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Rainfall–runoff modelling of Bua River basin, Malawi

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Abstract This paper investigates the applicability of three systems-types of models: a Simple Linear Model (SLM), a Seasonally Varying Runoff Coefficient Model (SVRC) and a Linear Perturbation Model (LPM), and two conceptual models: NAM and SMAR, in six sub-catchments of the Bua catchment located in the central region of Malawi. The results of the study indicate that the LPM performed better in simulating streamflow series in the study area than the other system-type models and the two conceptual models. Comparison of the performance of the two conceptual models indicated that the NAM model performed better than the SMAR model.

Key words Bua River; rainfall–runoff modelling; system type of model; lumped conceptual models

INTRODUCTION

Rainfall–runoff models can be classified into three categories depending on the degree of their physical abstraction from the real world system (Dawdy & O'Donnell, 1965; Clarke, 1994):

(a) Systems-based (black or grey box) models which make little or no attempt to simulate the individual constituent hydrological processes and which rely heavily on systems theory developed in other branches of engineering science.

(b) Distributed physically-based models which are based on the complex law of physics generally expressed as systems of nonlinear partial differential equations.

(c) Quasi-physical conceptual models which occupy an intermediate position between the other two types of models in terms of complexity, disaggregation and data requirements.

The physically-based distributed models are well suited to solving problems such as predicting the effects of land-use changes and studying the hazards of pollution (Beven, 1989, 1997; Beven *et al.*, 1995). The other two more conventional types of model are often too primitive to present scientifically sound solutions to such problems. Nevertheless, these last two types of models have often proved to be effective in the solution of a wide spectrum of important hydrological problems, such as river flow forecasting and the extension of hydrological records.

In this study, a comparison was carried out to investigate the adequacy of the application of lumped systems and conceptual models in Bua River basin in Malawi.

CANDIDATE SYSTEM AND LUMPED CONCEPTUAL MODELS

Three systems-types of models: the Simple Linear Model (SLM), Seasonally Varying Runoff Coefficient Model (SVRC) and Linear Perturbation Model (LPM), and two conceptual models: NAM and SMAR, were used in the study. Brief descriptions of these models are given below.

Systems Models

Simple Linear Model (SLM) The SLM postulates a linear time invariant relationship between the rainfall x and the discharge y. In discrete form, embodying a model fit error term e_i, it may be expressed as (Kachroo & Liang, 1992):

$$y_i = \sum_{j=1}^{m} h_j x_{i-j+1} + e_i \tag{1}$$

where h_j are the ordinates of the pulse response, and m is the memory length. The pulse response ordinates can be estimated by Ordinary Least Squares in a parametric form, using the well known gamma function model which was introduced by Nash (1957) as a general equation for the instantaneous unit hydrograph. The impulse response of the gamma function model $h(t)$ is given by:

$$h(t) = \frac{1}{K\Gamma(n)} e^{-(t/k)} \left(t/K \right)^{n-1} \tag{2}$$

where $\Gamma(n)$ is the gamma function of *n*. *K* is a constant having the dimension of time, *n* is a "shape" parameter constant. Moreover in this model form, *n* and the lag product *nk* are usually considered as parameters of this model rather than *n* and *k* (Kachroo & Liang, 1992).

Seasonally Varying Runoff Coefficient (SVRC) Model This model is introduced in the context of long-term variance to show how much can be achieved by a simple empirical tool. The SVRC model is given by (Kachroo & Liang, 1992):

$$y_i = C_d \sum_{j=1}^{m} h_j x_{i-j+1} + e_i \tag{3}$$

where C_d is a time-varying coefficient of runoff which is allowed to vary throughout the year in a periodic fashion.

The Linear Perturbation Model (LPM) The LPM was originally proposed by Nash & Barsi (1983). Perturbation models are used to account for the seasonality of the observed rainfall and the runoff. The model is based on the following assumptions:

(a) If, in a particular year, each input function, is equal, for each day of the year to its expected value for that date, then the output will also equal to its expectation for that date (Kachroo & Liang, 1992),

(b) Perturbations, or departures from the date of expected input values are linearly related to the corresponding perturbations or departures from the date of the expected output values.

The transformation is introduced in the observed data of rainfall x_i and the runoff y_i. The transformation has the form of;

$$Q'_i = y_i - y_s \quad \text{and} \quad R'_i = x_i - x_s \quad \text{for} \quad i = 1,2,3,......,n \quad \text{and} \quad s = 1,2,3,.....,365 \tag{4}$$

where x_s and y_s are the seasonal mean rainfall and discharge respectively. The linear relationship between the perturbations of the discharge Q'_i and rainfall R'_i with a gain factor *G* is given by:

$$Q'_i = G \sum_{j=1}^{m} R'_j \, h_{i-j+1} + e_i \tag{5}$$

Conceptual Models

The Soil Moisture Accounting and Routing (SMAR) model SMAR is a conceptual type of model that assumes that the catchment is analogous to a vertical stack of horizontal soil layers, which can contain various amounts of water. Evaporation from the top layer occurs at the potential rate. Evaporation from the second layer occurs after exhaustion of the first layer at the potential rate multiplied by a factor *C*. Evaporation from the third layer occurs at the potential rate multiplied by C^2. Thus a constant evaporation applied to the basin reduces the soil moisture in a roughly exponential manner. The capacity of each layer is taken as 25 mm and the total storage *Z* (mm) is the parameter to be optimized.

The NAM model The NAM model (DHI, 2000) is a deterministic, lumped conceptual rainfall–runoff model, which represents various components of the rainfall–runoff process by continuously accounting for the water content in five different and mutually interrelated storages where by each storage represents different physical elements of the catchment (Madsen, 2000). These storages are: snow storage, surface storage, lower zone storage, upper groundwater storage and lower groundwater storage.

DATA AVAILABLE

Data used in the study was from gauging stations located in six sub-catchments of Bua River basin. The details of the stations used in the study are presented in Table 1. Rainfall and

Table 1 Discharge data used in the study.

No.	Station code	Data length	Area (km^2)	Missing (%)	Mean daily flow (m^3 s^{-1})	Mean annual flow (mm)	Mean areal rainfall (mm)	Runoff coefficient
1	5C1	11	10654	12.39	44.577	137.24	901	0.147
2	5D2	9	6790	1.98	25.550	120.82	917	0.132
3	5D3	10	233	1.45	1.290	172.64	874	0.200
4	5D4	11	1394	5.95	6.731	152.21	879	0.173
5	5F1	11	2580	7.42	10.116	105.85	879	0.140
6	5E6	8	126	2.02	0.605	151.47	1090	0.138

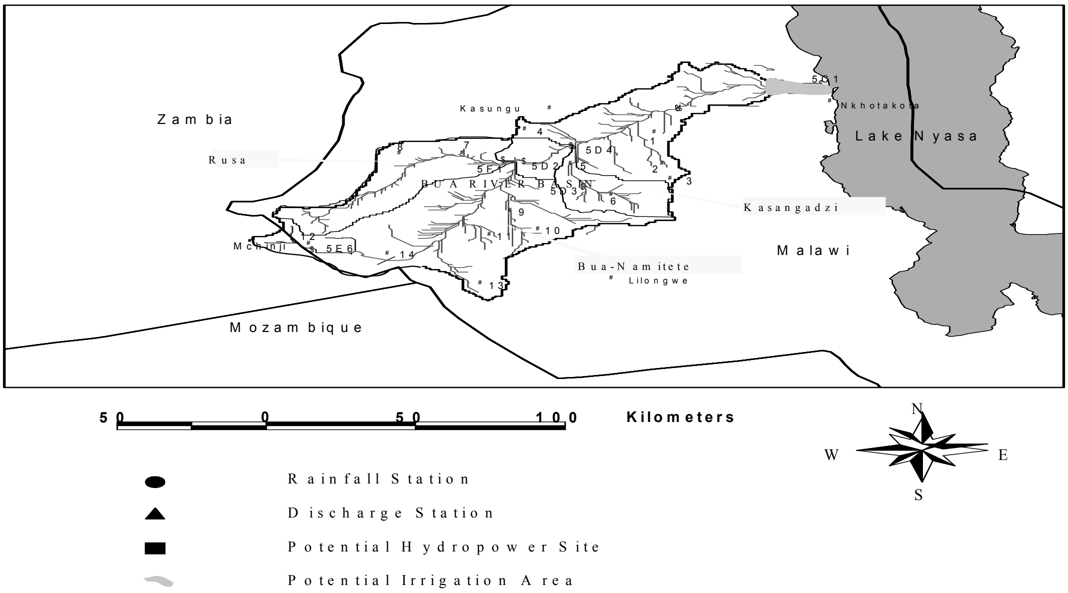

Fig. 1 Map of the Bua River showing the location of rainfall and river gauging stations.

temperature data used in the study were from 14 and 4 stations respectively. Rainfall and temperature data were collected from the Malawi Meteorological Department at Chileka Airport in Blantyre. Temperature data was used to estimate daily potential evaporation since daily climate data for the catchment was not available. Discharge data were obtained from the Ministry of Water (Malawi), Department of Hydrology in Lilongwe. Figure 1 shows the location of river gauging and rainfall stations.

MODEL APPLICATION

Each of the five models used in the study was applied to each of the six sub-catchments of the Bua River basin. The model application involved calibration and verification of the models whereby approximately four-fifths of the available data was used for calibration and one-fifth for verification. In the calibration process, the first portion of the data was used to compute the model parameters for each sub-catchment. During verification, the estimated discharge, sometimes referred to as discharge forecasts, were obtained by using rainfall in the second period as input to the model, which was already calibrated. The model efficiency in the verification period was the measure of the accuracy at which the model can make forecasts in the period different from that on which it was calibrated.

RESULTS AND DISCUSSION

The results of performances of the models used in the study are presented in Table 2 both for the calibration and verification periods. From the results presented, it is observed that the LPM model with its inherent component of seasonal variation performed consistently well in calibration mode

Table 2 Comparison of the results of the five models.

Catchment Area (km^2)	Bua-5C1 10654	Bua-5D2 6970	Bua-5F1 2580	Kasa-5D4 1394	Mtiti-5D3 233	Bua-5E6 126
Calibration efficiency (R^2)						
SLM	38.6	46.74	29.16	46.47	36.61	23.82
SVRC	56.56	67.74	56.78	42.36	42.18	26.19
LPM	65.12	78.93	74.05	38.85	63.84	36.86
SMAR	75.77	69.13	38.02	58.29	33.04	23.44
NAM	71.89	80.95	47.68	65.48	42.00	25.37
Verification efficiency (R^2)						
SLM	63.05	69.20	39.07	33.97	1.90	46.88
SVRC	45.26	53.73	56.37	12.19	–3.76	53.29
LPM	72.15	66.61	77.44	38.84	59.20	38.38
SMAR	56.89	67.45	48.02	58.58	11.05	49.59
NAM	62.14	87.02	57.69	8.41	18.02	50.40

in all cases applied, except for station 5D4, this model out performed all the other models at four (5F1, 5D3, 5E6 and 5D2) out of the six stations with the exception of one station (5D2) where the NAM model performed better. In two stations (5C1 and 5D4) out of six stations, both NAM and SMAR out performed the LPM. On the other hand, the NAM model also performed consistently well, second to the LPM model. Table 2 also shows the performance results of the models in calibration mode. It can be seen from Table 2 that the LPM consistently performed better than all the other models. The NAM and then the SMAR models followed the LPM model in better performance.

Figure 2 shows the observed hydrograph and the hydrograph simulated using the LPM model for one river gauging station (5D2) used in the study for the period 1981–1988. Figure 2 shows that the LPM model has managed to simulate the low flows adequately well. The model has also managed to simulate peak flows fairly well for most of the years. Failure to simulate peak flows is noted for the year 1981. The reason may be due to poor quality of the climatic data. The objective

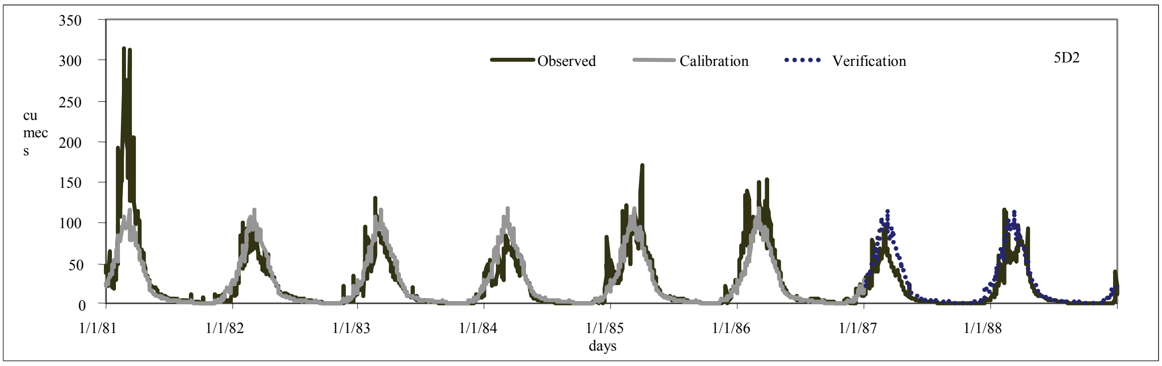

Fig. 2 Hydrographs of observed and estimated discharges by LPM (Station 5D2).

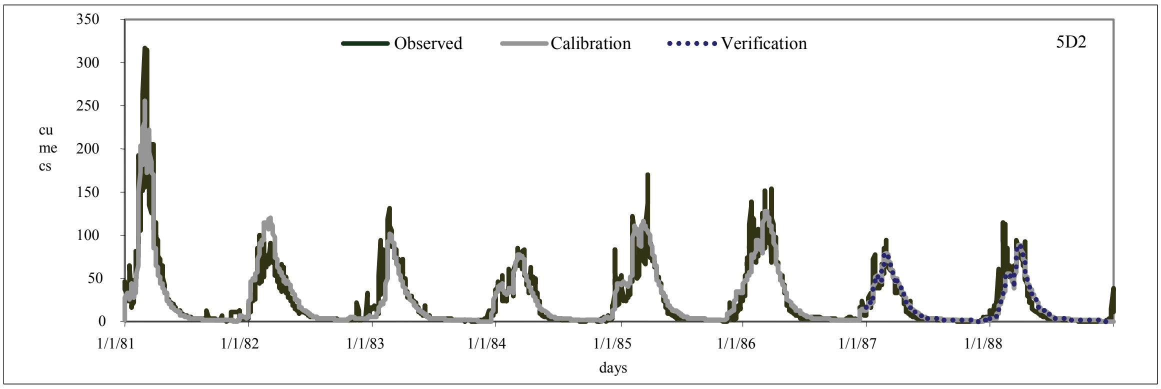

Fig. 3 Hydrographs of observed and estimated discharges by NAM Model (Station 5D2).

criterion of the explained variance gave a value of $R^2 = 79\%$ (Table 2). Figure 3 shows the observed hydrograph and the hydrograph simulated using the NAM model for one river gauging station (station 5D2). In Fig. 3, the NAM model has fitted the observed data very well. The model has managed to simulate the low flows adequately well as well as the peak flows. The objective criterion of explained variance gave a value of $R^2 = 81\%$ (Table 2).

CONCLUSION AND RECOMMENDATIONS

Conclusion

The results of the performance of the models used in the study indicate that the LPM model is a better model for application to simulate streamflow in the Bua River catchment. The second preferred model is the NAM model.

Recommendations

Based on the analysis of the results from the application of system and lumped conceptual models, further studies on rainfall runoff–runoff modelling in the study area should concentrate on other conceptual models, semi-distributed, and distributed models.

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