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Comparison of numerical and experimental study of dam-break induced mudflow

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Abstract The purpose of this preliminary study is to model mudflow movement in an idealized dam-break configuration. One-dimensional motion of a shallow mud layer over a rigid inclined bed is considered. The resulting shallow water equations are solved by finite volumes using the HLL scheme. A Bingham fluid model is chosen to describe the mudflow rheology. The simulations are validated by comparison with flume experiments. Unsteady mudflow movement is found to be reasonably well captured by the model. In addition, a decoupled algorithm is also employed in the present paper to compute the aggradation and degradation of bed-level elevation for Newtonian fluid by using the Manning-Strickler formula and Exner's relationship.

Key words dam-break flow; finite volume method; HLL scheme; mudflow

INTRODUCTION

The Chichi Earthquake on 21 September 1999 caused more than a dozen landslide dams to form across Taiwan streams, temporarily impounding large volumes of water (Chen, 1999). Once formed, these natural dams were highly exposed to catastrophic failure. Partial or complete failure can lead to severe flooding downstream, and possibly trigger further debris slides or mudflows. To understand the dam formation process and evaluate the potential consequences of subsequent failure, it is important to be able to model the dynamics of mudflows and their interactions with water currents. In this paper we attempt to develop a numerical model for simulating dam-break flows, mudflows, and aggradation and degradation of channel by using a finite volume method with the Godunov-type scheme.

NUMERICAL SCHEME

Governing equations

The Saint Venant equations are used to describe unsteady one-dimensional (1-D) open-channel flow. Continuity and momentum balance are respectively written (Abbott, 1979; Chaudhry, 1993):

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{h}+\frac{1}{2}gh^2\right)=gh\left(S_0-S_f\right), \tag{2}$$

where h is the depth of flow or mudflow above the rigid bed; $q = hu$ is the unit width discharge; u is the mean velocity in the longitudinal flow direction. Letting z_b denote the bed elevation above a reference datum, the slope S_0 can be written:

$$\frac{\partial z_b}{\partial x}=-S_0 \tag{3}$$

The friction slope S_f can be also expressed with a relationship established for uniform flow, by using the Manning-Strickler formula as follows:

$$S_f=\frac{q^2n^2}{h^{10/3}} \tag{4}$$

where n is the Manning coefficient, recalling that for a very wide channel the hydraulic radius is equal to the flow depth.

Equation (4) can only be used in the flow of clear water or a Newtonian fluid, i.e. the sediment concentration is low. However, at high sediment concentrations the mixture behaves as a non-Newtonian fluid such as a hyperconcentrated flow, mud flow or debris flow. The rheology of hyperconcentrated flows, mud and debris flows is frequently described using the Bingham model. Concerning the bottom friction of a Bingham fluid, Pastor *et al*. (2004) presented the shear stress τ_b at the bottom as related to the modulus of the depth averaged velocity u by:

$$u=\frac{\tau_b}{6\mu}h\left(1-\frac{\tau_y}{\tau_b}\right)^2\left(2+\frac{\tau_y}{\tau_b}\right) \tag{5}$$

where τ_y is the yield stress, μ is the fluid viscosity, and τ_y/τ_b should lie in the interval [0, 1]. Therefore, we can utilize equation (5) to compute the friction slope S_f of mud flow or debris flow if its rheology can be regarded as a Bingham fluid by $\tau_b = \rho_c ghS_f$, in which ρ_c is the density of mud.

To deal with regions where the mud layer is at rest, a condition for incipient motion must be provided. Following previous studies (Mei & Yuhi, 2001; Huang & Garcia, 1997), we assume that motion occurs wherever the gravitational pull and pressure gradient are sufficient to overcome the yield stress:

$$\rho_c gh\left(\sin\theta-\frac{\partial h}{\partial x}\cos\theta\right)>\tau_y \tag{6}$$

and where θ is the inclined angle of slope.

Hyperbolic term

In this study, the above shallow water equations are solved numerically using a finite volume approach, well suited for transient problems such as dam-break flows. We use an operator-splitting approach (see e.g. Toro, 1999) to separately treat the hyperbolic and source components. For both these operators, we adapt the procedure outlined by Harten *et al*. (1983) and Fraccarollo *et al*. (2003) for the computation of geomorphic dam-break waves. In the hyperbolic operator, we used the HLL scheme to deal with the partial differential equations, and an implicit backward Euler scheme to treat the source term.

HLL scheme

The HLL scheme of hyperbolic operator adopted in the present work is an extension of the HLL scheme proposed by Harten *et al*. (1983), widely used for shallow flows. Whereas the original HLL scheme applies to equations in full conservation form, the momentum equation feature non-conservative product associated with pressure along sloping bed. This term is treated following the approach proposed by Fraccarollo *et al*. (2003). Hence, we use the LHLL scheme (Fraccarollo *et al*., 2003) to discretize the momentum equation, which associates with the non-conservative product $gh\frac{\partial z_b}{\partial x}$. The source term associated with friction along the bed is treated further in the next section.

Source term

Consider now the source term operator. Using the Manning-Strickler formula to specify the friction slope S_f for the computation of clear water, the equation for the momentum source term can be written:

$$\frac{\partial q}{\partial t} = -gh\frac{q^2 n^2}{h^{10/3}} \tag{7}$$

Using an implicit backward Euler scheme, equation (7) is discretized as:

$$q_i^{t+\Delta t} = q_i^{t+\frac{1}{2}\Delta t} - \Delta t\left(gh^{-\frac{7}{3}} n^2 \left(q_i^{t+\Delta t}\right)^2\right) \tag{8}$$

Using the first component of the source operator, we have $\frac{\partial h}{\partial t} = 0$ hence $h_i^{t+\Delta t} = h_i^t$. Thus, one can solve equation (8) for the unit width discharge of clear water at the next time step.

Similarly, using the Bingham model to specify the friction slope of mud flow or debris flow by $S_f = \frac{\tau_b}{\rho_c gh}$, the equation for the momentum source term can be written as:

$$\frac{\partial q}{\partial t} = -\frac{\tau_b}{\rho_c}. \tag{9}$$

Since $h_i^{t+\Delta t} = h_i^t$, it follows that $q_i^{t+\Delta t} = h_i^t u_i^{t+\Delta t}$, and using equation (5) to substitute for the velocity at the next time step in the discretizing expression of equation (9) by an implicit backward Euler scheme, we derive:

$$\frac{h^2\tau_b}{6\mu}\left(1-\frac{\tau_y}{\tau_b}\right)^2\left(2+\frac{\tau_y}{\tau_b}\right)-q=-\frac{\Delta t}{\rho_c}\tau_b\,. \tag{10}$$

To obtain the solution of equation (10) directly is very difficult, so our strategy is to deal with it by using the Newton-Raphson method twice. In the first step, we can solve the shear stress τ_b from equation (5) using the results of a hyperbolic operator as the initial condition. Then the solution τ_b can be employed as the initial condition to solve equation (10) by the Newton-Raphson method for the second iteration.

To advance the solution at each time step, the hyperbolic operator is first applied to obtain a partial update. These results are then used as initial conditions for the source operator, yielding the complete update. Since the hyperbolic update is explicit, stability of the scheme is subject to the CFL condition on the time step:

$$\Delta t = \mathrm{CFL}\frac{\Delta x}{\max(|u|+c)} \tag{11}$$

where c is the wave celerity and u the velocity at any given grid point, and CFL is the Courant number. The value CFL should be smaller than 1 and it is found to be satisfactory in our case.

Aggradation and degradation

A decoupled algorithm has been used in the present paper to compute the aggradation and degradation of bed-level elevation for Newtonian fluid. The calculations start at time t = 0, when the bed-level elevations are known. The hydraulic parameters are calculated by solving the Saint Venant equations for water flow without considering sediment transport. Then the bed-load transport rate is calculated for all the elements. The balance of the sediments entering and leaving is subsequently calculated for all elements to find the volume of deposition or erosion. These volumes are then translated into bed-level modifications at the interfaces of elements by Exner's relationship:

$$(1-p)\frac{\partial z_b}{\partial t}+\frac{\partial q_s}{\partial x}=0 \tag{12}$$

where p is porosity of the bed layer, and q_s is volume sediment transport rate per unit width. This concludes the computational cycle for the time t = 0. The time is then advanced by Δt, and a new calculation concerning Saint Vennant equations is carried out with the new bed profile (Graf, 1998).

In this numerical model, the sediment transport rate is calculated by the bed-load transport formulae documented in Graf (1998), and an empirical power function of the flow velocity in which the empirical constants are specified by the user. To avoid too large a variation of bed level, the program compares the maximum relative variation $\Delta z_b/h$ with the maximum tolerated value as specified by the user, and prevents the relative variation from exceeding the tolerated value at the end of each time step. Moreover, it is often preferable to calculate Δz_b from equation (12) and split Δz_b as:

$$\Delta z_b = \alpha\left(\Delta z_b\right)_{i-1/2} + \left(1-\alpha\right)\left(\Delta z_b\right)_{i+1/2} \tag{13}$$

with $0 < \alpha < 1$ as a weighting factor between backward and forward differences (Julien, 2002).

RESULTS AND DISCUSSION

Dam-break experiment in prismatic channel

To test the numerical model for clear water, we compared the simulation with the laboratory experiments of the US Army Engineer Waterways Experiment Station (USACE, 1960) in Fig. 1. The experiments were conducted in a rectangular channel 122 m long and 1.22 m wide, with a bottom slope of 0.005 and Manning's coefficient of 0.009. The dam was placed at the middle of the channel, giving the initial water depth upstream of the dam H_1 = 0.305 m and downstream of the dam H_2 = 0.0 m. In this simulation, the uniform grid spacing Δx is 1.0 m and the Courant number CFL = 0.9. The agreement between the prediction and the experimental results is satisfactory. The results also indicate that the finite volume method can capture the surges well and the

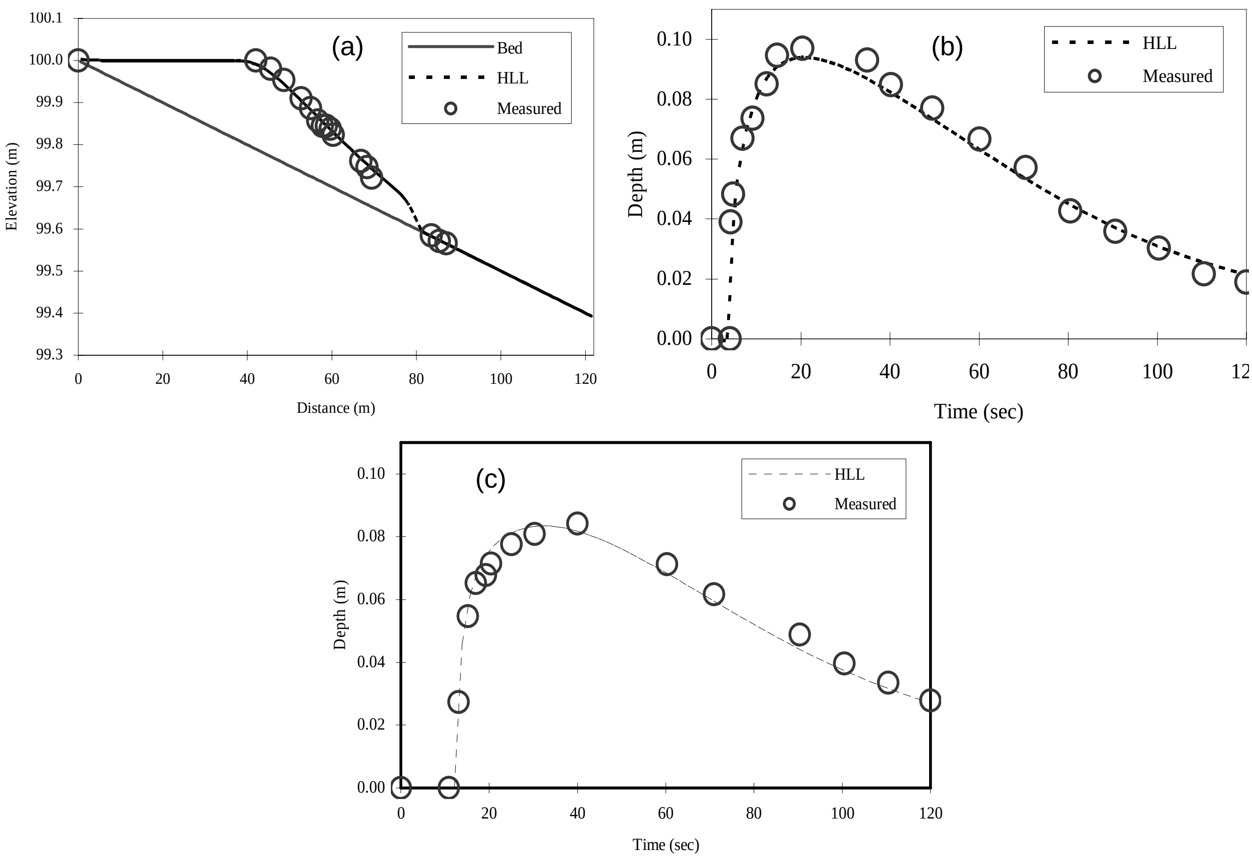

Fig. 1 Comparison with WES experiment: (a) water surface profile along the channel at t = 10 sec; (b) time evolution of water depth at x = 70.1 m; (c) time evolution of water depth at x = 85.4 m. (Source of measured data: USACE, 1960)

numerical model has a good ability to deal with the “contact points” at locations where the depth reaches zero depth.

Dam-break experiment for hyper-concentrated flows

Using this numerical model to compute the dam-break surge for hyper-concentrated flows, the comparison between the prediction and experimental results is shown in Fig. 2. Komatina & Dordevic (2004) performed a series of dam-break flow experiments in a 4.5 m long, 0.15 m wide glass-walled laboratory flume. The dam-break type of flow

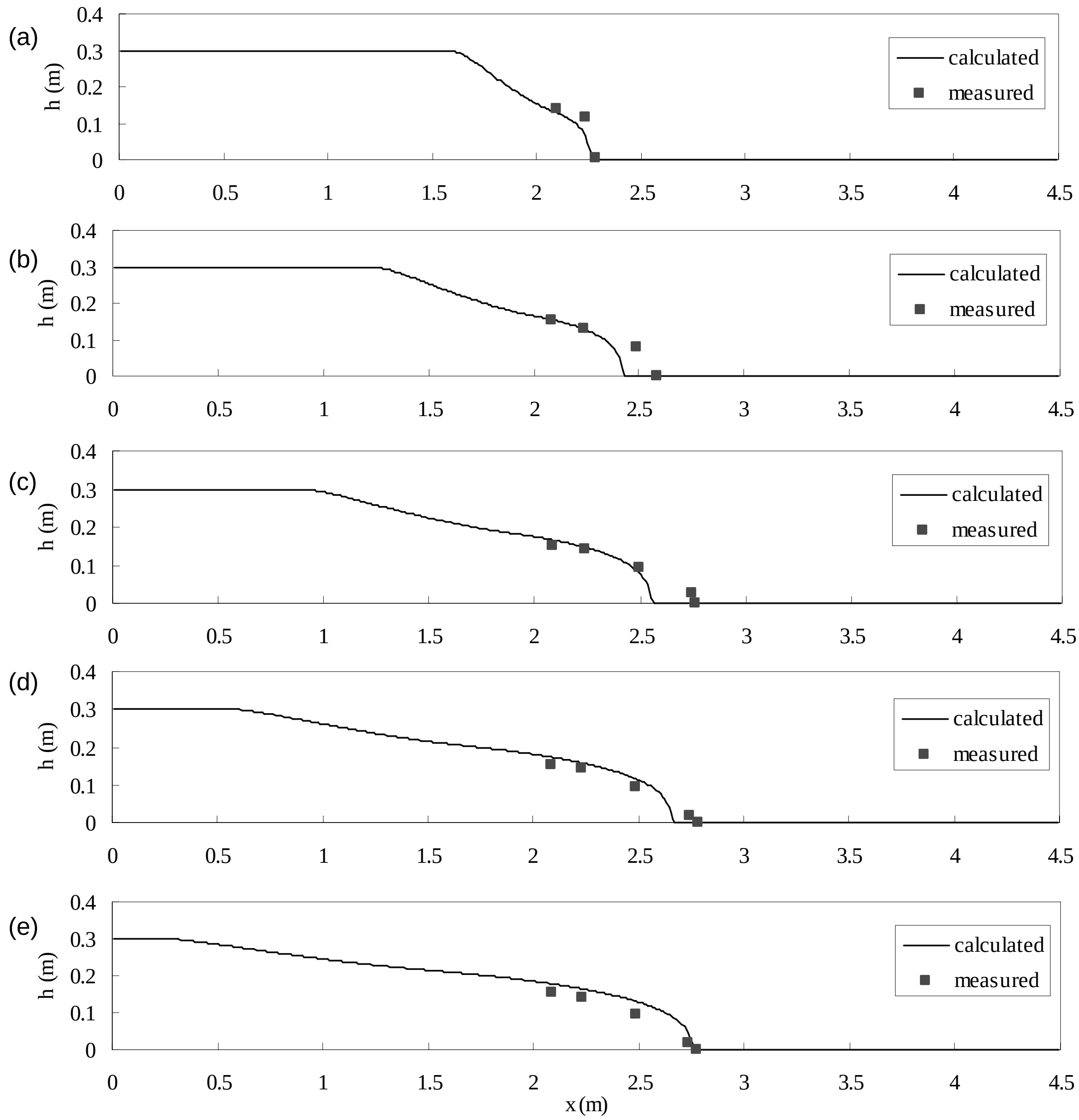

Fig. 2 Comparison of calculated and measured hyperconcentrated flow depth profiles: (a) t = 0.2 sec; (a) t = 0.4 sec; (a) t = 0.6 sec; (a) t = 0.8 sec; (a) t = 1 sec. (Source of measured data: Komatina & Dordevic, 2004)

was initiated by releasing mixtures from a 2 m long, 0.155 m wide reservoir. The rheological properties of mixtures are regarded as a Bingham fluid in the present paper and their parameters are shown as follows: density ρ_c = 1601 kg m^{-3}, yield stress τ_y = 102 Pa, and fluid viscosity μ = 32 Pa s. The results imply that the present numerical model not only computes dam-break flows for clear water but also simulates mudflows.

Knickpoint migration

A knickpoint is defined as an abrupt change in the longitudinal bottom profile of a channel (Chaudhry, 1993). The experimental data obtained from the paper proposed by Bhallamudi & Chaudhry (1991) was compared with the computed results (Fig. 3). The experimental channel is a 15.8 m long and 1.2 m wide flume and the slope of the channel was approximately equal to 0.00125. A fall of approximately 0.0305 m was provided at a distance of 10.8 m from the upstream end to simulate the knickpoint. The computational region was divided into 52 reaches ($\Delta x = 0.3048$ m). The initial and boundary conditions were included as q_0 = 0.0028 m^2 s^{-1} and h_0 = 0.0305 m. In addition, the median size of sand is assumed 0.67 mm and the Meyer-Peter bedload formula is chosen to compute the sediment discharge. The present model successfully simulated the knickpoint by combining the Saint Venant equations with Exner's relationship.

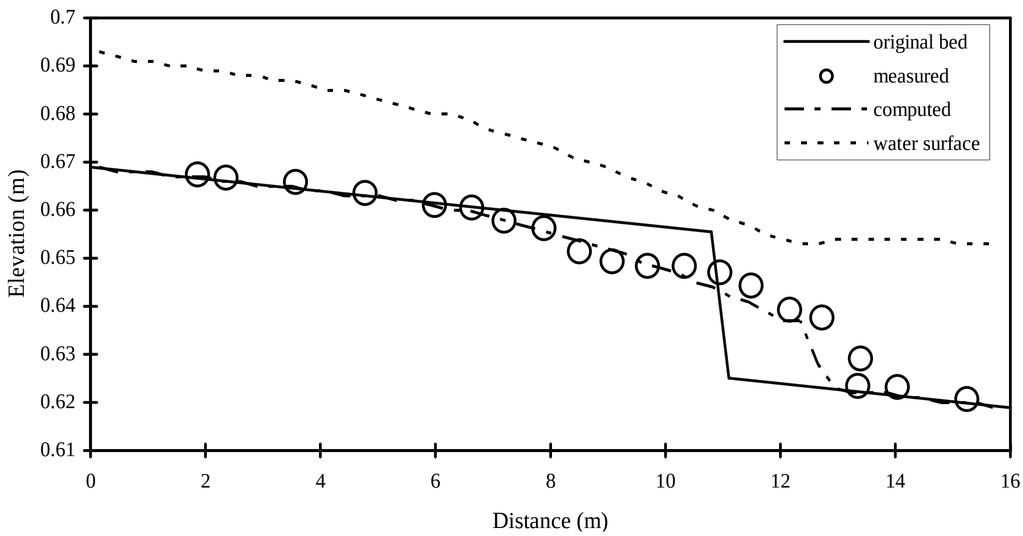

Fig. 3 Bed level variation due to knickpoint migration. (Source of measured data: Bhallamudi & Chaudhry, 1991).

CONCLUSIONS

This paper proposes a one-dimensional numerical model with the finite volume method based on the shallow water equations. A key feature of the model is the use of an operator-splitting method to divide the governing equations into hyperbolic and source terms. This approach provides an easy method for modelling Newtonian and non-Newtonian fluids, respectively. In addition, a decoupled algorithm is also employed to compute the aggradation and degradation of bed-level elevation for a Newtonian fluid by using the Manning-Strickler formula and Exner's relationship. The comparisons between the predictions and experimental results are all satisfactory.

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REFERENCES

Abbott, M. B. (1979) *Computational Hydraulics, Elements of the Theory of Free Surface Flows*. Pitman, London, UK.

Bhallamudi, S. M. & Chaudhry, M. H. (1991) Numerical modeling of aggradation and degradation in alluvial channels. *J. Hydraul. Engng* **117**(9), 1145–1164.

Chaudhry, M. H. (1993) *Open-Channel Flow*. Prentice-Hall, Englewood Cliffs, New Jersey, USA.

Chen, S. C. (1999) Failure mechanism and disaster mitigation on landslide-dammed lakes. *J. Chin. Soil Water Conserv.* **30**(4), 299–311 (in Chinese).

Fraccarollo, L. & Toro, E. F. (1995) Experimental and numerical assessment of the shallow water model for two-dimensional dam-break type problems. *J. Hydraul. Res.* **33**(6), 843–864.

Fraccarollo, L., Capart, H. & Zech, Y. (2003) A Godunov method for the computation of erosional shallow water transients. *Int. J. Numer. Meth. Fluids* **41**, 951–976.

Graf, W. H. (1998) *Fluvial Hydraulics*. John Wiley & Sons, Chichester, USA.

Harten, A., Lax, P. D. & van Leer, B. (1983) On upstream difference and Godunov-type schemes for hyperbolic conservation laws. *SIAM Review* **25**, 35–61.

Huang, X. & Garcia, M. H. (1997) A perturbation solution for Bingham-Plastic mudflows. *J. Hydraul. Engng* **123**(11), 986–994.

Julien, P. Y. (2002) *River Mechanics*. Cambridge University Press, Cambridge, UK.

Komatina, D. & Dordevic, D. (2004) Numerical simulation of hyper-concentrated flows. In: *Proc. Second Int. Conf. on Fluvial Hydraulics* (ed. by M. Greco, A. Carravetta & R. Della Morte), 1111–1120. Balkema, London, UK.

Mei, C. C. & Yuhi, M. (2001) Slow flow of a Bingham fluid in a shallow channel of finite width. *J. Fluid Mech.* **431**, 135–159.

Pastor, M., Quecedo, M., González, E., Herreros, M. I., Fernádez Merodo, J. A. & Mira, P. (2004) Simple approximation to bottom friction for Bingham fluid depth integrated models. *J. Hydraul. Engng* **130**(2), 149–155.

Toro, E. F. (1999) *Riemann Solvers and Numerical Methods for Fluid Dynamics: A Practical Introduction*. Springer, Berlin, Germany.

US Army Corps of Engineers (USACE) (1960) Flood resulting from suddenly breached dams. *Miscellaneous Paper, 2(374), Rep. 1*. US Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, USA.