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Uncertainties in water balance estimations due to scarce meteorological information: a case study for the White Volta catchment in West Africa

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Abstract Scientifically sound decisions in sustainable water management are usually based on hydrological modelling, which can only be accomplished with meteorological information. Especially in regions with a weak infrastructure, where meteorological data are not available at a sufficient spatial and temporal resolution, spatial interpolations of coarse-resolution meteorological point observations are afflicted with uncertainties. This particularly applies to discontinuous variables like precipitation. These input uncertainties are transferred to the hydrological simulations. The uncertainties resulting from precipitation interpolation and their effect on model-based water balance estimations are investigated. First, the results of different spatial interpolation techniques will be compared and analysed. Second, the results of the hydrological simulations driven by these meteorological fields shall be investigated to estimate the propagating effect of the precipitation uncertainties on water balance estimations. The area under study is the White Volta catchment (94 000 km^2) in the semi-arid environment in West Africa, for which basin-wide uncertainty estimates are required for sustainable water management decisions. The three geostatistical interpolation methods: inverse distance weighting, ordinary and external drift kriging, were applied for the calculation of areal precipitation. The results show that the interpolation technique selected influences the areal precipitation field. The results from the impact study of the differences in areal precipitation on hydrological modelling show that the different interpolation techniques produce small differences for aggregated variables and corresponding time series, but affect the spatial distribution of the water balance variables.

Key words geostatistical interpolation; impact; precipitation; water balance simulations

INTRODUCTION

Scientifically sound decisions in sustainable water management are usually based on hydrological modelling. Water balance simulation models generally require precipitation, temperature, humidity, wind speed, and solar radiation data as the driving meteorological information. Consequently, these observation data (point measurements) have to be converted to areal information using various interpolation techniques, e.g. geostatistical methods like inverse distance weighting or kriging. The selection of an appropriate method depends on a number of factors, including the time available, the density of the gauge network, and the spatial variability of the meteorological parameter. In general, the accuracy of all methods for estimating areal information increases with: (a) the density of gauges, (b) the length of the period considered, and (c) the size of the area (Robinson, 2005). In regions with a weak infrastructure, usually only sparse hydro-meteorological information is available, which additionally may contain large data gaps. Considering the requirements for an increasing accuracy of areal estimations, regions with a weak infrastructure are usually handicapped due to: (a) a sparse density of gauges, and (b) a variable length of periods containing large data gaps. This means that areal estimations in general are less accurate in regions with a weak infrastructure. Consequently, the areal estimations are afflicted with larger uncertainties. This problem arises particularly for discontinuous and spatially highly variable parameters, such as precipitation in hydrological modelling. Precipitation, however, is the basic component of the water balance. Therefore, areal estimation of precipitation is a major task in hydrological simulations. In this study, the geostatistical interpolation techniques of inverse distance weighting and kriging will be applied for the areal estimation of precipitation, and their effects on hydrological simulations shall be investigated in a region with only little hydro-meteorological information.

GEOSTATISTICAL METHODS ESTIMATING AREAL PRECIPITATION

The accuracy of the estimation at each point or grid is a function of the distance from the nearest gauges. The simplest technique is an inverse distance weighting of gauge values for each point. Weights are calculated depending on the distances between the location requiring an estimate and the locations of the observations. The interpolated value $z^*(u)$ at location u is calculated by:

$$z^*(u) = \sum_{i=1}^{N} (w_i \cdot z(u_i)) \tag{1}$$

where w_i is the weight of the value $z(u_i)$ observed at station i. The sum of weights w_i equals one, and each weight w_i is calculated as the weighted, reciprocal distance between station i and location u. N stands for the number of observations in the surrounding area of $z^*(u)$ defined by a maximum distance.

Kriging is a geostatistical method that uses the variogram of the precipitation field (i.e. the variance between pairs of points that lie different distances apart) to estimate interpolated values. The resulting precipitation field is optimal in the sense of identifying gauge weightings to minimise the estimation error (Robinson, 2005). Kriging calculates the “best” estimate of the values (BLUE: Best Linear Unbiased Estimator). Kriging incorporates the spatial structure of the variable (variogram) in the estimation. The layout of the observation network relative to the interpolation grid is considered. The reliability of the results is calculated as kriging error (estimation variance) for each grid point (Schafmeister, 1999). First, kriging requires an experimental variogram analysis for the estimation of the spatial variability of the variable. In this study, the precipitation data observed were normalised prior to experimental variogram analysis. This transformation is aimed at avoiding negative interpolated precipitation, which has to be set to zero, and finally results in an overestimation of the precipitation sums. The experimental variogram was calculated with distance classes, e.g. with a constant class width of 6 km. Afterwards, a monotonisation of the class values was performed. In general, the experimental variogram is fitted by a theoretical function $\gamma(h)$ which allows estimation of the variogram analytically for any distance h. For this study, the experimental variogram was fitted to a combination of a nugget effect (C_0) and a spherical model (equation (2)) to determine the sill ($C_0 + C_1$) and range (a) of the theoretical variogram:

$$\gamma(h) = \begin{cases} C_0 + C_1(1.5(h/a) - 0.5(h/a)^3) & \text{if } |h| \le a \\ C_0 + C_1 & \text{if } |h| > a \end{cases} \tag{2}$$

With the results of the variogram analysis, precipitation values at unsampled locations can be estimated using kriging. Kriging normally uses a linear estimator (equation (1)). It determines the values of the weights by minimising the variance of the estimation error $[z^*(u) - z(u)]^2$. With the results of the variogram analysis, the additional constraint $\Sigma w_j = 1$, and the introduction of a Lagrange multiplier μ, minimising of the estimation error leads to the following linear kriging equation system (equation (3)) with $N + 1$ equations (Schafmeister, 1999):

$$\sum_{j=1}^{N} w_j \gamma(u_i - u_j) + \mu = \gamma(u_i - u) \ \text{ for } i = 1,\ldots,N \ \text{ and } \sum_{j=1}^{N} w_j = 1 \tag{3}$$

Apart from ordinary kriging described above, external drift kriging (Ahmed & de Marsily, 1987) was applied, where external knowledge is incorporated in the system as external drift. Here, it is supposed that an additional variable $y(u)$ exists that is linearly related to $z(u)$. The estimator thus depends on the additional variable $y(u)$. Therefore, $y(u)$ has to be available at a high spatial resolution, preferably as a regular grid.

AREA UNDER STUDY

The area under study is one of the main tributaries of the Volta basin, the White Volta catchment (94 000 km^2) which is situated upstream of Lake Volta in Northern Ghana and Burkina Faso (Fig. 1). Lake Volta is one of the largest artificial lakes in the world and hydropower generation at the Akosombo Dam is the major energy source in Ghana. In the basin, rain-fed agriculture is the major source of livelihood. Due to high population growth, the demand for water, food and energy increases continuously. This means that the livelihood of the population depends mainly on rainfall, in particular on rainfall variability. In general, precipitation intensities as well as annual rainfall amounts show a strong inter-annual and inter-decadal variability in West Africa. Rainfall variability in the White Volta catchment is also very high. Mean annual precipitation ranges from less than 500 mm (north) to more than 1500 mm in the south, of which around 80% falls between July and September. Furthermore, small-scale rainfall variability is very high. Friesen (2003) estimated the coefficient of variation of 9 × 9 km^2 intra-scale rainfall variability to be between 0.25 and 0.4 in northern Ghana. Spatial precipitation distribution in the White Volta catchment is characterised by a strong latitudinal dependence which is taken into account in the

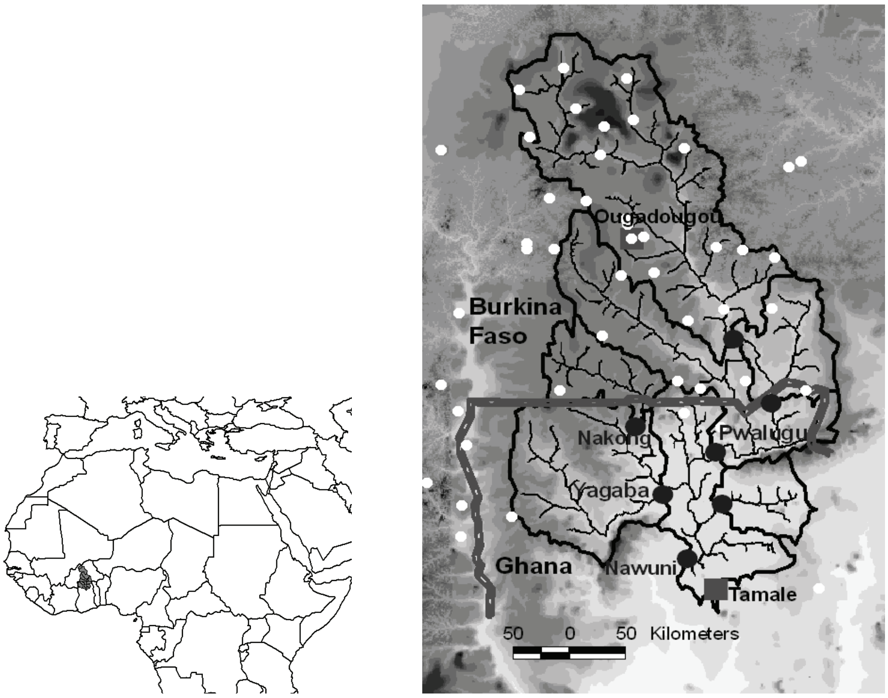

Fig. 1 Digital elevation model (1 km), location of the precipitation stations (white points) and sub-catchments in the White Volta basin.

simulations by applying an anisotropy factor. Climatologically, the White Volta basin is situated in the semi-arid climate zone with a mean annual temperature between 27 and 36°C. Evapotranspiration is a very important factor in this region. The mean annual potential evaporation lies between 2500 mm in the north and 1500 mm in the south. Approximately 80% of the precipitation is lost to evapotranspiration during the rainy season (Oguntunde, 2004).

The White Volta catchment is very flat, in particular in the southern part (<0.1%). The predominant land-use types are Guinea savannah in the southern, and Sudan savannah in the northern part. The main geological systems of the basin are a Precambrian platform and a sedimentary layer, the Voltaian sandstone basin. The predominant soil types are lixisols in the southern and arenosols in the northern part (Jung, 2005). Since 1993, the natural flow regime of the White Volta catchment has been disturbed by a dam and hydropower generation in Bagré in the south of Burkina Faso. For this reason and data availability, the hydrological model was calibrated for the year 1968 and validated for 1961–1967 (Wagner, 2006). During this period, both meteorological and hydrological measurement data were available for both Burkina Faso and Ghana. Therefore, the time period for this case study is 1968, when meteorological measurements of daily temporal resolution were available from six stations in Ghana and approximately 30 stations in Burkina Faso.

AREAL PRECIPITATION RESULTS

In this study, areal precipitation was calculated using inverse distance weighting, ordinary and external drift kriging as described above. First, the time series of

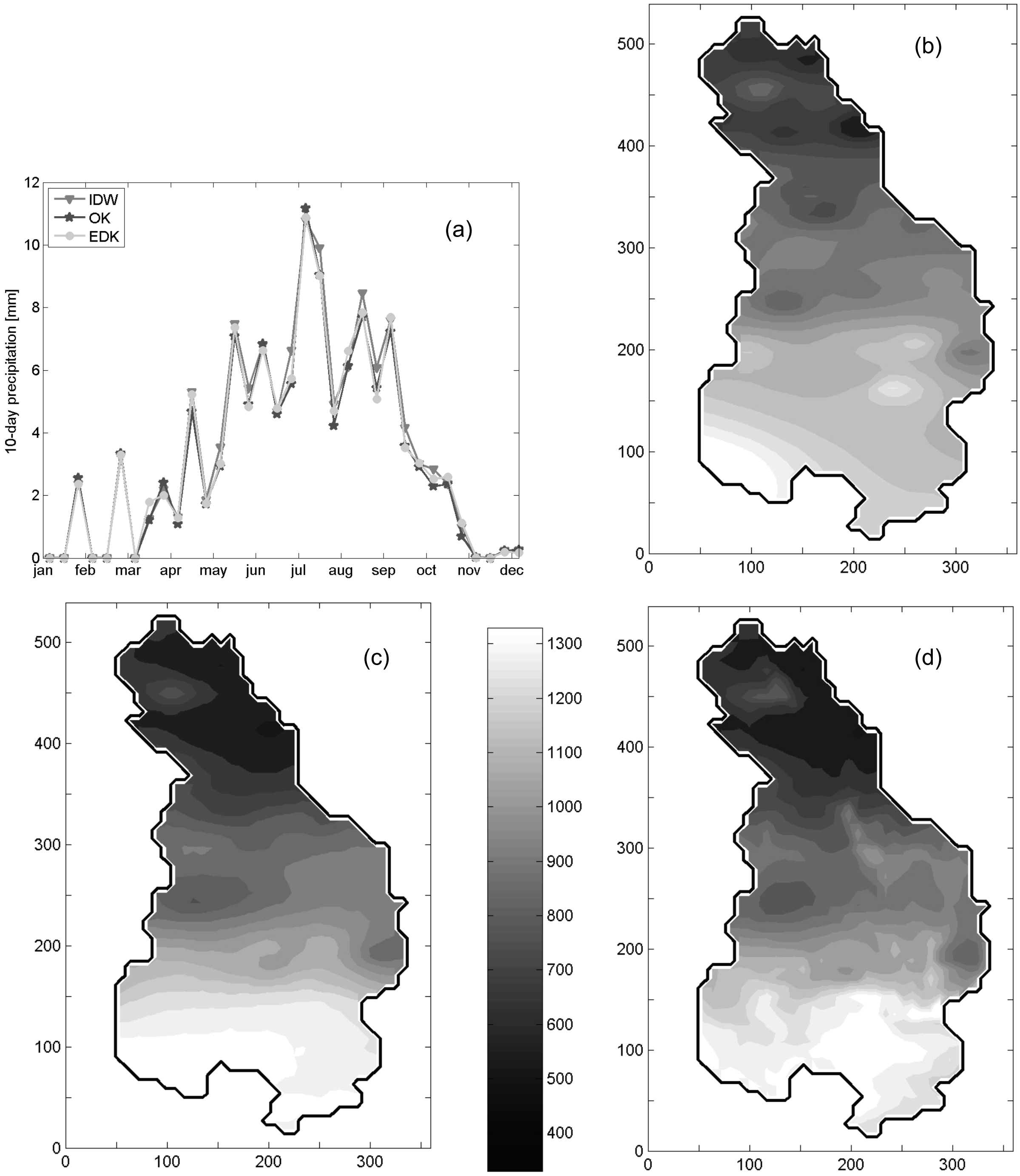

Fig. 2 (a) Time series of precipitation (10-day mean) averaged over the entire catchment as obtained from the interpolation techniques of inverse distance weighting, ordinary kriging, and external drift kriging. Annual precipitation (mm) for 1968 using (b) inverse distance, (c) ordinary kriging, and (d) external drift kriging interpolation techniques.

precipitation (10-day mean) averaged over the entire catchment are shown in Fig. 2(a) for the three interpolation techniques. In addition, Fig. 2 shows the spatial distribution of annual precipitation for 1968 using: (a) inverse distance weighting, (b) ordinary

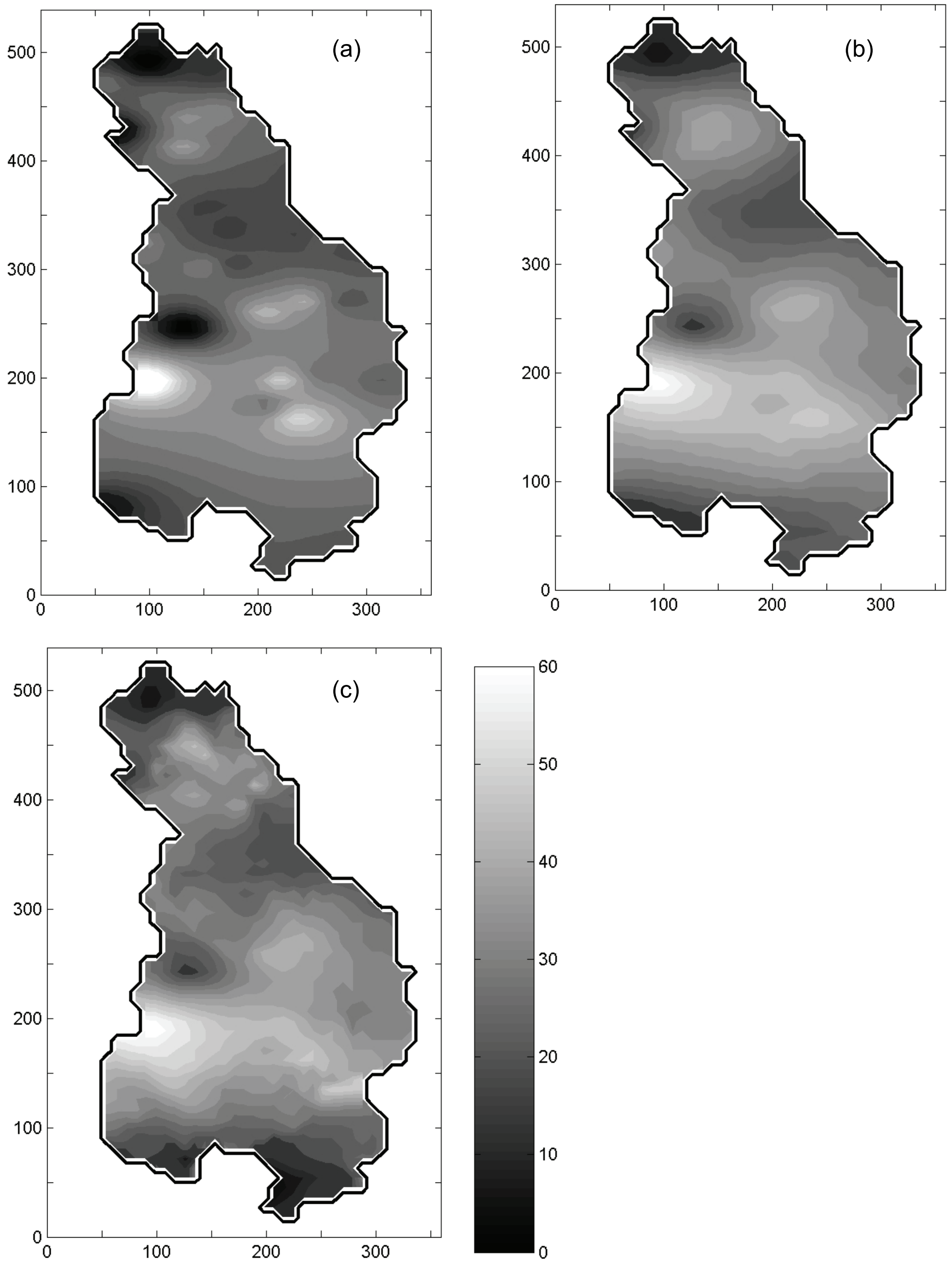

Fig. 3 Precipitation (mm) for 19 July using: (a) inverse distance, (b) ordinary kriging, and (c) external drift kriging as the interpolation technique.

kriging, and (c) external drift kriging with the digital elevation model (Fig. 1) as external drift. The results show that the interpolation technique selected has minor impact on the time series averaged over the entire catchment, but influences the areal precipitation field. Although the anisotropy factor is constant, the effect is more pronounced for the kriged precipitation fields. Due to the elevation model as external

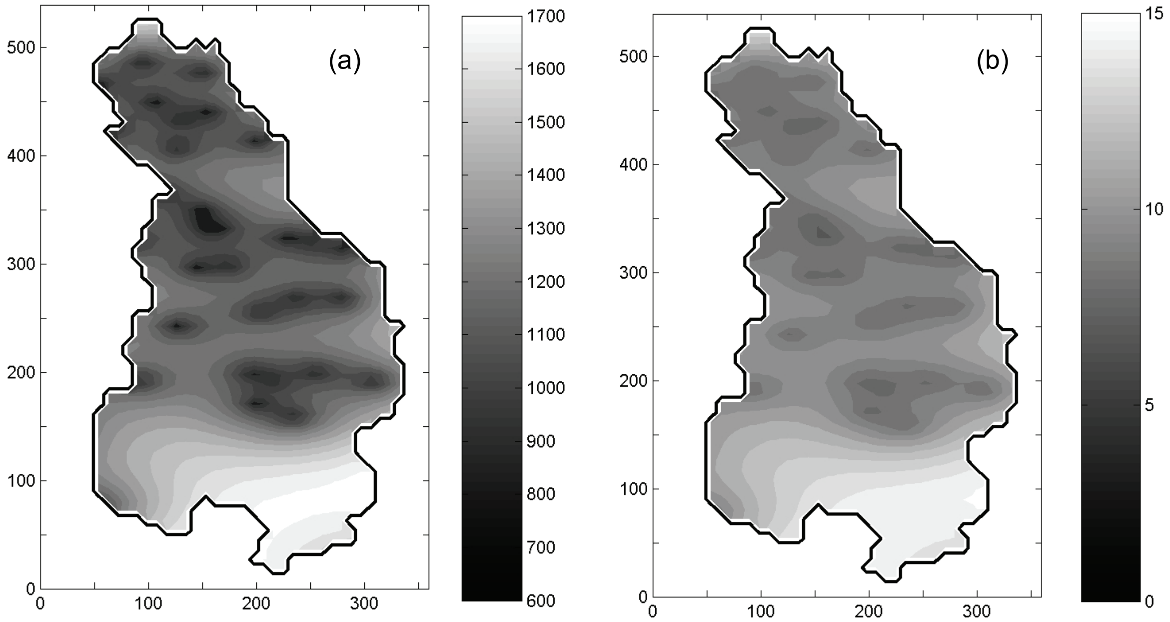

Fig. 4 Square root of the estimation variances of ordinary kriging for: (a) annual precipitation in 1968 and (b) precipitation on 19 July (mm).

drift, the annual precipitation grid calculated by external drift kriging follows the valleys and the few mountain ridges in the basin. In addition to annual precipitation, the spatial distribution of the areal precipitation is shown for 19 July, when the basin-wide daily maximum precipitation sums were measured. Figure 3 again shows the areal precipitation calculated by the three above-mentioned methods. Comparison of the precipitation fields calculated by inverse distance weighting and ordinary kriging shows that the transitions of the contours are smoother for ordinary kriging because of the nugget effect which allows for spatial variability close to the observations. In both figures, external drift increases spatial variability of the precipitation fields based on external drift. In Fig. 4, the square root of the estimation variances calculated by the ordinary kriging method are plotted for the annual precipitation 1968 and for 19 July. In both figures, the errors are small and found to be around the observations. In the southern part of the catchment the errors are larger due to the sparse observational stations. These results represent important information for the evaluation of the simulations and for the selection of new locations of measurement gauges.

IMPACT OF PRECIPITATION UNCERTAINTIES ON HYDROLOGICAL MODELLING

For hydrological simulations, the Water balance Simulation Model WaSiM-ETH (Schulla & Jasper, 2000) was used. It is a deterministic, fully distributed modular model for the simulation of the hydrologically important parts of the water balance and uses physically-based algorithms for most processes. Fluxes in the unsaturated zone are calculated with the Richards equation (Richards, 1931). The potential evapotranspiration is calculated according to Penman-Monteith (Monteith, 1975) and the real evapotranspiration is estimated by using a relation between soil moisture and actual capillary pressure. The calculation of interception is based on a simple bucket approach. Groundwater fluxes are calculated by a two-dimensional flow model which is dynamically coupled to the unsaturated zone. Discharge routing is based on a kinematic wave approach.

For the simulations, the White Volta catchment was subdivided into seven sub-catchments. The outlets of the sub-catchments are located at hydrological stations, such that simulated discharges can be compared with measurements, if available. The White Volta flows into Lake Volta, but because of backwater effects which cannot be calculated by the model, the outlet of the model set-up is not Lake Volta, but the station of Nawuni. The spatial resolution of this study is 1×1 km^2 which results in a regular grid of 411×631 grid points. The temporal resolution is 24 h. Vertically, the soil is represented by 20 layers, and each is 1 m thick. The model requires digital elevation data, gridded soil properties (derived from the global FAO soil map), land-use and hydrogeological information.

For the interpolation of the meteorological input data observed, WaSiM-ETH provides several interpolation techniques: Thiessen polygons, inverse distance weighting interpolation, altitude-dependent regression, and bilinear interpolation. As vegetation is independent of altitude in this region, cf. for example, Europe, the inverse distance weighting method was applied as an interpolation method for calibration (Wagner, 2006). For the simulations driven by kriged areal precipitation, regular grids of 9×9 km^2 resolution are imported.

Results

The impact of the different interpolation techniques for areal precipitation on hydrological modelling are shown for real evapotranspiration and routed discharge. First, the time series of real evapotranspiration (10-day mean) averaged over the entire catchment are shown in Fig. 5(a) for the three interpolation techniques. The general behaviours of all time series are comparable. During the rainy season, however, the real evapotranspiration rates are lower in the simulation with areal precipitation calculated by inverse distance weighting, compared to those calculated by kriging. The differences between ordinary and external drift kriging are minor during the rainy season. Figure 5(b)–(d) shows the differences of the spatially distributed annual evapotranspiration sums resulting from the calculation of areal precipitation by (b) inverse distance weighting and ordinary kriging, (c) inverse distance weighting and external drift kriging, and (d) ordinary and external drift kriging. In analogy to the results of the time series, the differences between inverse distance weighting and kriging are larger than between the two kriging methods. The second variable investigated is the routed discharge which is the only parameter for which observations are available for validation purposes. Considering the large catchment size and weak data availability, the simulation quality is satisfying in general. Figure 6(a) shows the 10-day mean discharge hydrographs for the source basin Nakong using the three different calculations of areal precipitation. The general discharge hydrographs simulated are similar for all three methods. For the maximum peak in July, however, the simulation with areal precipitation calculated by inverse distance weighting overestimates

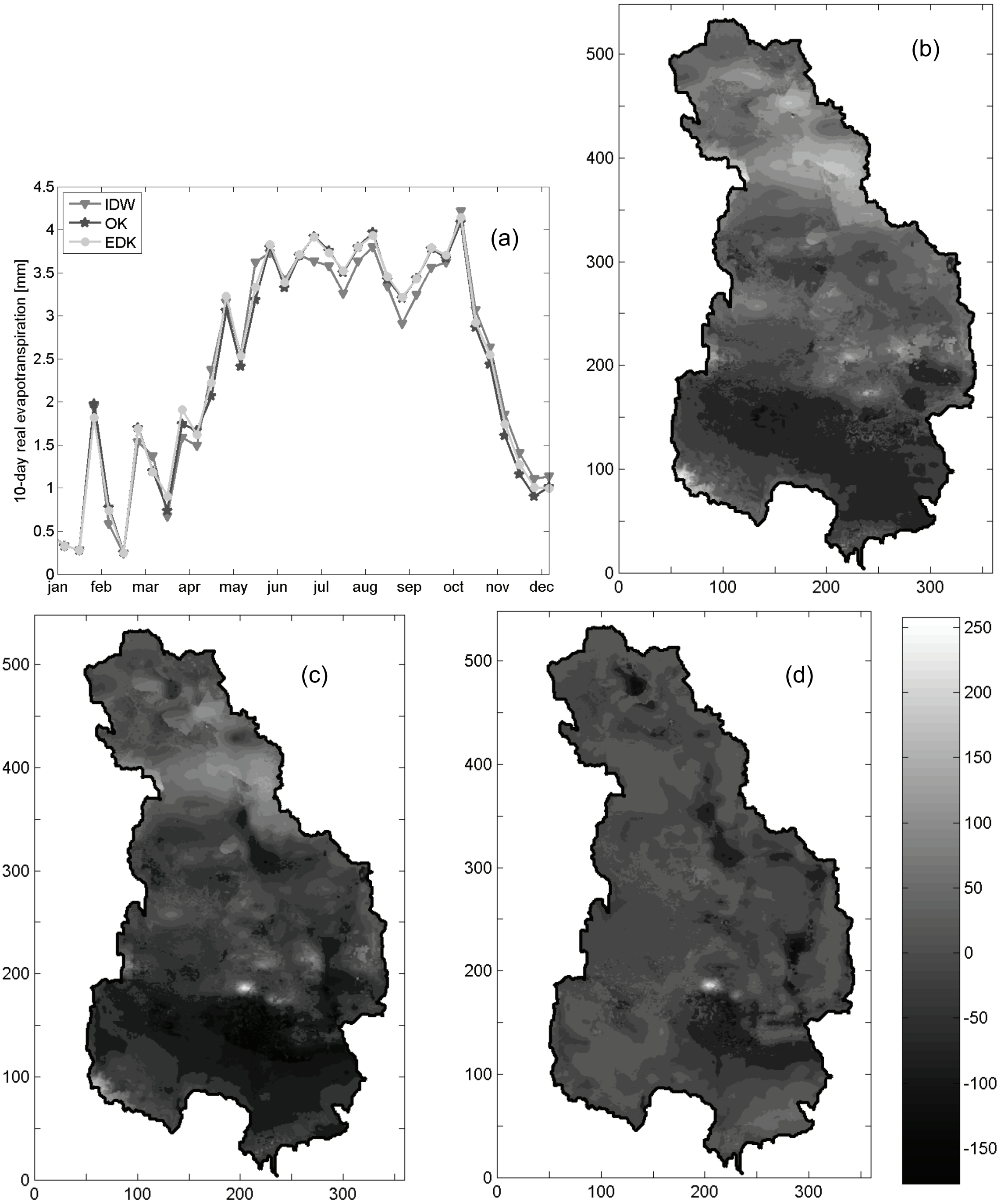

Fig. 5 (a) Time series of real evapotranspiration (10-day mean) averaged over the entire catchment as obtained from the interpolation techniques of inverse distance weighting, ordinary kriging, and external drift kriging. Differences of the spatially distributed annual evapotranspiration sums (mm) resulting from the calculation of areal precipitation by (b) inverse distance weighting and ordinary kriging, (c) inverse distance weighting and external drift kriging, and (d) ordinary and external drift kriging.

the peak flow less than the simulations using kriged precipitation fields. Furthermore, the first rain event in April was simulated using only the areal precipitation calculated by inverse distance weighting. Figure 6(b) shows the routed 10-day mean discharges

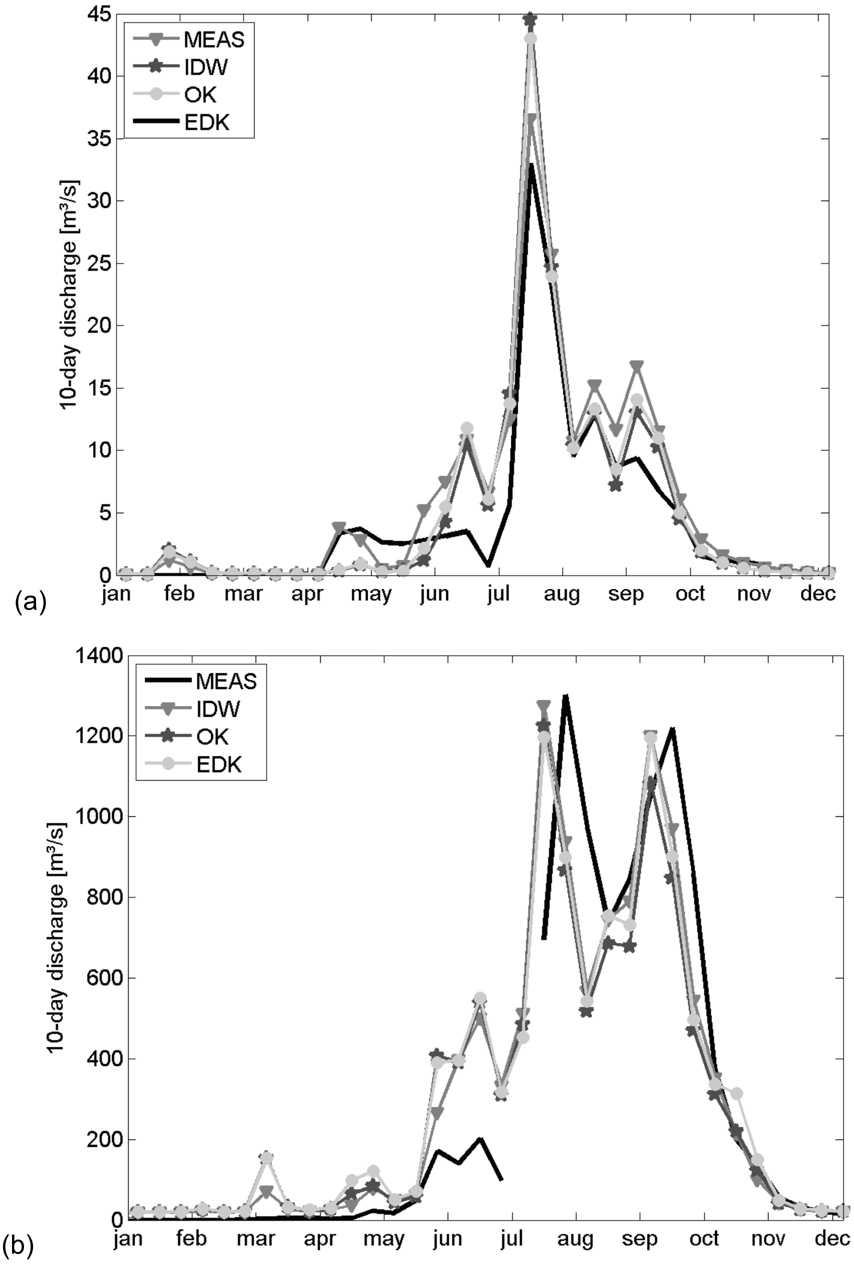

Fig. 6 Time series of the routed discharge (10-day mean) for: (a) the source catchment of Nakong and (b) the complete basin in Nawuni as obtained from the interpolation techniques of inverse distance weighting, ordinary kriging, and external drift kriging.

for Nawuni, the outlet of the complete basin, for the same three interpolation techniques for areal precipitation. Again, the general discharge hydrographs are similar. The two maximum peaks are simulated to be in the same range for all three cases. Table 1 shows a comparison of Nash-Sutcliffe coefficients of routed discharges for Nakong, Nawuni and two further available sub-catchments as obtained from using the different interpolation techniques for areal precipitation. For Nakong and Nawuni, the best model performances are achieved using inverse distance weighting. For Yagaba and Pwalugu, the kriged precipitation fields result in better model

Table 1 Nash-Sutcliffe coefficients of the daily and 10-day mean routed discharges (March1968–December 1968) in Nakong, Nawuni, Pwalugu and Yagaba, as obtained from the interpolation techniques of inverse distance weighting, ordinary kriging, and external drift kriging.

	Daily:			10-day:		
	IDW	OK	EDK	IDW	OK	EDK
Nakong	0.677	0.576	0.594	0.786	0.741	0.747
Nawuni	0.765	0.707	0.725	0.793	0.741	0.762
Pwalugu	0.237	0.400	0.380	0.275	0.520	0.487
Yagaba	0.777	0.817	0.794	0.809	0.870	0.851

performances. In general, the model performances are better for 10-day mean values compared to daily ones.

CONCLUSIONS

In this study, the geostatistical methods inverse distance weighting, ordinary and external drift kriging were applied for the areal estimation of precipitation in the White Volta catchment, a region where only a little hydro-meteorological information is available. The results of the areal estimation of both daily and annual precipitation show that the method selected influences the areal precipitation field. Compared to inverse distance weighting, ordinary kriging calculates smoother contours due to the nugget effect. External drift kriging increases spatial variability based on the selected external drift. Apart from areal precipitation, kriging calculates a spatially-distributed estimation variance which provides important information for the evaluation of the simulations. The results of the impact study of the different interpolation techniques for areal precipitation on hydrological modelling show that the general patterns of the time series of water balance variables presented are comparable. But the comparison of Nash-Sutcliffe coefficients shows varying model efficiencies for the different interpolation methods. The spatial distribution of the annual sum of real evapotranspiration is influenced by the interpolation method selected for the calculation of areal precipitation.

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