Evaluation of the hillslope-storage Boussinesq model with leakage

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Abstract The hillslope-storage Boussinesq (hsB) model (Troch *et al.*, 2003) has been extended to allow for leakage through a hypothetical aquitard at the hillslope bottom. A leakage term has been incorporated by extending the mass balance and combining it with the Boussinesq equation. To evaluate the extended hsB model, simulated water table levels are compared with results of a three-dimensional surface–subsurface flow model solving the Richards equation. Numerical experiments are performed on an inclined straight hillslope. Results from the extended hsB model are in reasonable agreement with those from the benchmark model, indicating that the hsB model can be used to simulate recharge to a deep semi-confined aquifer. The leakage rates calculated by the 3-D model show significant spatial variability, indicating the requirement for further extension of the hsB model to account for spatially distributed leakage.

Key words subsurface flow; hillslope drainage; Boussinesq equation; leakage

INTRODUCTION

Coupled surface water–groundwater models are increasingly used in studies of water cycle dynamics at the watershed scale. These models often incorporate a parsimonious representation of aquifer flow, including subsurface and deep groundwater contributions to river flow. A possible candidate to the simulation of subsurface flow is the one-dimensional (1-D) hillslope-storage Boussinesq (hsB) model, which has recently been the object of significant extensions and improvements. The latest versions of the hsB model now consider non-constant bedrock slopes (Hilberts *et al.*, 2004), as well as drainable porosity variations in the unsaturated zone (Hilberts *et al.*, 2005). The low-dimensional nature, physical basis, and sparse parameter needs of the hsB model make it appealing for application to a large variety of real-world hydrogeological problems. The increasing interest in modelling regional-scale catchments for integrated water management purposes motivates further development of the hsB model for an eventual coupling with a surface hydrological model and with a deep aquifer model.

This paper deals with the extension of the hsB model (1-D) to account for leakage through the hillslope bottom, expanding the model's applicability to simulate recharge to deep aquifers through the incorporation of an additional source/sink term in the subsurface flow mass balance equation. This question has been addressed by Koussis *et al.* (1998), but solely for the linearized Boussinesq model. Simulated water tables are compared to those calculated with CATHY (CATchment HYdrology), a coupled surface–subsurface model (Bixio *et al.*, 2000) based on the 3-D Richards equation, and taken to be the benchmark for comparison purposes (Paniconi *et al.*, 2003).

MATERIALS AND METHODS

Incorporation of the leakage term

The leakage concept has been extensively explored and applied in different contexts: in urban water management to assess sewage water exfiltration from pipes to the soil and *vice versa* (Karpf & Krebs, 2005); in geotechnical engineering to predict possible aquifer contamination below landfill liners (Foose *et al.*, 2001); and in hydrogeology for pumping test analysis in leaky aquifers (Hantush & Jacob, 1954).

The leakage concept is based on Darcy's law, and allows a vertical groundwater transfer between two aquifers bounding an aquitard:

$$L = -K_v \frac{h_2 - h_1}{D} = C(h_2 - h_1) \tag{1}$$

where L [L T^{-1}] is the leakage flux, K_v [L T^{-1}] and D [L] are the vertical hydraulic conductivity and thickness of the aquitard, and h_1 and h_2 [L] are the heads in the aquifers bounding the aquitard. Parameter C is the conductance, commonly called the leakage coefficient [T^{-1}].

In the hillslope-storage Boussinesq model a width function was incorporated into the classical Boussinesq equation in order to extend its applicability to hillslopes of arbitrary geometry. In this work, a leakage term has now been added, so that the mass balance equation describing subsurface flow along a hillslope of variable geometry can be written as:

$$\frac{\partial S}{\partial t} = -\frac{\partial Q}{\partial x} + Nw - Lw \tag{2}$$

where S [-] is the storage, Q [L³/T] is the subsurface flux, N [L/T] is the recharge, L [L/T] is the leakage through the hillslope bottom, w [L] is the hillslope width, t [T] is the time, and x [L] is the distance to the outlet along the hillslope.

The subsurface flux for the hillslope-storage Boussinesq model is:

$$Q = -\frac{KS}{f}\left[\cos i \frac{\partial}{\partial x}\left(\frac{S}{fw}\right) + \sin i\right] \tag{3}$$

where K [L/T] is the hydraulic conductivity, f [-] is the drainable porosity, and i [rad] is the slope angle.

The extended hsB equation accounting for leakage through the hillslope bottom is obtained by a combination of equations (2) and (3):

$$f\frac{\partial S}{\partial t} = \frac{K\cos i}{f}\frac{\partial}{\partial x}\left[\frac{S}{w}\left(\frac{\partial S}{\partial x} - \frac{S}{w}\frac{\partial w}{\partial x}\right)\right] + K\sin i\frac{\partial S}{\partial x} + fNw - fLw \tag{4}$$

In this study, the drainable porosity is kept constant, and therefore independent of the leakage. Equation (4) is discretized in the spatial coordinate ($\Delta x = 1$ m) and then solved using a variable-order ordinary differential equation solver based on the numerical differentiation formulas. It should be noted that leakage is a source/sink term, thereby providing the possibility to represent return flow from a possible deep aquifer.

Behaviour of the hsB model with leakage

Figure 1 depicts water tables and outflow rates calculated by the hsB model with constant leakage rates applied over the entire simulation time of 100 days, with a dry hillslope as initial condition,

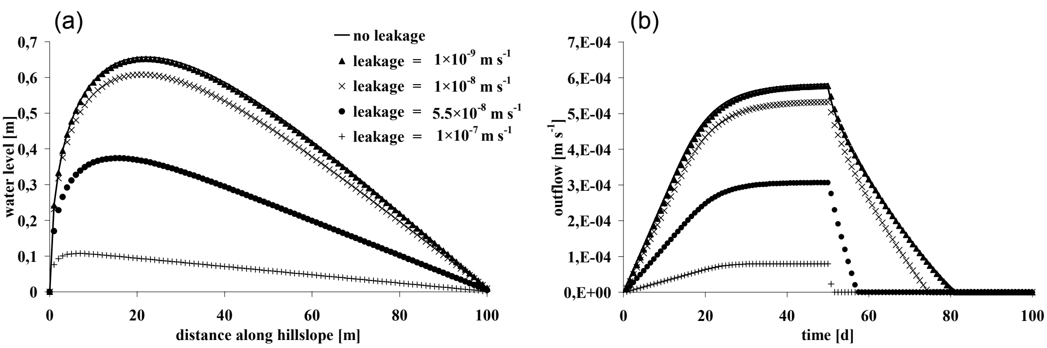

Fig. 1 Comparison of water table profiles at t = 50 days (a), and outflow rates (b), calculated with the hsB model for different constant leakage rates.

and recharge of 10 mm/day applied for the first 50 days, and zero recharge until the end of the simulation, conditions selected to best demonstrate the hsB model performance. As one can expect, leakage can have a significant impact on calculated water tables and outflow rates.

Model set-up for comparison of hsB and CATHY models

Preliminary evaluation of the 1-D hsB model with leakage was performed on a straight hillslope with planar geometry and a constant bedrock slope of 5% (Fig. 2(a)). The hillslope has a length of 100 m and a 2 m soil depth, corresponding to a shallow phreatic aquifer. The spatial discretization is $\Delta x = 1$ m. The hsB simulations are run for 10 days, with a 1 h time step. In this paper, a drainage scenario is performed.

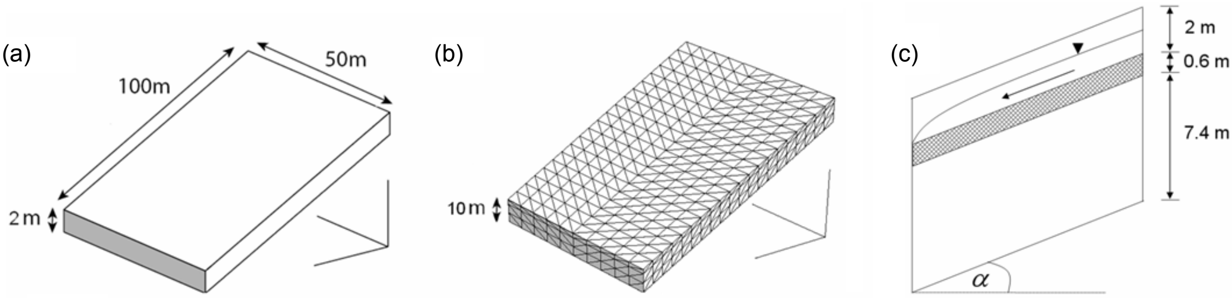

Fig. 2 Discretization of the straight hillslope for: (a) the hsB model, (b) the CATHY model (adapted from Hilberts *et al.*, 2007), and (c) vertical cross-section of the CATHY model.

The set-up for the 3-D CATHY model consists of a hillslope with the same horizontal extent (Fig. 2(b)). Vertically, a total depth of 10 m is applied, discretized in 20 layers with varying thicknesses and including the aquitard. The first 2 m of the hillslope represent the equivalent of the hsB hillslope, where unconfined groundwater flow occurs in a shallow phreatic aquifer. The next 0.6 m represents the aquitard, followed by 7.40 m representing the deep aquifer (Fig. 2(c)). Horizontally, a $\Delta x = 0.50$ m was used. The entire domain consists of 144 000 tetrahedral elements and 29 547 nodes.

In the hsB model, the storage is set to zero at the downslope limit. In the CATHY model, a fixed head boundary condition (Dirichlet type) ranging from 2 m to 2.80 m depth is assigned at the downslope end, corresponding to a river with a depth of 0.80 m cutting the aquifer. No flow conditions were applied for both models at the hillslope upper and lateral boundaries. The initial water table is set to 0.40 m in the hsB model, and to 8.40 m in the CATHY model (i.e. 0.40 m height within the unconfined aquifer). A sandy soil type was used for the shallow phreatic aquifer and for the deep aquifer in CATHY. Its hydraulic conductivity is 2.8×10^{-4} m s^{-1}, and its drainable porosity is 0.30. The aquitard is represented with three different hydraulic conductivities, ranging from 1×10^{-6} to 1×10^{-8} m s^{-1}.

Free drainage was first simulated with CATHY for a 10-day period. The simulated leakage rates through the aquitard were spatially averaged for each time step, adapted to the hsB reference frame to account for the hillslope inclination, and used as the leakage term (L) in a 10-day free-drainage simulation with the hsB model. Further hsB developments will provide for a stand-alone version in which the calculation of leakage is accomplished directly in hsB.

RESULTS

Figure 3 illustrates simulated water tables from the hsB and CATHY models for the free-drainage simulation. The hydraulic conductivity of the aquitard has a large influence on the simulated water table. Reasonable matches between the two models are obtained for low-conductivity aquitards for

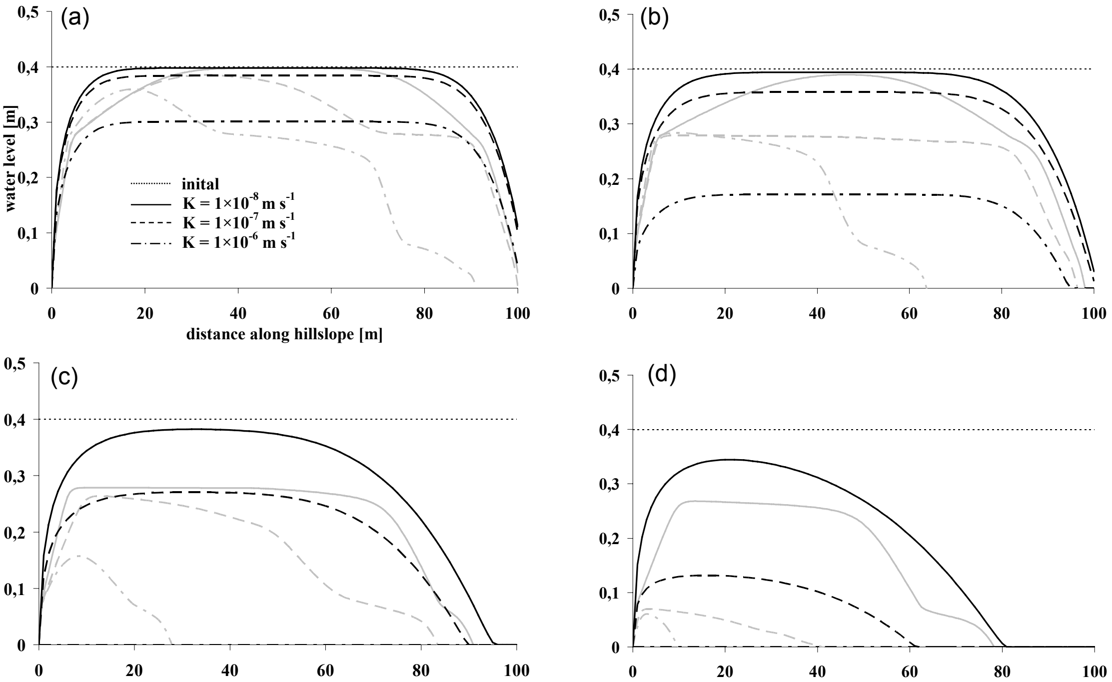

Fig. 3 Comparison of water table profiles calculated with hsB (black lines) and CATHY (grey lines) for different hydraulic conductivities of the aquitard after 1 day (a), 2 days (b), 5 days (c), and 10 days (d).

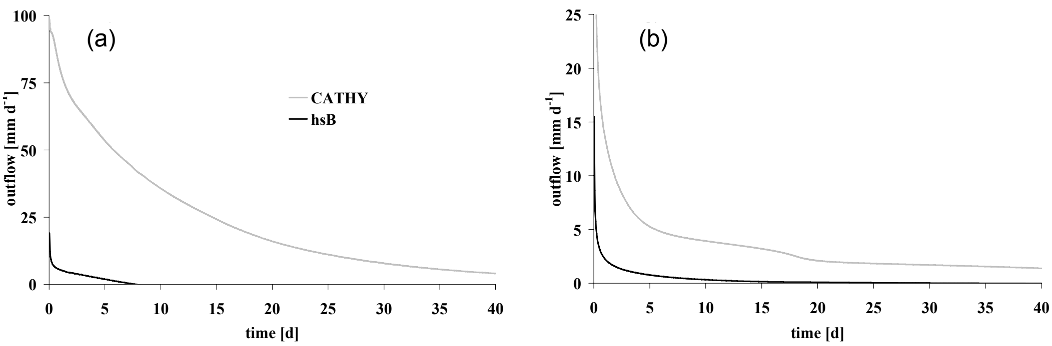

Fig. 4 Calculated outflow rates of CATHY and hsB for 5% (a) and 0.2% (b) hillslope inclination.

all time steps. When hydraulic conductivity increases, the shape and timing of the response curves are less similar.

It should be noted that the configuration of the two models is not exactly the same, thus there are several issues that require further investigation in comparing the results. For example, the outlet is represented by zero storage at the lower boundary in the hsB model, whereas in the CATHY model the Dirichlet nodes represent a river of 0.80 m depth. Tests have shown that simulated leakage rates and heads from the CATHY model are influenced by the position of the Dirichlet nodes. These differences in parameterization might explain the consistently lower CATHY water tables compared to those of the hsB model. The set-up of the Dirichlet nodes, combined with the applied hydraulic conductivity of the aquitard, causes a water table drop throughout the hillslope in comparison to those calculated by the hsB model. Additionally, the hsB model used in this study does not consider storage-dependent drainable porosity, so that systematic overestimation of water table heights with the hsB model is to be expected (Hilberts *et al.*, 2005). The wave shaped water tables calculated by CATHY are due to numerical artefacts, and are expected to disappear with a higher spatial resolution of the modelling grid.

Figure 4 depicts calculated outflow hydrographs from the two models (aquitard conductivity = 1×10^{-7} m s^{-1}). With a lower slope, the hsB outflow rates match slightly better results from CATHY, demonstrating that with an increasing slope a larger part of the deep saturated aquifer contributes to outflow. Furthermore, the CATHY model provides additional water by the unsaturated zone and capillary fringe contributing to higher outflow rates.

Figure 5 shows that nodal leakage rates along the hillslope can vary up to two orders of magnitude and differences become negligible with decreasing aquitard conductivity. The magnitude of leakage rates is controlled by the actual hydraulic heads in the unconfined aquifer, which are affected by the boundary conditions in the CATHY model. We note, for instance, that at the downslope end return flow towards the unconfined aquifer occurs. Use of a spatially averaged leakage rate in hsB therefore has an impact on the simulated water table. Both models are highly sensitive to the leaky layer parameterization (hydraulic conductivity and thickness).

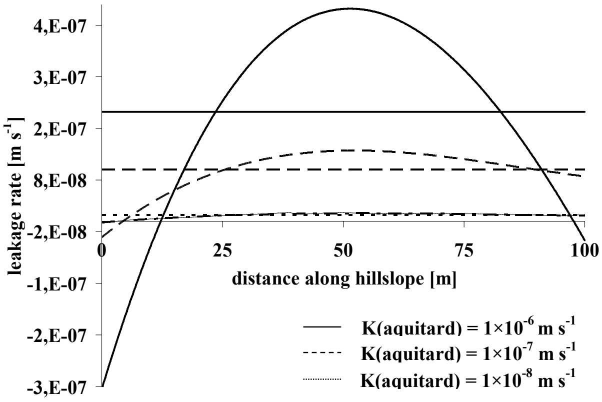

Fig. 5 Spatially averaged (straight lines) *vs* spatially distributed (curves) leakage rates calculated by CATHY at t = 5 days for different aquitard conductivities.

CONCLUSIONS AND OUTLOOK

Extension of the hsB model to allow for leakage provides the opportunity to apply this approach in real world catchments with layered unconfined/confined units. Results from this work are encouraging, but indicate the need to investigate corresponding parameterization techniques (aquitard K) as well as initial and boundary conditions in the CATHY benchmark model. These issues are the subject of ongoing work.

In a next step, a Darcy-type aquifer will be added below the aquitard in the hsB model, leading to the direct calculation of leakage flows between the shallow phreatic aquifer and the deep aquifer. Depending on the conceptualization of the deep aquifer, it might be important to account for spatially discretized leakage rates in hsB.

Further applications will focus on idealized hillslopes (straight, concave, convex) as well as on a local-scale real-world hillslopes, leading to a complete leaky hsB model. The final goal of this study is the incorporation of the hsB approach into the HYDROTEL surface flow model (Fortin *et al.*, 2001) to improve subsurface flow representation. Applications on local and regional scale catchments will be performed to test this application.

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