A_i, I = \overline{1,N}$ is the number of overland states whose total area covers the entire catchment (say, we neglect the actual surface of channelized patterns), and c_i defines any channel link of the catchment (N is the total number of links), all the paths are supposed to originate within hillslopes i.e. $A_\mathrm{i} \rightarrow c_\mathrm{i} \rightarrow c_\Omega$, where Ω is the conventional notation for the outlet of the basin. The above rules specify the spatial distribution of pathways available for hydrological runoff through an arbitrary network of channel and overland regions. The travel time spent by a particle along any one of the above paths is composed by the sum of the residence times within each of the states actually composing the considered path. Nevertheless, the time T_x that a particle spends in state x ($x = A_i$ or $x = c_i$) is a random variable which can be described by probability density functions (pdfs) $f_x(t)$. Obviously, for different states x and y, T_x and T_y can have different pdfs, $f_x(t) \neq f_y(t)$ and we assume that T_x and T_y are statistically independent for $x \neq y$. For a path $\gamma\in\Gamma$ defined by the collection of states $\gamma = \langle x_1,\ldots x_k \rangle$ (where, in turn, $x_1,\ldots x_k \in(A_1,\ldots, A_\Omega, c_1,\ldots, c_\Omega)$ we define a travel time T_γ through the path γ as:

$$T_\gamma = T_{x_1} + \ldots.. + T_{x_k} \tag{19}$$

From the statistical independence of the random variables T_{x_i} it follows that the derived distribution $f_\gamma(t)$ of the sum of the (independent) residence times T_{x_i} is the convolution of the individual pdfs:

$$f_\gamma(t) = f_{x_1} * ..... * f_{x_k} \tag{20}$$

where the asterisk $*$ denotes the convolution operator.

Travel time distributions $f(t)$ at the outlet of a system whose input mass is distributed over the entire domain are obtained by randomization over all possible paths (Rodriguez-Iturbe & Valdes, 1979; Gupta *et al*., 1980):

$$f(t) = \sum_{\gamma \in \Gamma} p(\gamma) f_\gamma(t) \tag{21}$$

where γ is the arbitrary path from source to outlet constituted of states $\langle x_1, ... x_k \rangle$, f_γ, is the path travel time distribution as given by equation 20, $p(\gamma)$ is the path probability, i.e. $\Sigma_{\gamma\in\Gamma}\, p(\gamma) = 1$, defining the relative proportion of particles in γ.

We now define (and generalize) different types of path probabilities. In the simplest case, the path probabilities may be simply defined as $p(\gamma) = A_\gamma/A$, where A_γ is the contributing area draining into the first channel state of any given path γ. In such a case $\Sigma_{\gamma\in\Gamma}\, A_\gamma = A$, where A is the total area drained by the channel network, and the path probability is solely determined by geomorphology. The above time-independent determination of the path probabilities is tantamount to assuming uniform rainfall in space, and this severely constrains the size of the catchment to be modelled, which is related to the basic scale of spatial heterogeneity of rainfall patterns.

Where rainfall patterns, say $j(\mathbf{x},t)$, are distributed in space and time, the path probabilities would be simply dictated by the relative fraction of rainfall, i.e.:

$$p(\gamma,t) = \frac{\int_{A_\gamma} j(\mathbf{x},t)d\mathbf{x}}{\int_A j(\mathbf{x},t)d\mathbf{x}} = \frac{J(\gamma,t)}{J(t)} \tag{22}$$

(where $J(\gamma,t)dt = dt\int_{A_y} j(\mathbf{x},t)d\mathbf{x}$ is the total quantity of rainfall entering the system in $(t - dt, t)$) through the path γ, and $J(t)dt$ the total rainfall injected in the same period over the entire watershed), which allows one to embed any rainfall pattern in space and time routing them through the catchment at each time interval. This capability is central to the innovation contained in our model, and constitutes a new and relevant extension of traditional GIUH approaches.

Whether a pattern in space and time of $j(\mathbf{x},t)$ derives from the characters of rainfall or of runoff production will be seen elsewhere. Notice that we may derive arbitrary rainfall fields either by kriging of point rainfall measurements, or by assuming stochastic patterns derived from theoretical models. Hence one might derive the rainfall-weighted path probabilities in the general case by simple quadratures. A reliable operational procedure consists of isolating through suitable drainage directions on digital terrain maps a spanning set of sub-basins of size considerably smaller than the macroscales of intense rainfall patterns, thereby defining spanning sets of landing areas γ where one can assume locally constant rainfall intensity $J(\gamma,t)$. This procedure is tantamount to a coarse-graining of the original rainfall patterns from the pixel size to that of a collection of thousands of them, with much improved computational efficiency at no cost of predictive loss. Moreover, any spatially distributed model of runoff production would result in distributions of input $j(\mathbf{x},t)$ more markedly heterogeneous in space.

Moreover, whether or not one needs to modify travel times depending on the intensity of the hydrological events (e.g. geomorphoclimatically) depends on the modes of hydrological transport, say when dominated by storage rather than kinematic effects, but the basic formal machinery remains unaffected. Many papers have addressed the characterization of travel times and the related hydrological response. We will not review them here. Suffice here to say that the description of hillslope transport is of great importance (e.g. Rinaldo *et al.*, 1995b; Robinson *et al.*, 1995; Botter & Rinaldo, 2003). In fact, hillslope residence times are responsible not only for key lags (and rather complex mechanisms like preferential pathways to runoff) in the overall routing, but are also important to the understanding of derived transport processes, chiefly solute generation and transport to runoff waters. The above matter, jointly with the physical problem of characterizing well where channels begin, still needs to be resolved satisfactorily.

In the framework previously depicted, flow rates are obtained by propagating spatially distributed, time-dependent net rainfall impulses by the use of linear invariant hydrological responses. The basic formulation of the geomorphic theory of the hydrological response is thus given by the following, commonly adopted convolution integral:

$$Q_w(t) = \int_0^t dt_0 J(t_0) \sum_{\gamma\in\Gamma} p(\gamma, t_0)\, f_\gamma(t-t_0) \tag{23}$$

In the occurrence of spatially uniform, time varying net rainfall intensity $J(t)$ one has:

$$Q_w(t) = \int_0^t dt_0 J(t_0) \sum_{\gamma\in\Gamma} p(\gamma)\, f_\gamma(t-t_0) = \int_0^t dt_0 J(t_0)\, f\ (t-t_0) \tag{24}$$

because $f(t) = \sum_{\gamma\in\Gamma} p(\gamma)\, f_\gamma(t)$, and we recover the usual GIUH relationship which is employed in several practical cases. It should be stressed that the general formulation of equation (23) uses rainfall patterns in space and time both for determining the path probabilities $p(\gamma t)$ and for filtering the net contribution $J(t)$.

The convolution integrals up to equations (23) and (24) may be solved exactly for a number of cases (Rinaldo *et al.*, 1991) where the dynamical parameters determining the propagation of the flood wave are assumed to be uniform. Alternatively, we may allow arbitrary variations in celerity and hydrodynamic dispersion, and thus numerical convolutions are often in order. In such cases, arbitrary travel time distributions may be used depending on the hydraulics and suitable numerical techniques (typically employing integral transforms) are used to accurately convolute in time. A strong control over the numerical machinery is obviously provided by continuity, given that $\int_0^\infty f_\gamma(\tau)d\tau \equiv 1 \forall\gamma$.

We note that the key identification of the paths $\gamma \in \Gamma$ may be done directly from digital terrain maps, hence exploiting our capabilities of extracting useful geomorphic information from them and chiefly the extent of the channelized portion of the basin (see e.g. Rodriguez-Iturbe & Rinaldo, 1997). This is why these approaches constitute the geomorphic flood studies.

From the results of the previous Section, solute mass discharge at the basin scale is given in the following form:

$$Q_w(t) = \int_0^t dt_0 J(t_0) \sum_{\gamma \in \Gamma} p(\gamma, t_0) C_\gamma (t - t_0, t_0) f_\gamma (t - t_0) \tag{25}$$

where C_γ is a "path" resident concentration. In the case of water flow one simply has $C_\gamma = \rho$, the density of water. In this case $Q_s(t)/\rho$ becomes a flow rate, Q_w [L^3T^{-1}], and equations (23) and (24) are straightforward to recover.

The particular formulation of a mass-response function (MRF) approach depends on the number and the arrangement of the reacting states. A (relatively) simple case is that of a path (say $\gamma = x_1 \rightarrow \ldots \rightarrow x_\Omega$, where x_Ω denotes, as usual, the terminal reach of the catchment), where the state x_1 generates solute mass to the mobile phase (hence one has a mobile and immobile concentrations in x_1 denoted by $C_{x_1}(t, \tau), N_{x_1}(t)$, and all other states (from x_2 to x_Ω) route the transported matter without further exchanges. In this case one has in equation (25):

$$C_\gamma(t,0) f_\gamma(t) = f_{x_1}(t,0) * f_{x_2} * \ldots * f_{x_\Omega} \tag{26}$$

In the general case where x_1 is a "generation" state (wherein solutes are transferred from the immobile to the mobile phase) and $x_2, x_3, \ldots, x_\Omega$ are reactive states where the solutes transported by the carrier may be retarded owing to chemical processes occurring with other immobile phases (e.g. bed sediment or dead zones that define chemical, biological or physical reactions), the mass response function may be expressed as:

$$C_\gamma(t,0) f_\gamma(t) = f_{x_1} C_{x_1}(t,0) * f_{x_2} \lambda_{x_2} * \ldots * f_{x_\Omega} \lambda_{x_\Omega} \tag{27}$$

where $\lambda_{x_i}.(i = \overline{2,k})$ represents the gain/loss function within each reactive state forced by a non-null input flux concentration of solute $C_{x_i}^{F,in}(t) \neq 0$:

$$\lambda_{x_i}(t - t_0, t_0) = \frac{C_{x_i}(t - t_0, t_0)}{C_{x_i}^{F,in}(t_0)} \tag{28}$$

Obviously when downstream states route the matter without sorption, $\lambda_{x_i} \equiv 1$. The notation C_{x_i} and λ_{x_i} should not surprise, as we argued that for each state where gain/loss processes occur one needs to carry out a global mass balance to determine the instantaneous fraction of matter stored in immobile phases $N_{x_i}(t)$. We argue that equation (27) is the general form of Mass Response Function (MRF) which, in different forms that reduce to particular cases of (27), has been known for some time (e.g. Rinaldo & Marani, 1987).

On this basis alone one needs to weigh carefully the spatial and temporal scales relevant to a mathematical model of transport at catchment scales. All possible combinations of states generating, losing or simply routing solutes may thus be explored, thus straightforwardly extending the geomorphic theory of the hydrological response to solute transport.

The linkage of travel times with the global basin-scale contact times between phases controlling mass exchanges provides a leap forward in our operational capabilities of describing large-scale transport processes. Indeed a complex catchment entails a nested structure of geomorphic states where the spatial pathways of any rain-driven particle moving through the network of channel and overland regions define the control volumes for which one needs to carry out mass balances and compute travel and lifetime distributions.

We shall discuss a few examples with the scope of clarifying the structure of mass response functions. The examples are kept to a minimum of geomorphic and hydrological complexity to avoid clouding the main issue. Rainfall is assumed constant in space, i.e. $p(\gamma,t) = p(\gamma)$. Figure 2 shows the chosen setup, composed of five source areas and five channels. Overall, the topological order is $\Omega = 2$.

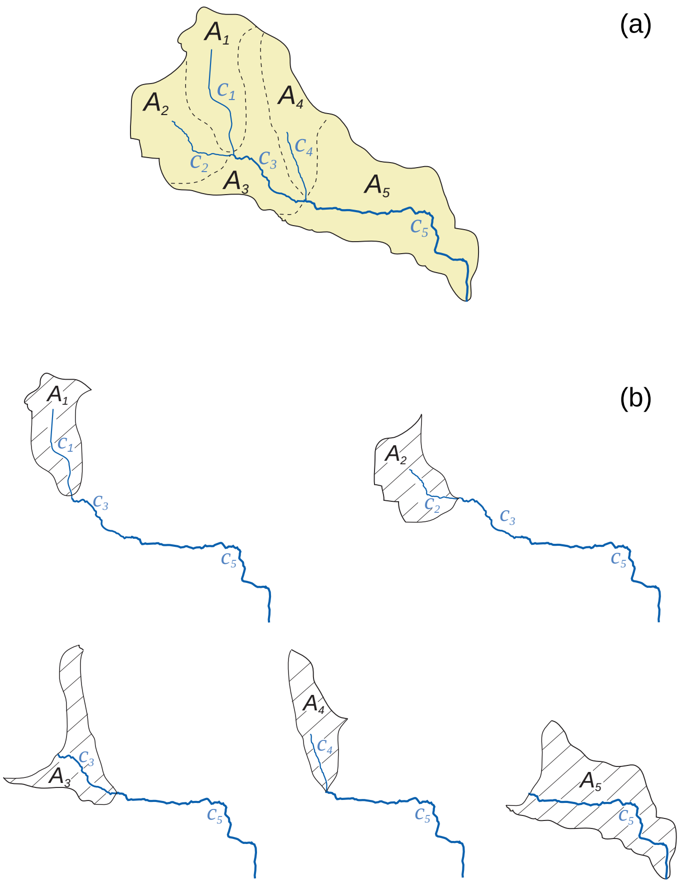


Fig. 2 (a) Sample of a relatively simple geomorphic structure of a river basin and notation for the theoretical model. The basic elements of the geomorphic approach for basin-scale transport are provided. Notice that the set Γ of all possible paths to the outlet defined by the geomorphic structure is made up by 10 states, five overland states and five channels (e.g. transitions to overland areas A_i to their outlet channel c_i and then to ensuing transitions ($c_i \rightarrow c_k \rightarrow \ldots \rightarrow c_5$) towards the closure—the endpoint of channel c_5). Notice the treatment of the *i*-th source area A_i as a well-mixed reactor. Here we assume that all sources areas A_1 to A_5 act as generators of solutes to the mobile phase. (b) The set of independent paths available for hydrological runoff is enumerated and shown (after Rinaldo *et al*., 2006a).

The complete set Γ of paths to the outlet (see Fig. 2) is the following:

$A_1 \to c_1 \to c_3 \to c_5$

$A_2 \to c_2 \to c_3 \to c_5$

$A_3 \to c_3 \to c_5$

$A_4 \to c_4 \to c_5$

$A_5 \to c_5$

The states where paths originate are labelled by an area A_i, so that the total catchment area A obeys the relation $A = A_1 + \dots A_5$ and path probabilities are defined by $p(1) = A_1/A$; … ; $p(5) = A_5/A$, thereby assuming that the rainfall is spatially uniform—this is tantamount to assuming that the watershed "width" is smaller than the correlation scale of rainfall events. Under the circumstances shown in Fig. 2, equation (21) and (26) apply with:

$$f(t) = \frac{A_1}{A} f_{A_1} * f_{c_1} * f_{c_3} * f_{c_5} + \frac{A_2}{A} f_{A_2} * f_{c_2} * f_{c_3} * f_{c_5} + \frac{A_3}{A} f_{A_3} * f_{c_3} * f_{c_5} + \frac{A_4}{A} f_{A_4} * f_{c_4} * f_{c_5} + \frac{A_5}{A} f_{A_5} * f_{c_5} \tag{29}$$

(where we have neglected for the sake of simplicity the probability for a particle to land directly on a channel state).

Figure 3(a) shows the individual and compounded travel time distributions for the path γ_1 defined by the transitions: $A_1 \to c_1 \to c_3 \to c_5$. Also shown (Fig. 3(b)) is a comparison of the path, $f_\gamma(t)$, and the basin, $f(t)$, travel time distributions needed for the general definition of fluxes. The comparison shows the obvious blending of different arrivals that reflect the geomorphic complexity of the pathways to the outlet.

Mass response functions are easily determined when parallel generation states occur. If we assume that every hillslope A_i acts as a generator of solute matter to runoff (a usual assumption in nonpoint source pollution studies), we have, for the water pulse injected at $t_0 = 0$ (i.e. $\tau = 0$):

$$\sum_\gamma p(\gamma) C_\gamma(t,0) f_\gamma(t) = \frac{A_1}{A} f_{A_1} C_{A_1}(t,0) * f_{c_1} * f_{c_3} * f_{c_5} + \frac{A_2}{A} f_{A_2} C_{A_2}(t,0) * f_{c_2} * f_{c_3} * f_{c_5} + \frac{A_3}{A} f_{A_3} C_{A_3}(t,0) * f_{c_3} * f_{c_5} + \frac{A_4}{A} f_{A_4} C_{A_4}(t,0) * f_{c_4} * f_{c_5} + \frac{A_5}{A} f_{A_5} C_{A_5}(t,0) * f_{c_5} \tag{30}$$

which defines the mass-response function for the basin shown in Fig. 2. Note that for a unit pulse of rainfall one has $Q_s(t) = \sum_\gamma p(\gamma) C_\gamma(t,0) f_\gamma(t)$ and the flux concentration is $C^F(t) = Q_s/Q_w$, while for compound inputs of rainfall $J(t)$ one has to solve equation (25).

Sample computations are shown in Fig. 4, where results for an instantaneous unit pulse of effective rainfall $J(t) = \delta(t)$ are reported in *a* and *b*. Figure 4(a) shows the connected behaviour of the resident mobile, C, and immobile, N, concentrations in state A_1 obtained by solving equation (11) with a given initial concentration $N(0)$ and initially zero concentration in mobile phase $C(0,0) = 0$. Note that the particular choice

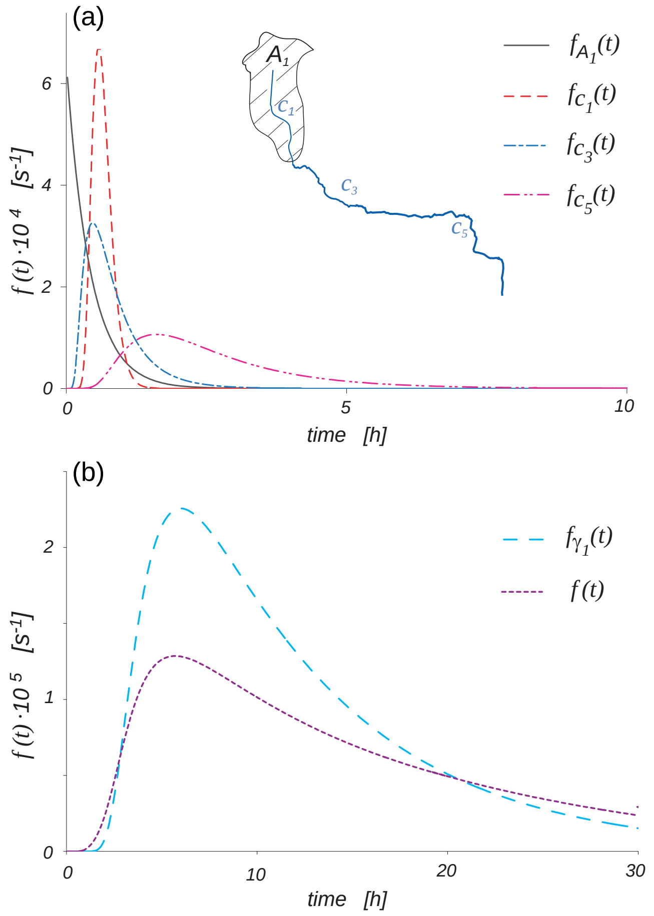


Fig. 3 (a) Example of individual travel time distributions along the path $A_1 \rightarrow \ldots c_5$. (b) Travel time distribution $f_\gamma(t)$ obtained by convolution of the individual pdfs, and catchment travel time distribution $f\,(t)$ (after Rinaldo *et al.*, 2006a).

of numerical value of $N(0)$ (here about 5.5 kg ha^{-1}) is immaterial. The flux concentration at the outlet is obtained by solving five mass balance equations of the type (11) for the five generating states A_i to determine five different path concentrations $C_\gamma(\tau,t_o)$, and then posing $Q_s(t) = \Sigma_\gamma p(\gamma) C_\gamma(t,0) f_\gamma(t)$ and $C^F(t) = Q_s/Q_w$, which is the final result shown in Fig. 4(b). Notice the difference in the timescales with respect to the travel time $f(t)$ shown in Fig. 2 due to chromatographic effects induced by the reaction kinetics. Figure 4(c), instead, describes a case where a sequence of rainfall inputs $J(t)$ (shown in the upper plot) drives a complex chain of events, thus requiring more complex computations. In the lower plot of Fig. 4(c) we show the behaviour of $N(t)$ in one of the generating states, evidencing the effect of solute leaching due to the sequence of rainfall impulses. One may also notice the reduced rates of solute generation to runoff for the late-coming pulses (most of the mass had been leached previously), which reflect the lack of translational invariance postulated by the dependence of resident concentrations onto two different timescales, i.e. $C = C(\tau,t_0)$. The plot reported in Fig. 4(d) has been obtained by solving equation (25) with the sequence of $J(t)$ reported in Fig. 4(c), in the case of parallel generation and transport of solutes.

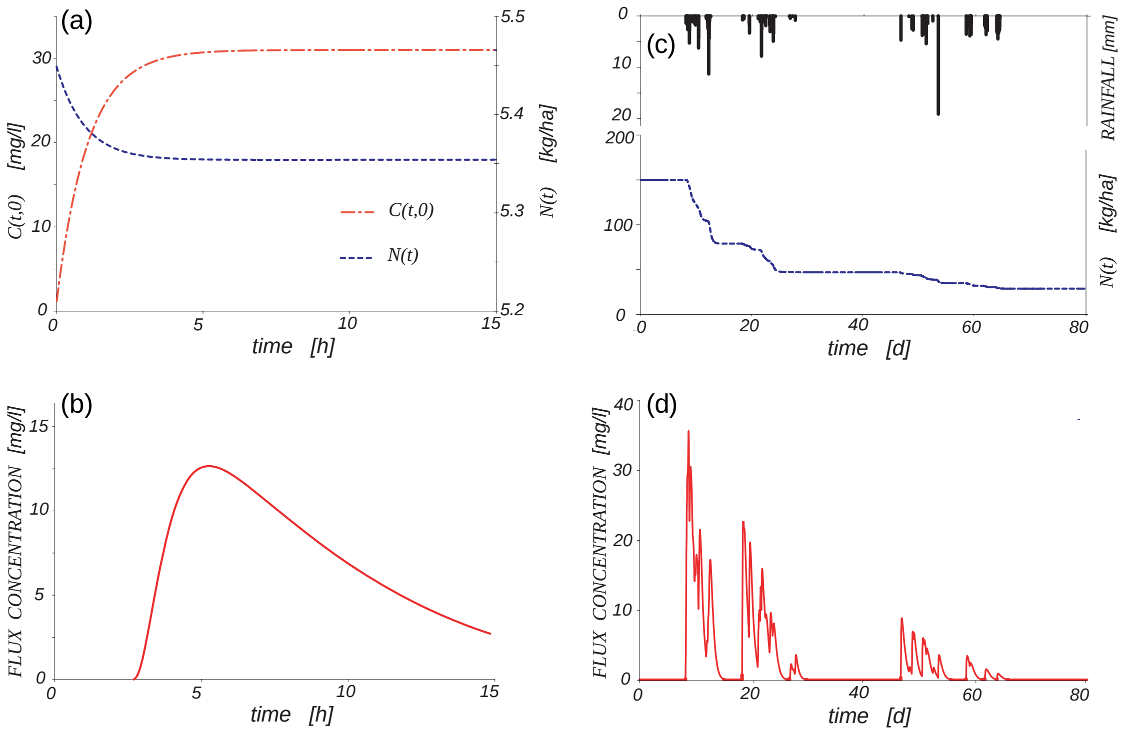


Fig. 4 (a) Resident concentration $C_{A_1}(t,0)$ and its corresponding immobile phase concentration $N_{A_1}(t)$ (expressed in kg ha^{-1}) *vs* t for an instantaneous pulse; (b) flux concentration at the outlet of the basin $C^F_{c_5}(t)$; (c) temporal evolution of the rainfall depths (upper plot) and corresponding immobile phase concentration $N_{A_1}(t)$ for a sequence of intermittent rainfall pulses, a case typical of transport in the hydrological runoff. Also shown, in (d), is the corresponding flux concentration at the outlet of the catchment, $C^F_{c_5}(t)$ (after Rinaldo *et al*., 2006a).

A second example, involving serial transport, is more complex. If we assume that mass loss/gain processes are significant in serial states (two hillslopes and a stream channel, see Fig. 5), one may specifically assume that:

(i) the overland states A_1 and A_4 are generation states, e.g. agricultural areas where fertilization occurs;

(ii) the stream channel c_5 is a relatively vegetated, high-residence time channel reach where reaction processes matter. In this case the travel time distribution is the same as for the case above, whereas the MRF for the water pulse injected at $t_0 = 0$ ($\tau = t$):

$$\sum_{\gamma} p(\gamma) C_\gamma(t,0) f_\gamma(t) = \frac{A_1}{A} f_{A_1} C_{A_1}(t,0) * f_{c_1} * f_{c_3} * f_{c_5} \frac{C_{c_5}}{C^{F,in}_{c_5}} + \frac{A_4}{A} f_{A_4} C_{A_4}(t,0) * f_{c_4} * f_{c_5} \frac{C_{c_5}}{C^{F,in}_{c_5}} \tag{31}$$

where the resident concentrations in states that follow generation (the channel 5) are properly normalized by the inflowing flux concentrations. Note that only the contributions of "source" states explicitly appear in the MRF, whereas large dilutions

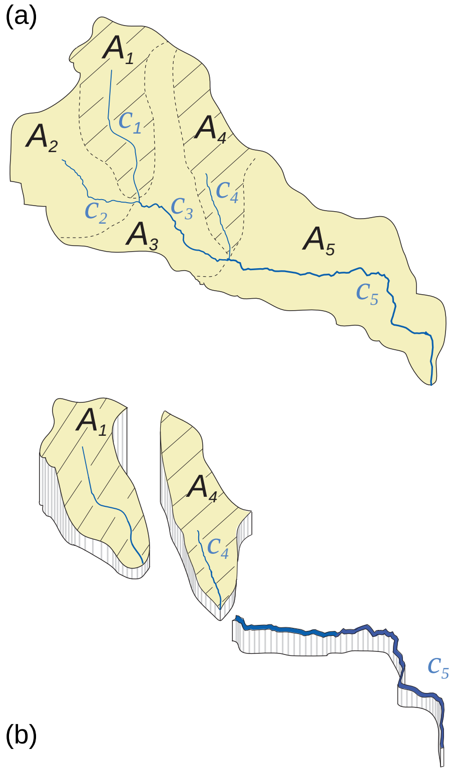


Fig. 5 (a) Geomorphic structure of the test catchment: here we assume that only sources areas A_1 and A_4 act as generators of solutes to the mobile phase. Moreover, we assume that mass transfer processes also occur in state c_5 owing to its travel times and nature. Reactive states (A_1, A_4 and c_5) are isolated in (b) (after Rinaldo *et al*., 2006a).

determined by all the states generating clean runoff are reflected by lower flux concentration along the stream network. Needless to say, the serial arrangement is considerably more involved computationally.

Every possible combination is thus tackled. Figure 6 shows an example of compounded application of the model above to the case of nitrate transport in the hydrological response of the basin shown in Fig. 5 (Rinaldo *et al.*, 2006b). Although by necessity several details are missing (they are reported elsewhere, see caption to Fig. 6), I chose to show it because the capability of the model to reproduce complex hydrological events and the associated transport features seems notable. Thus theoretical consistency and practical applicability are claimed to characterize the geomorphic approach.

4. TRANSPORT ON FRACTAL NETWORKS

Transport on fractal networks is a relatively old enterprise. In the context of flood studies, perhaps relevant is the exact nature of certain solutions to a Wiener process in the framework of Section 2 (Rinaldo *et al.*, 1991). Of particular interest therein is the derivation of the concept of geomorphic dispersion, i.e. the rate of the variance of the arrival time distribution that can be directly attributed to the heterogeneity of the flow paths from any source to the outlet—properly, the geomorphic contribution. Also interesting is the proof (Rinaldo *et al.*, 1991) that the contribution of other processes

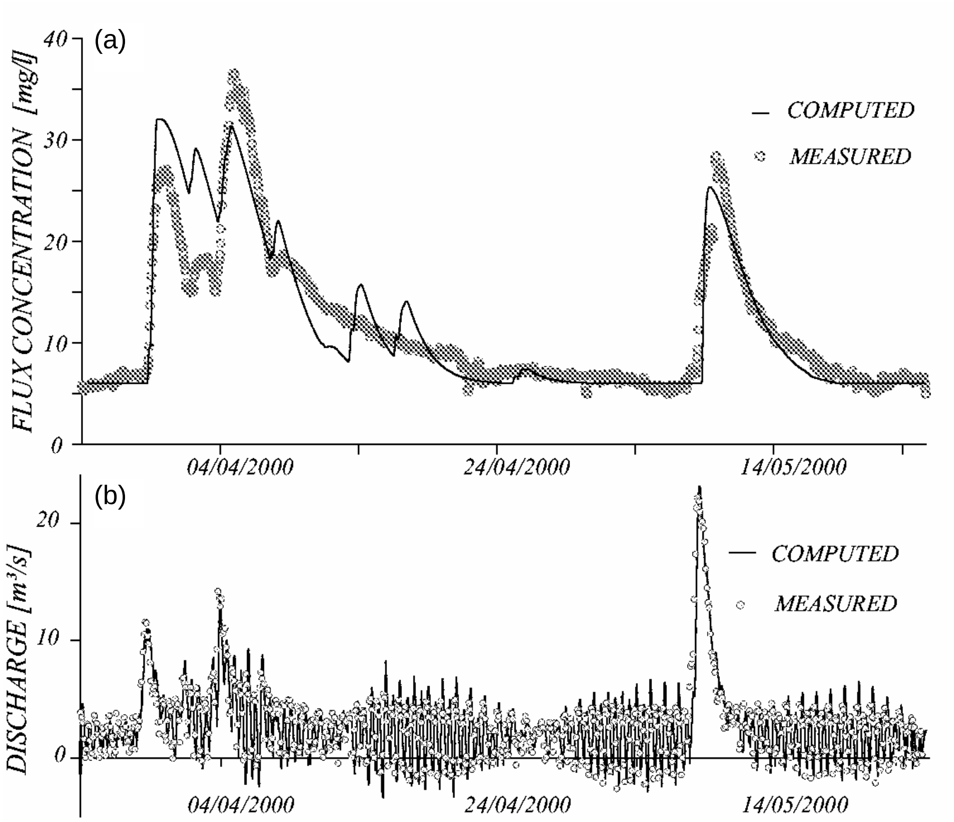


Fig. 6 Sample of extended computations for the basin shown in Fig. 5. While the necessary details are described elsewhere (Rinaldo *et al.*, 2005, 2006a)—indeed an involved technological machinery is needed whose description is clearly outside the scope of this paper—we must note that the observational data were collected by a downstream receiving stream where fluctuations (even reversal) in the flows were induced by tidal forcings at the outlet. The tools described here thus produce the flux boundary condition for flow and transport to a numerical code carrying out a mass and momentum balance, thus partitioning hydrological and hydrodynamic modelling domains. Indeed in the case shown here the partition was also physical owing to a small chute at the end of channel c_5 where critical depths are steadily maintained. Nevertheless, it should be noted that quite complex and distinct patterns of fluctuations for flow and transport can be described by the tools introduced in Sections 2 and 3 (after Botter *et al.*, 2005).

(like, e.g. the cumulative effect of hydrodynamic dispersion within individual channel links, or the overall effects of reaction processes) to the variance of the arrivals is additive. Moving from that result, it was observed that geomorphic dispersion tends to dominate the hydrological response.

Later this powerful result was smoothed by the observation that, quite unexpectedly, hillslope processes tend to produce signatures on the travel time distribution, and thus arguably the hydrological response, that last more than expected (Rinaldo *et al.*, 1995a,b; Robinson *et al.*, 1995). Indeed one would think that as the mean travel time through states external to the channelized portion of the catchment becomes much smaller than the mean overall travel time, hillslope effects would tend to become progressively negligible. This is not true for moments higher than the first for a wide range of sizes and geomorphologies. Typically, studies on transport through

Scheidegger, Peano and random networks have addressed the above issues exactly (Marani *et al.*, 1991; Takayasu *et al.*, 1991; Flammini & Colaiori, 1996; Colaiori *et al.*, 1997). Let me briefly mention, for example, that the exact mapping of the width function of Peano's curve into a Besicovitch binomial multiplicative process (Marani *et al.*, 1991) sheds light on a variety of fractal and multifractal results.

More interesting for the scope of this Kovacs Lecture is perhaps the recent interest in transport through fractal networks for modelling migration fronts, with a specific application to colonization of the United States during the 19th century (Campos *et al.*, 2006). The main tenet there is that landscape heterogeneities should have strongly affected such process, as the need for water forced the colonizers to follow the routes provided by river networks to settle their cities. Indeed it is well known that the US transition westwards was not characterized by the regularly patterned pathways predicted by classic homogeneous models, but rather tailored to the structure of the great river basins. The model employed in the study (Campos *et al.*, 2006) was a reaction-diffusion model needed to produce the creation of a front whose speed of propagation is a concept making sense. Thus in this model population fronts spread onto new regions and population density saturates behind the front.

The parameters here reduce to two: the speed of the front, which has been "measured" through historic demographic records, and the parameter of the logistic growth that dictates how rapidly population saturates behind the front. Figure 7 shows that almost any model of network geometry and topology does a job much better than

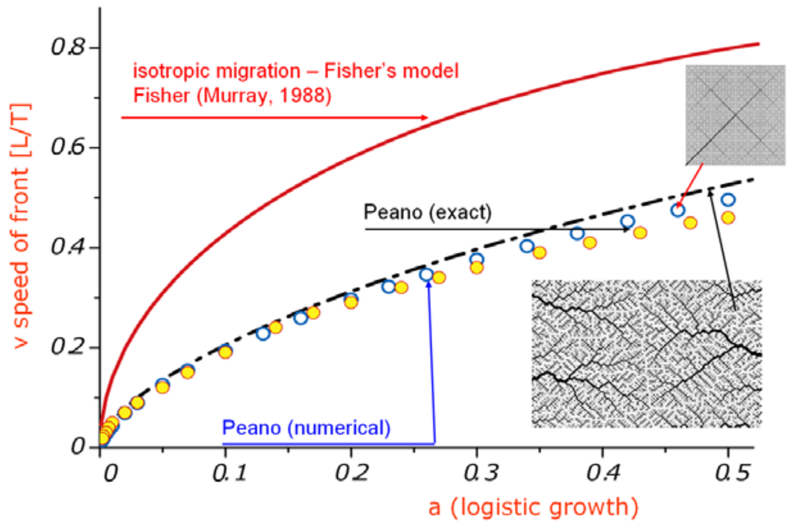


Fig. 7 A plot of computational results (redrawn from Campos *et al.*, 2006) on the speed of propagation of a front developed by a general reaction–diffusion process applied to the migration of population in the 19th century United States. To produce an analogue, Campos *et al.* (2006) have assumed that jump distances covered by settlers were derived from archive and demographic distributions of the distances covered by colonizers from their birthplace to the place where they had landed 25 years after. Distances were taken only in the E–W direction. The fraction of people that after 25 years still remained in their birthplace was likewise estimated and used. Here we show the observational plot of speed of front propagation *vs* the growth parameter of the model. The continuous line is the isotropic, classic Fisher's model; the dotted line represents observational points; circles represent the solution computed for Peano's and optimal networks (Fig. 1) of different order.

any classic homogeneous model, thus supporting the claim of the authors. So, for instance, details of the generation of the Peano basin do not really matter, and optimal networks explain the observed phenomena better than anything else. In any case, geometric constraints imposed by the network involve strong corrections over the speed of the fronts, and the classical isotropic propagation clearly overestimates the observed speed of the migrating fronts. Thus geomorphic constraints must have played a decisive role in regulating human migrations—heterogeneities substantially reduce propagation rates.

My guess is that we shall soon see more on this, in particular for the study of hydrochory, i.e. the spreading of species along corridors.

5. CONCLUSIONS

In this paper a review has been attempted of outstanding issues relevant to geomorphic flood research, including:

(1) a unified formulation of transport by travel time distributions that encompasses both flow and transport while using fully geometric and topological information derived from geomorphology;

(2) an overview of possible schemes for application of such a general scheme, typically distinguishing geomorphic transport states arranged in series and/or in parallel;

(3) a survey of issues concerning transport through fractal networks where I believe action will be seen, in the near future, in a few fields ranging from population biology to ecology.

Acknowledgements The writer gratefully acknowledges support from RIMOF-2 (Fondazione CaRiVrViBlAn), and from the AQUATERRA Project no. EU-555-774-05.

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