A variable fuzzy-sets assessment model and its application to regional water resources assessment

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Abstract Regional water resources assessments aim at estimating the current status and trends of water resources, providing a scientific basis for the development, utilization and protection of water resources, and optimizing a sustainable utilization plan. Therefore, it is essential to conduct a holistic water resources assessment. Most of the current assessment models for regional water resources only focus on water quantity and water quality. The assessment of hydroenergy resources is usually ignored. Furthermore the assessment result is only compared with that of other models without analysing the stability itself. To target these issues, a variable fuzzy-sets model for regional water resources assessment was developed. Water quantity, water quality, and hydroenergy are considered in this model, and its stability is also analysed with varying parameters. The given case study focusing on water quality shows that the variable fuzzy-sets model is useful to guide regional water resources assessment.

Key words hydroenergy; variable fuzzy-sets assessment model; water quality; water quantity; water resources

INTRODUCTION

Water is an important natural resource for human living, production, and societal development. River basins with abundant water of high quality, together with the convenience for utilization, are historically preferred places for human habitation and development. Today, unwise use of water resources has led to many problems, such as eco-environmental damage, water pollution and abnormal hydrological cycles, etc. Thus, water resources utilization should be well planned. This calls for a need to assess water resources reasonably, scientifically and holistically.

The earliest national water resources assessment was established in the USA in 1965. Later, other countries carried out similar projects, e.g. Japan (1975), India (1975), and China (1985) (Wang, *et al.*, 2006). Many models or methods were established with the development of water resources assessment, e.g. the DEA method (Charnes *et al.*, 1978), the method of fuzzy comprehensive evaluation (He, 1983), the matter-element model (Cai, 1990), the grey system theory (Deng, 1991) and the fuzzy pattern recognition theory (Chen, 1997), etc. The models were applied widely. In this paper, the variable fuzzy-sets assessment (VFSA) model (Chen, 2005) for regional water resources assessments is described. This model also considers hydroenergy besides water quantity and water quality. Stability analysis is conducted with varying parameters. A case study for the model, focusing on water quality, is applied to the water resources assessment of the Three Gorges reach. This study is anticipated to help guide future regional water resources assessments.

Principle of VFSA

The law of contradiction points out that opposite and unity relations are concurrent between contradictions, which are the power of events movement and development.

Definition of variable fuzzy sets (VFS)

Supposing that U is an opposite fuzzy concept (alternative or phenomenon), $\underset{\sim}{A}$ and $\underset{\sim}{A}^c$ denote the attractability and repellency, to any element u ($u \in U$), which falls into the intervals of $[1,0]$ ($\underset{\sim}{A}$) and $[0,1]$ ($\underset{\sim}{A}^c$), their relative membership degrees (RMD) of attraction and repellence are denoted by $\mu_{\underset{\sim}{A}}(u)$ and $\mu_{\underset{\sim}{A}^c}(u)$, respectively (Chen, 2005). Let:

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$$\mu_{\underset{\sim}{A}}(u)+\mu_{\underset{\sim}{A}^c}(u)=1 \tag{1}$$

The boundary conditions are:

$$0\le\mu_{\underset{\sim}{A}}(u)\le 1,\ 0\le\mu_{\underset{\sim}{A}^c}(u)\le 1 \tag{2}$$

Let:

$$\underset{\sim}{V}=\{(u,\mu)\mid u\in U,\mu_{\underset{\sim}{A}}(u)+\mu_{\underset{\sim}{A}^c}(u)=1,\mu\in(0,1]\} \tag{3}$$

Here $\underset{\sim}{V}$ is the VFS of U. The attraction sets, repellence sets and balance boundary or qualitative change boundary are:

$$A_{+}=\{u\mid u\in U,\mu_{\underset{\sim}{A}}(u)>\mu_{\underset{\sim}{A}^c}(u)\} \tag{4}$$

$$A_{-}=\{u\mid u\in U,\mu_{\underset{\sim}{A}}(u)<\mu_{\underset{\sim}{A}^c}(u)\} \tag{5}$$

$$A_{0}=\{u\mid u\in U,\mu_{\underset{\sim}{A}}(u)=\mu_{\underset{\sim}{A}^c}(u)\} \tag{6}$$

Suppose that C is the variable factor sets of $\underset{\sim}{V}$:

$$C=\{C_A,C_B,C_C\} \tag{7}$$

where C_A are the model variable sets; C_B are the model parameters variable sets (e.g. weight); C_c are a variable other factors sets, except model and its parameters.

The qualitative change sets of VFS $\underset{\sim}{V}$ to variable elements sets C are:

$$A^{+}=C(A_{-})=\{u\mid u\in U,\mu_{\underset{\sim}{A}}(u)<\mu_{\underset{\sim}{A}^c}(u),\mu_{\underset{\sim}{A}}(c(u))>\mu_{\underset{\sim}{A}^c}(c(u))\} \tag{8}$$

$$A^{-}=C(A_{+})=\{u\mid u\in U,\mu_{\underset{\sim}{A}}(u)>\mu_{\underset{\sim}{A}^c}(u),\mu_{\underset{\sim}{A}}(c(u))<\mu_{\underset{\sim}{A}^c}(c(u))\} \tag{9}$$

The quantitative change sets of VFS $\underset{\sim}{V}$ to variable elements sets C are:

$$A_{(+)}=C(A_{+})=\{u\mid u\in U,\mu_{\underset{\sim}{A}}(u)>\mu_{\underset{\sim}{A}^c}(u),\mu_{\underset{\sim}{A}}(c(u))>\mu_{\underset{\sim}{A}^c}(c(u))\} \tag{10}$$

$$A_{(-)}=C(A_{-})=\{u\mid u\in U,\mu_{\underset{\sim}{A}}(u)<\mu_{\underset{\sim}{A}^c}(u),\mu_{\underset{\sim}{A}}(c(u))<\mu_{\underset{\sim}{A}^c}(c(u))\} \tag{11}$$

Model of relative membership function (RMF)

The model of RMF considering intervals can be expressed as follows (Chen, 2005). We suppose that $X_0=[a,b]$ is the attraction sets of VFS $\underset{\sim}{V}$ on the real axis, i.e. interval of $\mu_{\underset{\sim}{A}}(u)>\mu_{\underset{\sim}{A}^c}(u)$, and $X=[c,d]$ is a certain interval containing X_0 ($X_0\subset X$). Relationships are shown in Fig. 1.

According to the definition of VFS, intervals $[c,a]$ and $[b,d]$ are repelling sets of VFS, i.e. interval of $\mu_{\underset{\sim}{A}}(u)<\mu_{\underset{\sim}{A}^c}(u)$. Suppose that M is the point of $\mu_{\underset{\sim}{A}}(u)=1$ in attraction set $[a,b]$, according to actual problems M may not be the median value of interval $[a,b]$.

It is assumed that x is a random point in interval X, and if x locates on the left side of M, the model of RMF is as follows:

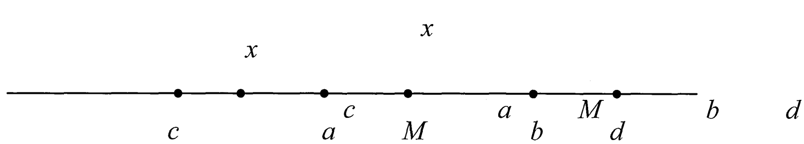

Fig. 1 Relationships between points x , M and intervals X_0, X.

$$\mu_{\underset{\sim}{A}}(u) = \begin{cases} 0.5 + (x-a)/2(M-a) & x \in [a, M] \\ 0.5 - (x-a)/2(c-a) & x \in [c, a] \end{cases} \tag{12}$$

If x locates at right side of M, the model of RMF is as follows:

$$\mu_{\underset{\sim}{A}}(u) = \begin{cases} 0.5 + (x-b)/2(M-b) & x \in [M, b] \\ 0.5 - (x-b)/2(d-b) & x \in [b, d] \end{cases} \tag{13}$$

If x locates out of interval X, the model of RMF is:

$$\mu_{\underset{\sim}{A}}(u) = 0 \qquad x \notin [c, d] \tag{14}$$

The model of VFSA

The model VFSA is expressed as follows:

$$v_{\underset{\sim}{A}}(u)_h = \left[1 + \left(d_{gh}/d_{bh}\right)^{\alpha}\right]^{-1} \tag{15}$$

where α is the rule parameter, h is grade number and h = 1, 2, …, c. Through calculating $v_{\underset{\sim}{A}}(u)_h$ of each grade, we get synthetic RMD vector $v_{\underset{\sim}{A}}(u)$.

$$d_{gh} = \left\{ \sum_{i=1}^{m} \left[w_i \left(1 - \mu_{\underset{\sim}{A}}(u)_{ih}\right) \right]^p \right\}^{1/p} \tag{16}$$

$$d_{bh} = \left\{ \sum_{i=1}^{m} \left(w_i \mu_{\underset{\sim}{A}}(u)_{ih} \right)^p \right\}^{1/p} \tag{17}$$

where i is an index number and I = 1, 2, …, m, p is distance parameter.

Generally, α = 1, 2 and p = 1, 2. α = 1 is least single model and α = 2 is least square model; p = 1 is hamming distance and p = 2 is Euclidean distance. Therefore, α and p can be arranged in four pairs: (a), $\alpha = 1, p = 1$; (b), $\alpha = 1, p = 2$; (c), $\alpha = 2, p = 1$; (d), $\alpha = 2, p = 2$.

Through normalizing $v_{\underset{\sim}{A}}(u)$, RMD $v^{\circ}_{\underset{\sim}{A}}(u)$ can be obtained, and then the rank feature values (RFV) can be calculated by the RFV equation.

$$\mathrm{H} = (1, 2, \ldots c)\ v^{\circ}_{\underset{\sim}{A}}(u)^T \tag{18}$$

In the VFSA model, model variability and model parameters variability can be used to analyse the assessment results stability.

Case study

Three sub-systems i.e. water quality, water quantity and hydroenergy, should be synthetically considered in regional water resources assessments (Fig. 2).

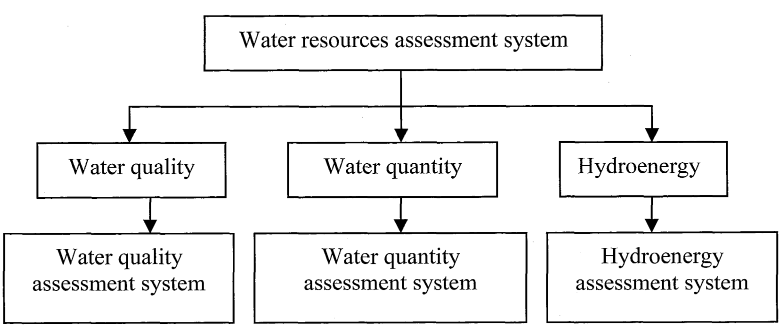

Fig. 2 Water resources assessing system.

Table 1 Index values for water resources assessment (mg/L).

No.	Cross-section	COD_{mn}	BOD_5	NH_3-N
1	Qingxichang (flood period)	8.26	1.80	1.79
2	Tuokou (flood period)	7.54	1.12	1.98
3	Saiwangba (flood period)	8.00	2.20	2.10
4	Fengjie (flood period)	8.13	2.18	2.05
5	Qingxichang (dry period)	2.71	1.79	1.98
6	Tuokou (dry period)	2.49	1.67	2.08
7	Saiwangba (dry period)	2.65	1.54	2.12
8	Fengjie (dry period)	2.54	2.01	2.19

Table 2 The grades intervals of assessment indices.

Index	Grade				
	I	II	III	IV	V
COD_{mn}	0~2	2~4	4~6	6~10	10~15
BOD_5	0~3	3~3	3~4	4~6	6~10
NH_3-N	0~0.15	0.15~0.5	0.5~1.0	1~1.5	1.5~2

The VFSA model was applied for water resources assessment to the Three Gorges reach. Wang (2004) monitored and collected to the data for the four cross-sections of the Three Gorges reach, viz. Qingxichang, Tuokou, Saiwangba and Fengjie. Owing to the limitation of referring data and criteria, only water quality assessment is analysed in this paper, i.e. $w_2 = w_3 = 0$. According to referring data, three indices are used (Table 1).

Based on the State Criterion of Surface Water Environmental Quality (GB3838-2002), assessment indices and their grades standard intervals are shown in Table 2.

According to Table 2 and Chen (2005), the matrixes $\mathbf{I}_{[a,b]}$, $\mathbf{I}_{[c,d]}$ and $\mathbf{I}_M$ are set up.

$$\mathbf{I}_{[a,b]} = \begin{bmatrix} [0,2] & [2,4] & [4,6] & [6,10] & [10,15] \\ [0,3] & [3,3] & [3,4] & [4,6] & [6,10] \\ [0,0.15] & [0.15,0.5] & [0.5,1] & [1,1.5] & [1.5,2] \end{bmatrix}$$

$$\mathbf{I}_{[c,d]} = \begin{bmatrix} [0,4] & [0,6] & [2,10] & [4,15] & [6,15] \\ [0,3] & [0,4] & [3,6] & [3,10] & [4,10] \\ [0,0.5] & [0,1] & [0.15,1.5] & [0.5,2] & [1,2] \end{bmatrix}$$

$$\mathbf{I}_M = \begin{bmatrix} 0 & 2 & 5 & 10 & 15 \\ 0 & 3 & 3.5 & 6 & 10 \\ 0 & 0.15 & 0.75 & 1.5 & 2 \end{bmatrix}$$

Using both equations (12) and (13), we can obtain RMDs $\mu_{\underset{\sim}{A}}(u)_{ih}$, here i is assessment index number and $i = 1,2,3$; h is grade number and $h = 1,2,3,4,5$. For example, RMD $\mu_{\underset{\sim}{A}}(u)_{ih}$ of sample 1 is:

$$\mu_{\underset{\sim}{A}}(u)_{ih(3\times5)} = \begin{bmatrix} 0.00 & 0.00 & 0.22 & 0.78 & 0.28 \\ 0.70 & 0.30 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.21 & 0.79 \end{bmatrix}$$

The weight vector (Wang *et al.*, 2004) is:

$$\vec{w} = \begin{pmatrix} 1/3, & 1/3, & 1/3 \end{pmatrix}$$

Then we can use the VFSA model (equation (15)) to obtain synthetic RMDs to five grades. Here synthetic RMDs of sample 1 are calculated under two conditions: $\alpha = 1, p = 1$ and $\alpha = 1, p = 2$.

When $\alpha = 1, p = 1$, synthetic RMD vector $\mathrm{v}_{\underset{\sim}{A}}(u)$ can be calculated:

$$\mathrm{v}_{\underset{\sim}{A}}(u) = \begin{pmatrix} 0.233 & 0.100 & 0.073 & 0.331 & 0.358 \end{pmatrix}$$

After normalization, $\mathrm{v}^{\circ}_{\underset{\sim}{A}}(u)$ is obtained.

$$\mathrm{v}^{\circ}_{\underset{\sim}{A}}(u)_k = \begin{pmatrix} 0.213 & 0.091 & 0.066 & 0.302 & 0.327 \end{pmatrix}$$

Using RFV equation (18), we calculate RFV of water quality:

$$\mathrm{H} = (1, 2, \ldots c)\ \mathrm{v}^{\circ}_{\underset{\sim}{A}}(u)^T$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} 0.213 & 0.091 & 0.066 & 0.302 & 0.327 \end{pmatrix}^T = 3.438$$

So, the RFV of sample 1 is 3.438. Similarly, we can generate RFV of sample 1 under $\alpha = 1, p = 2$ (see Table 3). With the same process, the assessment results of other samples can be calculated (see Table 3).

Table 3 Assessment results of water quality with $\vec{w}$.

	1	2	3	4	5	6	7	8
$\alpha = 1, p = 1$	3.438	3.390	3.497	3.509	2.642	2.599	2.624	2.626
$\alpha = 1, p = 2$	3.271	3.267	3.317	3.324	2.734	2.693	2.710	2.718
mean	3.354	3.328	3.407	3.416	2.688	2.646	2.667	2.672

H intervals corresponding to water quality grades are:

(a) $H \in [1.0, \ 1.5]$: I grade;

(b) $H \in (1.5, \ 2.5]$: II grade;

(c) $H \in (2.5, \ 3.5]$: III grade;

(d) $H \in (3.5, \ 4.5]$: IV grade;

(e) $H \in (4.5, \ 5.0]$: V grade.

Hence, from Table 3, we observe that the four cross-sections' water quality (samples 1–4) is between grade III and grade IV, and part of water quality is grade IV during flood period; while during dry periods, four cross-sections water quality (samples 5–8) are generally between grade II and grade III with partial to grade III. We also can see that the water quality of the Tuokou cross-section is the best in the four cross-sections. The assessment using VFSA model is similar to Wang's (2004), but, water quality assessment with grades is more accurate.

Generally, the weights vector defined by the theories and experience exhibits the importance of indices. Undoubtedly, it contains the subjectivity of decision-makers. Aiming at the above problem, this paper obtains the weight matrix, $\mu_{\underset{\sim}{A}}(u)$. Based on $\mu_{\underset{\sim}{A}}(u)$ and the principle of larger value $\mu_{\underset{\sim}{A}}(u)$ obtain larger weights, the variable weight matrix w' of three assessment indices of eight samples is obtained:

$$\mathrm{w}' = \begin{bmatrix} 0.399 & 0.388 & 0.386 & 0.387 & 0.231 & 0.224 & 0.230 & 0.224 \\ 0.128 & 0.118 & 0.132 & 0.131 & 0.159 & 0.158 & 0.155 & 0.163 \\ 0.472 & 0.495 & 0.482 & 0.481 & 0.610 & 0.618 & 0.615 & 0.612 \end{bmatrix}$$

Table 4 Assessment results of water quality with w'.

No.	1	2	3	4	5	6	7	8
$\alpha = 1, p = 1$	4.048	4.101	4.123	4.134	3.556	3.551	3.569	3.550
$\alpha = 1, p = 2$	3.936	3.968	3.972	3.983	3.672	3.671	3.673	3.673
mean	3.992	4.034	4.048	4.058	3.614	3.611	3.621	3.611

Following the above, the calculating process i.e. the assessment results, are shown in (Table 4).

We found that four cross-sections water quality (samples 1–4) belongs to grade IV during a flood period; while during a dry period, four cross-sections of water quality (sample 5–8) generally fall between grade III and grade IV, with partial to grade IV. Here, COD_{mn} and NH_3-N concentrations are high, over grade V. The weights of these 2 indices are enhanced. Therefore comparing to assessment with $\vec{w}$, this assessment is reliable.

CONCLUSIONS

It is essential to integrate water quantity, water quality, and hydroenergy into water resource assessment. The VFSA model for regional water resources assessment is introduced and a tidy water resources assessment system is established in this paper. The four cross-sections of the Three Gorges reach are selected for a case study. Because regional water resources assessment is complex, nonlinear and fuzzy, the VFSA model, a developed fuzzy tool, is valuable, compared to mathematical models. The stability of the model is also analysed by variable parameters. The case study focusing on water quality shows that the VFSA model is useful for regional water resources assessment.

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