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The use of physical basin properties and runoff generation concepts as an aid to parameter quantification in conceptual type rainfall–runoff models

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Abstract A physically-based approach to estimating the soil moisture accounting and runoff parameters of a conceptual, monthly time-step rainfall–runoff model is proposed. The approach is based on a conceptual interpretation of the model parameters taking into account the spatial and temporal scales used in typical model applications. The results of applying the approach to five test basins in South Africa suggest that while very different parameter sets are obtained compared to existing parameter regionalization methods, the simulated flows are similar. While current estimates of the physical basin properties are somewhat subjective, the estimation approaches have the potential to make use of remotely sensed soil and soil moisture information that is likely to become more readily accessible in the future.

Key words hydrological models; parameter estimation

INTRODUCTION

For the past three decades many water resource planning and management decisions in southern Africa have been based on the original Pitman monthly rainfall–runoff model (Pitman, 1973) or recent revisions (Hughes, 2004). It is a conceptual type model and within South Africa, parameter estimation approaches for ungauged catchments have traditionally been based on somewhat subjective regionalizations using areas of assumed hydrological similarity (Midgley *et al.*, 1994), supported by calibration against a limited amount of gauged streamflow data. While alternative parameter estimation approaches have been proposed (Mazvimavi, 2003) they have either been applied to limited parts of the region, or have met with limited success. The direct estimation of the parameters of this type of model without calibration (Johansson, 1994) has not been adequately explored but could have potential in the light of new remote sensing data products that are becoming more readily accessible worldwide (Ottlé & Vidal-Madjar, 1994). To realize this potential it would be necessary to establish linkages between model parameter values and basin physical properties. This paper presents part of a study to re-investigate the physical meaning of the model parameters, taking into account the conceptual framework, as well as the spatial and temporal scales at which the model is designed to function.

The structure of the model has been described in previous publications (Pitman, 1973; Hughes, 2004; Hughes *et al.*, 2006) and is not repeated here. There are three runoff mechanisms: *surface runoff* based on a triangular distribution function approach to catchment adsorption; *soil moisture runoff* generated as a nonlinear function of the main moisture storage, which also determines groundwater recharge; *groundwater runoff* based on recharge to a conceptual ground water store (see Hughes, 2004, for details). There are a total of 23 parameters in the version of the model being used and the focus of this paper is on the six parameters that control the soil moisture accounting, runoff and recharge routines.

SOIL MOISTURE ACCOUNTING, RUNOFF AND RECHARGE ROUTINES

Within the Pitman model the main soil moisture runoff (SQ) and groundwater recharge (RQ) routines are controlled by two similar nonlinear equations:

$$SQ = FT \times [S/ST]^{POW} \tag{1}$$

$$RQ = GW \times [(S - SL)/ST]^{GPOW} \tag{2}$$

where ST is maximum soil moisture storage (mm), S is current soil moisture storage (mm), SL is soil moisture storage (mm) below which recharge equals = 0, FT is maximum soil moisture runoff (mm) at $S = ST$, and GW is maximum recharge (mm) at $S = ST$. POW and $GPOW$ are the powers of the relationships.

The objective of the study is to determine physically-based approaches to estimate the parameters of these equations, given information about the basin soil, geology and topographic characteristics. However, a review of the conceptual understanding of the soil moisture runoff function is first required. Large parts of southern Africa are underlain by fractured rock aquifers with unsaturated zones of quite substantial depth (>10 m). The recharge process is likely to occur primarily through the fracture zones, many of which can be saturated during wet periods. In areas of relatively steep topography there is expected to be a lateral flow component which could result in some of the recharging water re-emerging in valley bottoms and contributing directly to streamflow. The volumes of water that could contribute to this unsaturated zone interflow will depend on the fracture density and alignment as well as the surface topography. The implication from the point of view of the conceptual model framework is that ST represents storage in the soil and the unsaturated zones (i.e. $ST = ST_{\text{soil}} + ST_{\text{unsat}}$) and that FT represents the maximum flow out of both zones (i.e. $FT = FT_{\text{soil}} + FT_{\text{unsat}}$).

Estimating *ST* The soil moisture component of ST will be related to soil depth and texture. Table 1 suggests a scheme for estimating ST_{soil} based on soil depth and porosity relationships with texture (Rawls *et al.*, 1982; Schulze *et al.*, 1985). The unsaturated zone component (ST_{unsat}) will be related to the degree of fracturing, average depth to water table and basin topography. The latter has been included as it will only be that part of the fracture storage that can potentially contribute to lateral flow that will be relevant to the estimate.

Estimating *FT* The assumptions used for the estimation of the "soil" component of FT is that when the basin is saturated (at ST), soil water outflow will be occurring through both banks of the active channel, with a contributing area A ($\text{m}^2\ \text{km}^{-2}$) of:

$$A = 2 \times DD \times SD \tag{3}$$

Table 1 Suggested scheme for estimating the soil moisture component of *ST* (ST_{soil} in mm of storage).

Soil texture	Soil depth: Shallow	Shallow – moderate	Moderate	Moderate – deep	Deep
Loamy Sand	<80	80–200	200–401	401–802	>802
Sand Clay Loam	<66	66–165	165–330	330–660	>660
Clay Loam	<78	78–190	190–390	390–780	>780

Note: The following soil depths are assumed: shallow <0.2 m, shallow to moderate 0.2–0.5 m, moderate 0.5–1.0 m, moderate to deep 1.0–2.0 m, deep >2.0 m.

Table 2 Suggested scheme for estimating the soil moisture component of *FT* (values in mm month^{-1}).

Drainage density	Low = 1.5			High = 2.5		
Soil depth	Shallow	Moderate	Deep	Shallow	Moderate	Deep
Texture and hillslope gradient						
Loamy Sand/5%	1.0	3.0	6.0	1.7	5.0	10.0
Loamy Sand/10%	2.0	6.0	12.0	3.4	10.0	20.0
Loamy Sand/20%	4.0	12.0	24.0	6.7	20.2	40.5
Sand Clay Loam/5%	0.6	1.7	3.4	0.9	2.8	5.6
Sand Clay Loam/10%	1.1	3.4	6.7	1.9	5.6	11.2
Sand Clay Loam/20%	2.2	6.8	13.5	3.7	11.2	22.5
Clay Loam/5%	0.4	1.2	2.4	0.7	2.0	4.0
Clay Loam/10%	0.8	2.4	4.7	1.3	4.0	7.9
Clay Loam/20%	1.6	4.7	9.4	2.6	7.9	15.8

Note: The following soil depths are assumed: shallow 0.25 m, moderate 0.75 m, deep 1.5m.

The drainage density (*DD*) represents the non-dimensional channel length (km km^{-1}), while the soil depth (*SD*) represents the maximum possible depth of the contributing soil layer (mm). The rate of outflow (mm month^{-1}) will be the product of contributing area, saturated hydraulic conductivity (*K* in units of m d^{-1}) and hillslope gradient (*HG*):

$$FT_{\mathrm{soil}} = A \times K \times HG \times 30/1000 \tag{4}$$

Table 2 suggests values based on typical hydraulic conductivities (Cosby *et al.*, 1984) and the assumption that appropriate values for drainage density would be high and include many contour convergence zones experiencing surface flow during saturated conditions.

Estimating the unsaturated zone component of *FT* is more difficult as the physical concepts of runoff generation are less well defined and typical hydraulic conductivities of fracture zones are not well documented. While the contributing area (depth to groundwater) and the conductivity of individual factures may be high, the effective discharge will be much lower as the saturated fractures will only represent a small proportion of the total contributing area. Values of FT_{unsat} in the range of 0 to 20 mm month^{-1} are suggested, depending upon the fracture density and pressure head in the fractures of the unsaturated zone.

Estimating *POW* *POW* is the power of the relationship between *SQ* and *S* and is made up of two components associated with soil water and the unsaturated zone runoff. The proposed approach to the soil water component is to assume that for a given mean soil water content (*S*) there will be a frequency distribution that represents the spatial variation of water content within the basin (similar to the probability distributed principle of Moore, 1985). The variability is assumed to be low at both low and high mean moisture contents, when the basin is either uniformly dry or wet, and highest at moderate mean moisture contents. It should be possible to define the distribution shapes from detailed field observations, but such information is not currently available and a simple approach has been initially adopted. The distribution is defined by two combined triangles, one with a base length 1.5 times and height 0.5 times the other. The base length (BL_1) of the narrow triangle varies between a maximum value (BL_1max) at a mean moisture content (S/ST) of 0.5 and a minimum value (BL_1min) when the mean moisture content is both 0.0 and 1.0. Between these extremes the base length is assumed to vary using a sine curve function. The triangle heights are calculated on the assumption that the combined triangle area is equal to 1.0, representing the total basin area.

A value of $S/ST = 0.9$ is defined (fairly arbitrarily) as close to saturation and the threshold for soil moisture runoff to occur. Simple geometry can be used to find the proportion of the area of the triangles (and therefore the basin) that exceeds this value for any mean moisture content. This area is assumed to be SQ/FT. Figure 1 shows several examples of the resulting relationship between mean moisture content and relative runoff, as well as equivalent curves using the model algorithm ($SQ/FT = (S/ST)^{POW}$). It is more difficult to suggest an approach for the unsaturated zone runoff-moisture content relationship, and in the absence of better information, it has been assumed to follow a fixed nonlinear relationship of the form $SQ_{\text{unsat}}/FT_{\text{unsat}} = (S/ST)^2$.

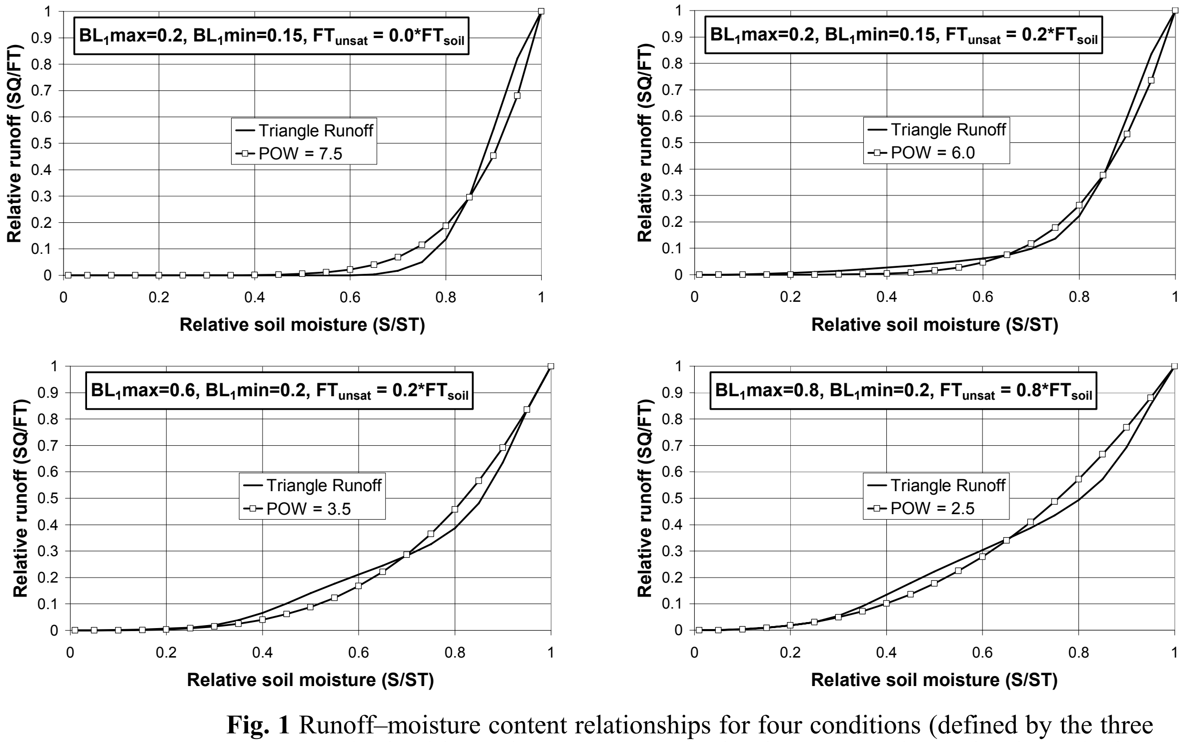

Fig. 1 Runoff–moisture content relationships for four conditions (defined by the three parameters in the titles). The *POW* parameter has been set to give a closely equivalent relationship.

Figure 1 illustrates the results for different combinations of the BL_1max and BL_1min parameters in the triangle base length equation and FT_{soil}/FT_{unsat} proportions. The assumption is that the parameters used to determine variation in triangle base length will vary with the physical characteristics of the basin and the way in which they affect the rate of moisture re-distribution. Basins with low relief and/or soils with low conductivities will be expected to have low rates of re-distribution and therefore low BL_1max and BL_1min parameters (top left in Fig. 1). Conversely, steep catchments with highly conductive soils will have a high BL_1max parameter (bottom right in Fig. 1, for example). Figure 1 includes the model relationship using appropriate *POW* values and it is worth noting that some of the *POW* values are much higher than have been traditionally used with the Pitman model.

Estimating *GW* and *GPOW* The proposed approach to estimating *GPOW* is the same as for *POW*, except that the relative moisture (S/ST) threshold for the calculation of the contributing basin area is set to a value appropriate to the field capacity of the basin soils. Estimating the value of *GW* is more difficult as it involves the complexities of vertical drainage through the total unsaturated zone. There are, however, existing estimates of mean annual recharge available for South African basins (Conrad, pers. comm.) which can be used to calibrate the value of *GW*.

TEST CASE STUDIES

While the main study is investigating a large number of basins, there is only space in this contribution to refer to several examples. Five gauged basins are briefly described in Table 3, while Table 4 presents the previous and revised parameter values, as well as estimates of the physical basin properties. Most of the previous parameter values have been taken from Midgley *et al.* (1994), while the additional parameters associated with the revised groundwater functions of the model (Hughes, 2004) have been based on information provided by Conrad (pers. comm.) from the Groundwater Resource Assessment project of the Department of Water Affairs and Forestry. Apart from the parameters listed in Table 4, all other parameter values are the same for the two model runs. The same hydrometeorological inputs (rainfall and evaporation demand) have been used for the different versions of the model. Table 4 includes two statistics of correspondence between observed and simulated monthly flows; the coefficient of efficiency (Nash & Sutcliffe, 1970) based on un-transformed (CE) and natural log transformed (CE{ln}) data.

The drainage density values were estimated from drainage lines identified on 1:250 000 scale maps and corrected using sample checks from 1:50 000 maps. On average a correction factor of three times the 1:250 000 estimate was found to be appropriate. Valley side gradients were also estimated from topographic maps. The estimates of slopes and drainage density could clearly be improved through the use of digital terrain data. Soil depths were estimated from personal experience and information provided in Midgley *et al.* (1994). The soil depths given in Table 4 were used in the estimation of FT_{soil}, while reduced values (60%) were used in the estimation of ST_{soil} to account for lower depths appropriate to basin averages. Soil hydraulic conductivity (K_{soil}) and porosity were estimated using soil texture information provided in Midgley *et al.* (1994), FAO soil maps and Cosby *et al.* (1984). The

Table 3 Example basins.

Gauge/basin code	Area (km^2)	Description
K4H003 / K40A	72	Steep topography, shallow sandy and stony soils, fractured quartzite. Present day impacts of pine plantations.
V7H016 / V70B	121	Steep topography, moderate to shallow clayey soils, interbedded mudstones, shales and sandstones.
C1H004 / C12D	898	Undulating topography, moderate to deep clayey soils, interbedded shales and sandstones.
X1H016 / X12A-C	581	Undulating topography, moderate to deep sandy loam soils, gneiss and ultrametamorphic geology
W5H005 / W52A-C	804	Undulating topography, moderate sandy loam soils, interbedded sandstones and shales.

Table 4 WR90 and revised parameters, estimated physical properties and model results.

Basin	K4H003	V7H016	C1H004	X1H016	W5H005
MAP (mm)	702	1093	661	868	832
WR90 parameters and results					
ST	100	100	45	150	180
FT	50	50	2	24	15
POW	2.0	3.0	3.0	2.0	3.0
GW	50	20	5	18	15
ZMIN	0	999	999	999	0
ZMAX	200	999	999	999	900
CE : CE{ln}	0.66 : 0.57	0.69 : 0.79	0.61 : 0.52	0.24 : 0.61	0.62 : 0.72
Estimated physical properties, revised parameters and results					
DD	2.1	2.3	1.4	1.8	1.6
SD (m)	0.6	0.8	1.5	1.8	1.0
K_{soil} (m d^{-1})	1.85	0.47	0.16	0.81	0.81
HG	0.30	0.30	0.10	0.12	0.10
Porosity	0.41	0.35	0.35	0.35	0.35
ST_{soil}	148	224	420	378	210
FT_{soil}	41.5	15.5	2.0	18.5	8.0
GW depth (m)	30	20	10	15	10
K_{unsat} (m d^{-1})	0.02	0.01	0.015	0.05	0.01
GW storativity	0.002	0.002	0.003	0.002	0.002
ST_{unsat}	60	40	30	34	20
FT_{unsat}	7.5	2.5	0.5	3.0	0.5
POW	2.0	2.8	5.0	3.0	3.5
GW	50	20	10	22	15
ZMIN	10	30	50	50	0
ZMAX	200	400	350	550	650
CE : CE{ln}	0.66 : 0.67	0.68 : 0.80	0.63 : 0.60	0.36 : 0.69	0.58 : 0.73

groundwater physical properties are extremely difficult to estimate as there is almost no appropriate information available. The values given in Table 4 are therefore very uncertain approximations based on perceived degrees of fracturing. *POW* values were estimated using the approach described above and illustrated in Fig. 1, with the BL_1max and BL_1min values based on soil texture and hillslope gradients. Once the *ST*, *FT* and *POW* parameters were established (and any adjustments made to *GW* to ensure similar recharge estimates), the surface runoff parameters (ZMIN and ZMAX) were adjusted to ensure correspondence between observed and simulated mean monthly flow volumes.

The CE and CE{ln} statistics given in Table 4 suggest that in almost all the examples the revised parameters are at least as acceptable as the original parameter sets. In all cases the revised *ST* values are substantially higher, while the *FT* values are generally lower. In the revised parameter set these differences have been compensated for by generating more runoff through the surface runoff function involving ZMIN and ZMAX.

DISCUSSION AND CONCLUSION

Perhaps the first observation is that there is very little information readily available with which to apply the new estimation procedures. There is therefore a great deal of uncertainty about the estimates of the physical basin properties and specifically those related to the unsaturated zone components of *ST* and *FT*. However, despite these problems it is thought that the revised parameter sets are generally more physically realistic than the original ones. For example, it is not really sensible to suggest that a basin with undulating topography and moderate to deep clayey soils (C1H004) would only have a storage depth of 45 mm and that no surface runoff would be generated in an area dominated by high intensity convective rainfall (C1H004 in Tables 3 and 4).

Clearly this initial part of the study has only addressed part of the parameter estimation question and there are a number of outstanding issues that need to be investigated, apart from extending the analysis to a greater number of basins. Estimation procedures have only been developed for some of the parameters (*ST*, *FT*, *POW*), while others have been manually re-calibrated (ZMIN and ZMAX) and others left unchanged from the original calibrations (Midgley *et al.*, 1994). It is important therefore to extend the estimation procedures to include at least ZMIN and ZMAX, as well as some of the parameters controlling the relationship between storage and evaporation losses. Rough comparisons of the re-calibrated ZMIN and ZMAX values given in Table 4 with basin characteristics suggest that the development of appropriate estimation procedures might be possible. The low values for the basin K4H003, for example, are consistent with the thin and stony hilltop soils found in this basin.

The biggest problem with a study of this type is the lack of readily available information on which to base consistent estimates of basin physical properties. The approach used during this initial study is far too subjective and further information coupled with more objective methods is required. It is expected that such information will become available for the southern Africa region within the near future. One example would be the use of remote sensing data to improve understanding of soil moisture spatial variations (Ottlé & Vidal-Madjar, 1994) and therefore the shape of *SQ*/*FT* versus *S*/*ST* relationships (see Fig. 1). It is assumed that it will always be relatively difficult to obtain appropriate information on the characteristics of the unsaturated zone (depth, fracture density, storativity, etc.), while soils data might be more readily available, but not always in a suitable form. A recently proposed project is aimed at improving the hydrological interpretation of existing soils data and developing new approaches for using such information in models. It is possible that similar initiatives may develop if the value of the resulting data can be realized through improved water resource estimation within the region.

The overall conclusion is that, despite the existing constraints on the application of the proposed method, the results are sufficiently encouraging to justify further research. The direction of the future research must be to identify potential sources of physical basin information and the development of objective and consistent methods of using such information.

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