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Ungauged catchments: how to make the most of a few streamflow measurements?

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Abstract This paper deals with an alternative to the classical regionalization approach used to address the problem of ungauged basins with rainfall–runoff models. We chose a new, more practical orientation, which consists of making the most of a few discharge measurements. Indeed, it is not unrealistic to obtain such measurements by sending a gauging team to the spot where a hydrological prediction is needed. In this paper we aim to identify the parameters of a daily lumped rainfall–runoff model, GR4J, and we look for a strategy to calibrate its parameters using a few streamflow measurements, which are combined with *a priori* knowledge of the parameters. Results show this approach to be much more efficient than classical regionalization studies, as soon as about thirty measurements can be made, at random, during a period of three to five years.

Key words point measurements; rainfall–runoff modelling; regionalization; ungauged basin

INTRODUCTION

All problems posed to the hydrologist require the representation of the natural system by a model. Without a model, nothing can be said, computed or predicted. The type of model mostly needed by hydrologists is of the rainfall–runoff type. The most versatile type of rainfall–runoff model is the lumped model. Such a model has to be calibrated using rainfall–runoff data to obtain its parameters. When no data are available, i.e. when the studied watershed is ungauged, hydrologists are at pains to identify appropriate parameter values. The conventional approach consists in generalizing for a whole region regression relationships established for many gauged catchments monitored in that region. However, several studies have shown that regressions are not really effective when addressing the problem at the scale of a country (Lee *et al.*, 2005). Merz & Blöschl (2004) and Parajka *et al.* (2005) developed alternatives based on spatial proximity.

Working with a very parsimonious model (just four free parameters) we found that we were unable to understand how basic characteristics, such as the catchment area or the aridity index (ratio of the mean annual evapotranspiration to the mean annual precipitation) could influence the values taken by the model parameters. Thus, we chose a new avenue of research which aims at making the most of a few discharge measurements. We assume that we are looking for the parameters of a daily lumped rainfall–runoff model and that one day of measurements allows one to know the daily flow on that day.

The paper is organized as follows: (a) the GR4J model to be applied on an ungauged catchment is presented; (b) the data that have been used in this research to gain some statistical knowledge of GR4J parameters and to test the methods developed for ungauged catchments are described; (c) the method suitable when very few measurements are available is presented; and (d) the results and a conclusion are given.

THE GR4J MODEL

The GR4J model, which we will use throughout this paper, is described in detail by Edijatno *et al.* (1999) and Perrin *et al.* (2003) and we refer readers to the latter article for a complete description of the model. GR4J has four parameters to be calibrated. It is composed of a soil-moisture accounting procedure (parameter X_1 in mm), a transfer function combining a routing reservoir (parameter X_2 in mm) and a unit hydrograph (parameter X_3 in days), and a gain-loss function (parameter X_4 in mm). In order to make the calibration more efficient, it is carried out with transformed values of these parameters such that transformed parameters, noted x_i, belong to the interval]–10,10[.

The transformations applied to the four parameters of the GR4J model during the calibration process are:

$$x_1 = \log(X_1)$$

$$x_2 = \log(X_2)$$

$$x_3 = \frac{X_3 - 5}{0.45}$$

$$x_4 = \sinh^{-1}(X_4)$$

Based on the large sample of catchments on which GR4J has been calibrated, we have gained a sound knowledge of the prior distribution of the four parameters. Mean values and standard deviations of the transformed parameters are shown in Table 1.

Table 1 First two moments of the transformed GR4J parameters (from a sample of 1111 catchments).

Parameter	Mean value	Standard deviation
X_1	6.2	1.1
X_2	3.9	1.5
X_3	–6.1	3.7
X_4	–0.1	1.7

As a first approach towards using GR4J on ungauged catchments, we tried to explain the transformed parameters using three catchment features: the catchment area (A, km^2), the mean daily potential evapotranspiration (E, mm) and the probability of having a daily precipitation larger than 0.1 mm (W). The explanatory relationships found are:

$$x_1 = 6.3 + 0.001\log(A) + 0.05\log(E) + 0.1\log(W) \tag{1}$$

$$x_2 = 5.3 - 0.1\log(A) - 0.7\log(E) + 0.5\log(W) \quad (2)$$

$$x_3 = -8.7 + 0.4\log(S) + 0.4\log(E) + 0.5\log(W) \quad (3)$$

$$x_4 = 1.2 - 0.07\log(S) + 0.03\log(E) + 1.3\log(W) \quad (4)$$

Note that all these regression relationships have very low coefficients of determination, even though some explanatory variables (in bold type) have acceptable Student ratios (Table 2). Thus, these relations would not be sufficient to apply GR4J to ungauged basins.

Table 2 Coefficients of regression, Student ratios and coefficients of determination of the regression relationships of the GR4J parameters

Parameter	coefficients of regression	Student ratios	coefficient of determination (R^2)
x_1	$a_0 = 6.25$	49.66	0.0008
	$a_1 = 0.001$	0.05	
	$a_2 = 0.05$	0.42	
	$a_3 = 0.12$	1.12	
x_2	$a_0 = 5.31$	31.53	0.14
	$a_1 = -0.06$	–3.62	
	$a_2 = 0.67$	–4.68	
	$a_3 = 0.46$	3.35	
x_3	$a_0 = -8.68$	–21.19	0.06
	$a_1 = 0.41$	9.78	
	$a_2 = 0.44$	1.28	
	$a_3 = 0.50$	1.50	
x_4	$a_0 = 1.16$	5.92	0.04
	$a_1 = -0.07$	–3.27	
	$a_2 = 0.30$	1.83	
	$a_3 = 1.33$	8.31	

TEST DATA

In order to test our approach to determine GR4J parameters on an ungauged catchment, we used a large sample of catchments spanning five continents, assembled for this study: it is comprised of 1111 catchments, with areas from 0.1 to 50600 km^2, located in the United States (500 catchments), France (305 catchments), Mexico (260 catchments), Australia (32 catchments), the Ivory Coast (10 catchments) and Brazil (4 catchments). Note that the 428 basins of the MOPEX US database (Schaake *et al.*, this issue) as well as the 40 basins of the MOPEX French database (Chahinian *et al.*, this issue) are part of this international sample.

METHOD

Each of the 1111 catchments is successively considered as ungauged (with only a limited number of point measurements available) during the first half of the available flow series. It is treated with our approach and its efficiency in control (on the second half of the flow series) is assessed by comparing the results to those of a full calibration operation. To make the most of all data available, the previous operation is carried out a second time using the two periods in reverse order.

The proposed approach

Our approach blends prior knowledge and the information contributed by a few discharge measurements taken randomly during a given period of time. It is embodied into the calibration criterion C given as:

$$C = \alpha \frac{1}{4} \sum_{i=1}^{4} \left(\frac{x_i - x_i^0}{\sigma_i} \right)^2 + (1-\alpha) \frac{\sum_{j=1}^{N} \left(\sqrt{Q_j^{obs}} - \sqrt{Q_j^{calc}} \right)^2}{\sum_{j=1}^{N} Q_j^{obs}} \tag{5}$$

where the parameters x_i are chosen to minimize C, σ_i is the standard deviation of parameter i as shown in the calibration process equations, x_i^0 is the set of prior parameters, Q^{obs} represents a measured daily flow at the basin outlet and Q^{calc} represents the flow calculated by the model using the set of parameters x_i. The parameter α is determined as a function of N, the number of daily measurements, in order to give the approach its maximum efficiency. The least square of the errors is calculated on the square roots of the outflow values in order to avoid giving to some large values an overriding role that could reduce the value of the other measurements. According to equation (5), when no measurements are available our best estimate is to use the prior set of parameters, x_i^0, which can be either the mean values displayed in calibration process equations or, alternatively, the values given by the four regression relationships given in equations (1) to (4).

Test of the proposed approach

We assume that the catchment of interest is now gauged and that a series of discharge data are available for a second period of time. Having determined the parameters x_i, by minimizing criterion C, we can assess the value of these parameter estimates against flow data available for the control period. We use the appraisal criterion F:

$$F = \frac{\sum_{k=1}^{K} \left[\left(Q_k^{obs} - \overline{Q}^{obs} \right)^2 - \left(Q_k^{obs} - Q_k^{calc} \right)^2 \right]}{\sum_{k=1}^{K} \left[\left(Q_k^{obs} - \overline{Q}^{obs} \right)^2 + \left(Q_k^{obs} - Q_k^{calc} \right)^2 \right]} \tag{6}$$

Criterion F, proposed by Mathevet *et al.* (2005) is the bounded version of the Nash-Sutcliffe efficiency but has the new property of belonging to the interval]–1,1]. This calculation can be made for each half of the available flow record (when reversing their roles) and subsequently for each of the 1111 catchments. The mean value of the 2222 individual F values is denoted $\overline{F}$ and will be used to assess the efficiency of our approach. The standard deviation of the 2222 F values is denoted σ_F.

What is the difference between a gauged and an ungauged catchment?

We can tell the difference by just looking at the $\overline{F}$ value for both situations: for our 1111 catchments as for ungauged we have obtained $\overline{F} = 0.13$ when using the mean values from Table 1, and $\overline{F} = 0.14$ when using the values derived from the regression relationships (equations (1) to (4)). Clearly, very little can be gained from the most significant features of a catchment. Opposed to this, when considering all those catchments as gauged and calibrating GR4J in the conventional way, we obtain a much higher value of $\overline{F} = 0.36$. Figure 1 shows the corresponding cumulative distribution of the F values instead of just their mean value. A shift of the distribution towards the right corresponds to an improvement. Our objective is to find a way to move the "ungauged" (left) distribution towards the "gauged" distribution with the help of a few daily measurements. Referring to the mean of the F values, the closer we will be to 0.36, the better our approach. Preliminary work on the same catchment sample (not reported here) showed that most models reported in the literature would yield, after full calibration, a $\overline{F}$ value in the range (0.19 to 0.36). Therefore, we could consider that a candidate method starts to be successful when $\overline{F}$ is greater than the threshold value of 0.19.

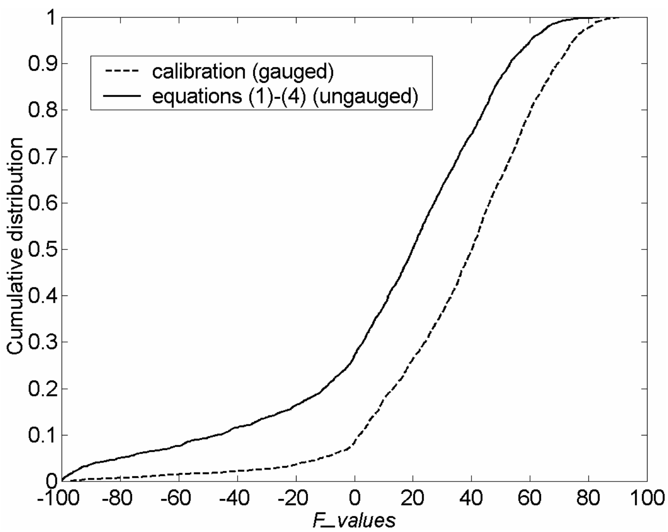

Fig. 1 Cumulative distribution of F values considering 1111 catchments either as gauged or as ungauged.

RESULTS

Using criterion *C*

Results corresponding to N successively equal to 5, 10, 20 and 50 are shown in Table 3.

Table 3 First results using criterion C.

N (number of available streamflow measurements)	α (respective weight of the prior knowledge)	$\overline{F}$ (efficiency criterion)
5	0.03	0.15
10	0.01	0.16
20	0.01	0.18
30	0.01	0.19
50	0.01	0.20

We see that with only 30 daily measurements, we can obtain the same statistical efficiency as with using the less efficient model reported in the literature but fully calibrated. These first results are encouraging.

Selection of the parameters to be calibrated

We had doubts concerning the σ_i used in equation (5). It is acknowledged that the standard deviations cannot properly perform their task of weighting the efforts with respect to each parameter. When the number of measurements is low, one is better advised to calibrate only one or two parameters.

The results bear out our concern with optimizing all four parameters of the GR4J model. Only when the number of measurements is greater than 50, does it become worthwhile to calibrate all four parameters.

Therefore, in our case of very few measurements, we dropped some parameters from criterion C. The corresponding results are shown in Table 4.

Table 4 Best selection of parameters to be calibrated as a function of the number of measurements available.

N (number of available streamflow measurements)	Parameters * optimized	α (respective weight of the prior knowledge, see equation (5))	$\overline{F}$ (efficiency criterion, see equation (6))
5	x_4, x_1	0.01	0.16
10	x_4, x_1	0.03	0.17
20	x_4, x_1, x_2	0.00	0.18
50	x_4, x_1, x_2	0.00	0.21

*Where: x_1, capacity of the soil-moisture accounting store; x_2, capacity of the routing reservoir; $x_{3,}$ unit hydrograph time base; $x_{4,}$ gain-loss function parameter.

CONCLUSION

To our knowledge, this is the first time that the problem posed by ungauged catchments has been broached by elaborating a strategy of direct discharge measurements at the point of interest. The first results presented in this paper show the value of this approach. Instead of measuring complex soil parameters, it seems more profitable to focus our efforts on acquiring a few direct measurements of the variable of interest, the discharge at the outlet of a catchment. The present article reports on the early stage of this strategy, when the first measurements are being acquired. We are continuing our work, aiming to increase the benefit that can be expected from a few direct streamflow measurements, and we will report further improvements later.

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