Testing similarity indices to reduce predictive uncertainty in ungauged basins

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Abstract In ungauged watersheds, the common approach for estimating the parameter values of lumped rainfall–runoff models consists of a two-step approach. First, relationships between watershed characteristics and parameter values are established on gauged sites, and second, these relationships are used to estimate parameter values on ungauged sites. However, several studies suggested that there are strong limitations to this approach and that consideration for similarity and/or proximity offered a better outlook. We propose here an original approach based both on similarity considerations and multi-model methodology. First, gauged watersheds are clustered into 27 classes depending on the values of three characteristics (either physical or hydro-climatic). Then, for each ungauged watershed, we used the calibrated parameters of similar gauged watersheds, i.e. from the same class. Then, a combination of the simulations obtained with the different sets of parameters was performed. The methodology is based on the GR4J rainfall–runoff applied on more than 1000 basins located in France. Results show that the physical similarity approach performs slightly better than the regression-based approach. Refinements of these two approaches, such as regional calibration of regressions or multi-model considerations for regionalization based on physical similarity, do not yield significant improvements.

Key words rainfall–runoff modelling; regionalization; ungauged catchments

INTRODUCTION

Predictive tools for water resources management are essentially data-driven, i.e. they need to be calibrated with observed flow data (case of gauged catchments). Major difficulties exist when these models are to be transferred to catchments for which no flow data are available, which prevents model calibration (case of ungauged catchments). Therefore, hydrological modellers have been developing strategies to estimate model parameters without calibration since the 1970s. The term regionalization takes its roots in the process of transferring parameters from neighbouring catchments to the ungauged catchment. Then, the concept of regionalization enlarged to encompass studies aimed at developing methodologies to estimate model parameter values on any ungauged catchment.

Two kinds of regionalization approaches are considered in this paper: regionalization based on regression and regionalization based on physical similarity.
To test these approaches, we used the GR4J model applied over a large set of 1040 catchments located in France, for which daily precipitation and streamflow data were available over a 10 year period. After determining the optimal settings of the regionalization schemes, we compare their efficiency in predicting daily streamflows on ungauged sites.

MATERIAL

The GR4J rainfall–runoff model

The model used here is GR4J, a parsimonious daily lumped continuous rainfall–runoff model with four parameters to calibrate. Perrin et al. (2003) provide a detailed description of the model. On gauged catchments, four parameters are calibrated (see Table 1) using a local search procedure. The objective function \((OF)\) is the Nash & Sutcliffe (1970) criterion computed on root square transformed flows \(\sqrt{Q}\), which was shown to be a good compromise between several alternative criteria (Oudin et al., 2006):

\[
OF(\%) = 100 \left( 1 - \frac{\sum_j \left( \sqrt{Q_j} - \sqrt{\hat{Q}_j} \right)^2}{\sum_j (\sqrt{Q_j} - \sqrt{\bar{Q}})^2} \right)
\]

in which the summation is made on all the time steps \(j\) of the period where observed \(Q_j\) and computed \(\hat{Q}_j\) flows are available and where \(\sqrt{\bar{Q}}\) is the mean of root square transformed flows over the test period.

| \(X_1\) | Capacity of the production reservoir (mm) |
| \(X_2\) | Intercatchment groundwater flows magnitude (mm) |
| \(X_3\) | Capacity of the nonlinear routing reservoir (mm) |
| \(X_4\) | Unit hydrograph time base (day) |

Catchment set

We used a database of 1040 French catchments (see map in Fig. 1) with daily rainfall, runoff and Penman potential evapotranspiration (PE) data over the 1995–2005 period. Hydrometeorological conditions and catchment physical characteristics are quite diverse. For example, the database contains 162 small Mediterranean catchments as well as larger, groundwater-dominated basins or highland catchments. Catchments where snow has a major hydrological role were excluded.
METHODOLOGY

Regionalization using regression

The approach based on regression is the most popular among regionalization methodologies. It is a reference against which any new method should be tested. Here, regression equations are based on only four catchment descriptors: catchment area ($A_c$), mean annual rainfall ($R_m$), mean annual potential evapotranspiration ($E_m$), aridity index ($E_m/R_m$) and the probability of having a rainy day ($P_w$). We chose not to include land cover characteristics since Oudin et al. (2006) showed that they are not significantly correlated with GR4J model parameter values. Other potentially interesting characteristics such as soil types, slope and elevation were not available over the catchment set. Two regression methodologies can be followed:

**Option 1: Regression with local calibration** It consists in first calibrating the model for each catchment independently and then trying to relate the calibrated model parameters with the catchment characteristics using regression methods. Some problems may arise in this approach if model parameters are not well-defined. In this case, different sets of parameters may give the same model performance and therefore, the relationship obtained is likely to be weak.

**Option 2: Regression with predefined calibration** To reduce this possible bias, we also tested the alternative approach proposed by Hundecha & Bárdossy (2004). First, we assume the functional form of the relationship between the catchment characteristics and the model parameters and then we calibrate the model for many
catchments simultaneously:

\[
OF = \sum_k OF_k(X_k)
\]

\[
X_k = a \cdot A_k^* + b \cdot R_m^k + c \cdot E_m + d \left( \frac{E_m}{R_m} \right)^k + e \cdot P_w + f
\]  

(2)

where \(OF\) is the sum of the objective functions obtained on all catchments and \(X_k\) is the vector of model parameters, which are linear combinations of catchments descriptors. During the calibration process, \(OF\) is optimized by varying the coefficients \((a, b, c, d, e, f)\) of the linear combination.

**Regionalization using catchment physical similarity**

The similarity approach consists in using the parameters obtained in gauged catchments that are similar to the ungauged catchment in terms of catchment descriptors. This method was developed and tested by Rojas-Serna (2004) who combined this \textit{a priori} approach with calibration on point flow measurement. It is also very similar to the approach followed by McIntyre \textit{et al.} (2005) who also included prior likelihood based on the GLUE framework (Beven & Freer, 2001).

Figure 2 presents a scheme of the method when two catchment characteristics are considered. We tested combinations of two to three characteristics among: catchment area \((A_c)\), mean annual rainfall \((R_m)\), mean annual potential evapotranspiration \((E_m)\), aridity index \((E_m/R_m)\) and the probability of having a rainy day \((P_w)\). For a given ungauged catchment and a selected catchment descriptor, we look for the nearest \(m\) catchments in the distribution of the 1039 catchment descriptors of the gauged catchments.

When using these donor catchments, two options are considered. The first one consists in averaging model parameters obtained by calibration for the donor catchments and the second one consists in averaging model outputs obtained with the sets of parameters calibrated over the donor catchments.

**Option 1: Model parameter averaging** A regional set of parameters is computed as the mean of the parameter sets of donor gauged catchments and is then applied on the ungauged catchment. Thus streamflow for day \(j\) is computed as:

\[
\hat{Q}(j) = \hat{Q} \left( \frac{\sum_{i=1}^{m} X_i}{m} \right)
\]

(3)

where \(m\) is the number of donor gauged catchments, the optimal value being 20 for our catchment set, and \(X_i\) is the vector of parameters for the donor gauged catchment \(i\).

**Option 2: Model outputs averaging** A regional computed streamflow is derived from the simulations obtained with the sets of parameters of the donor gauged catchments. Thus streamflow for day \(j\) is computed as:

\[
\hat{Q}(j) = \frac{1}{m} \sum_{i=1}^{m} \hat{Q}(j, X_i)
\]

(4)
Assessment

The criterion used to assess model efficiency on the validation periods is the Nash & Sutcliffe (1970) criterion:

\[
\text{NSE} = 100 \left[1 - \frac{\sum_j (Q_j - \hat{Q}_j)^2}{\sum_j (Q_j - \bar{Q})^2}\right]
\]

in which the summation is made on all the time steps \( j \) of the period where observed \( Q_j \) and computed \( \hat{Q}_j \) flows are available. This criterion varies between \(-\infty\) and 100\% (perfect simulation), 0 corresponding to the error level of a naïve model that simulates a flow equal to the mean observed daily flow at each time step.

RESULTS

In this section, we compare the performance of the several tested approaches for regionalization. Two reference efficiencies are also considered: the model performance in calibration and the model performance with the median of the four parameters over the 1040 catchments.

Figure 3 shows the range of model efficiency obtained with the similarity approach when choosing different catchment characteristics to assess the similarity between
two catchments. The optimal setting consists in using three combined descriptors: catchment area ($A_c$), aridity index ($E_m/R_m$) and potential evapotranspiration ($E_m$). However, note that many combinations can give as satisfactory efficiencies as the optimal one.
Figure 4 summarizes the distributions of the model efficiency obtained by the several regionalization approaches. The results indicate that methods based on physical similarity perform slightly better than regression-based methods. For the physical similarity approach, transposing the complete set of model parameters from donor catchments (i.e. output averaging) is a little less efficient than averaging model parameters. For regression based approaches, simultaneously calibrating all catchments (predefined calibration) yields similar results as using locally calibrated sets of parameters. This may be due to the fact that interactions between the four parameters of GR4J are weak given the low number of calibrated parameters.

CONCLUSION

The aim of this paper was to compare two classical options to regionalizing rainfall–runoff model on ungauged basins: regression-based approaches and an approach based on physical similarity. Results show that the two approaches only slightly improve the simplest regionalization scheme that consists in using median values for model parameters. One reason for the low performance of the regionalization approaches may be the absence of key physical descriptors, such as soil type information. This lack of information may affect, in particular, the regionalization of the parameters linked to the capacities of the two reservoirs ($X_1$ and $X_3$) of the GR4J model.

Regionalization based on physical similarity, which appears conceptually more satisfying, is slightly better than regression. The refinements of these methods like regional calibration of regressions or multi-model considerations for regionalization based on physical similarity, do not yield significant improvements.

The low level of success of the tested regionalization schemes is comparable to other studies on large sample of catchments (Merz & Blöschl, 2004; McIntyre et al., 2005; Young, 2006). When comparing results with other such studies, it should be acknowledged that we did not have information on soil types, which could allow improvement of the regionalization results.

Moreover, given the density of the basin network, a simple spatial proximity approach should be tested as a baseline for comparison. Tests to address this issue are ongoing.

REFERENCES


