Application of the grey self-memory neural network model for annual runoff forecasting

XIAOWEI ZHANG, LINGMEI HUANG & BING SHEN
Northwest Key Lab of Water Resources and Environment Ecology, Ministry of Education, Xi'an University of Technology, Xi'an 710048, China
dbh0913@163.com

Abstract A runoff time series represents the hydrological response to a catchment, which is a nonlinear, weakly dependent, and highly complicated dynamic system. The key to improving the accuracy of runoff forecasting is to dig the information from the limited sample sufficiently. Grey system modelling reveals the law of system dynamics by processing grey information; self-memory theory emphasizes the fore-and-aft relation of system status, which urges upon evolutional rules of system itself. Then differential equations of the dynamic system could be built for the self-memory models. Combination of grey theory and self-memory theory can effectively represent the trend of the extreme values for the time series, but there is a phase lag between observed values and predicted values. The neural network has an advantage of solving nonlinearity of the system. Based on the idea of integrative forecasting, the grey self-memory neural network model was established, which can be used to predict the annual runoff series.

Key words grey system; self-memory model; neural network; annual runoff

INTRODUCTION

Runoff forecasting, being a very important problem in the hydrology and water resources system, is the basic reference for basin planning and design, water resources assessment, reservoir regulation and inter-basin water transfer.

The hydrology system is a complicated nonlinear system (Li *et al.*, 2006) and the runoff process is a non-stationary random process with an annual period (Zhang *et al.*, 2005). To improve the forecasting precision, it is necessary that the information and the variation law in the runoff time series are sufficiently investigated.

For processing the small samples by grey theory, the new series are generated from the primordial data to find out the change in law within the system without the statistical characteristic values, based on distinguishing the difference among the systemic factors and the correlation analysis. It is short of the rigorously theoretic foundation, while solving the forecasting equation (Zhang *et al.*, 2002; Li *et al.*, 2005; Shang *et al.*, 2005). Much ameliorating arithmetic has been put forward, but they are not valid for the series with the obvious fluctuation. The development of self-memory theory originates from the retrieved model. Self-memory theory is a measure on the basis of time series to process the data, and is a realization of the two methods of Determinism and Uncertainty in mathematics (Cao, 2002). The combination of grey and self-memory theory can effectively reflect the extreme trend of series (Shen, 2005), but there are the phenomena of phase lag between the forecasting values and observed data. The neural network method is a highly nonlinear dynamical system with the stronger fault-tolerant, self-learning ability and adapting. Depending on the BP neural network to process the errors from the combination model with the grey theory, and self-memory theory to further explain the information from the runoff data, one can enhance the forecasting precision.

GREY SYSTEM THEORY AND MODELLING

The research object of the grey system is small samples. It reveals the dynamic laws in the system by processing grey information (Guo *et al.*, 2003). The grey system research attaches importance to the correlation analysis so that the system information can be fully used to make the disorderly and unsystematic data translate into ordered series that fit for differential equation modelling.

Suppose there is an original series $\boldsymbol{X}^0 = [\boldsymbol{x}^0(1), \boldsymbol{x}^0(2), \ldots, \boldsymbol{x}^0(n)]$, it becomes $\boldsymbol{X}^1 = [\boldsymbol{x}^1(1), \boldsymbol{x}^1(2), \ldots, \boldsymbol{x}^1(n)]$ by accumulating time series at a time. The differential equation with the whitening form is set up as follows:

$$dx^1 / dt + ax^1 = u \tag{1}$$

in which a stands for developing grey number and u stands for endogenesis controlling grey number. $\hat{\alpha} = [\boldsymbol{a}, \boldsymbol{u}]^T$, supposing $\hat{\alpha}$ is a parameter to be determined by using linear analysis. Then the following equation is acquired according to the least two multiplication:

$$\hat{\alpha} = (\boldsymbol{B}^T \boldsymbol{B})^{-1} \boldsymbol{B}^T \boldsymbol{Y}_N \tag{2}$$

in which $\boldsymbol{Y}_N = [\boldsymbol{x}^0(2), \boldsymbol{x}^0(3), \ldots, \boldsymbol{x}^0(N)]^T$; $\boldsymbol{B}$ is data matrix as follows:

$$\boldsymbol{B} = \begin{bmatrix} -0.5[\boldsymbol{x}^1(1) + \boldsymbol{x}^1(2)] & 1 \\ -0.5[\boldsymbol{x}^1(2) + \boldsymbol{x}^1(3)] & 1 \\ \vdots & \vdots \\ -0.5[\boldsymbol{x}^1(N-1) + \boldsymbol{x}^1(N)] & 1 \end{bmatrix}$$

if we suppose $dx^1 / dt = F$ in equation (1), then

$$F = -ax^1 + u \tag{3}$$

SELF-MEMORY THEORY AND MODELLING

The memory concept is brought forward when the irreversible process is studied, i.e. the future not only has something to do with the present, but also with times past (Cao, 1995). The self-memory principle can be used to set up a self-memory model corresponding according to the dynamical system, which can be described with differential equations.

Suppose the differential equation of system evolution is:

$$\frac{\partial x}{\partial t} = F(x, \lambda, r, t) \tag{4}$$

In which, x is variable, λ is parameter, r is space and t is time. Define a memory function $\beta(r,t)$ and $|\beta(r,t)| \le 1$. The inner product is defined in Hilbert space as follows:

$$(f, g) = \int_a^b f(\varepsilon) g(\varepsilon) d\varepsilon \qquad f, g \in L^2 \tag{5}$$

Transform the differential equation of system with equation (5), suppose x and β are continuous, differentiable, integrable, make $\beta_t \equiv \beta(t)$, $\beta_0 \equiv \beta(t_0)$, $x_t \equiv x(t)$, $x_0 \equiv x(t_0)$, and the rest is by analogy with this method. A backdate order of p finite difference-integral equation, named as memory equation, is obtained by means of subsection integral and mean value theorem as follows:

$$\beta_t x_t - \beta_{-p} x_{-p} - \sum_{i=-p}^{0} (\beta_{i+1} - \beta_i) x_i^m - \int_{-p}^{t} \beta(\tau) F(x, \lambda, r, \tau) d\tau = 0 \tag{6}$$

in which, x_i^m represents the mean value of t_i and t_{i+1}, ss $x_i^m \equiv x(t_m)$ when $t_i < t_m < t_{i+1}$. Equation (6) emphasizes the relationship between fore and after time of system status, that is, self-memory of the system itself. After determining memory function, simulation or forecasting can be carried out.

$dx^1/dt = F$, set up by grey system theory, is considered as the dynamic kernel, time series model is set up with system self-memory principle. X is replaced with x^1 in the equation of dynamic kernel. For equation (6), dispersion is carried out, integral is replaced by summation, differential becomes difference, mean value $x^m(t_i)$ is replaced by two moments, suppose $x(t-p) = x^m(t-p-1)$, $\beta(t-p-1) = 0$ and the moments are the same interval, suppose

$\Delta t = t_{i+1} - t_i = 1$, the dispersion form is obtained as follows:

$$x_t = \sum_{i=-p-1}^{-1} \alpha_i y_i + \sum_{i=-p}^{0} \theta_i F(x,i) \tag{7}$$

in which, α_I and β_I are coefficients.

$$x_i^m = 0.5(x_{i+1} + x_i) \equiv y_i \tag{8}$$

$$F(x,i) = -ax_i + u \tag{9}$$

According to the historical data, memory coefficients α_I and β_I are obtained with the least square method.

$\boldsymbol{X}_t = [\boldsymbol{x}_{t1}, \boldsymbol{x}_{t2}, \cdots \boldsymbol{x}_{tL}]^T$, $\boldsymbol{W} = [\alpha_{-p-1}, \alpha_{-p}, \cdots, \alpha_{-1}]^T$, $\Theta = [\theta_{-p}, \theta_{-p+1}, \cdots, \theta_0]^T$

$\boldsymbol{Y} = \begin{bmatrix} y_{-p,1} & y_{-p+1,1} & \cdots & y_{0,1} \\ y_{-p,2} & y_{-p+1,2} & \cdots & y_{0,2} \\ \vdots & \vdots & \vdots & \vdots \\ y_{-p,L} & y_{-p+1,L} & \cdots & y_{0,L} \end{bmatrix}$, Matrix $\boldsymbol{F}_{L\times(P+1)}$, made up of $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{i})$, can express similarly following the pattern of $\boldsymbol{Y}_{L\times(p+1)}$. Therefore equation (7) can be expressed as matrix form as follows:

$$\boldsymbol{X}_t = \boldsymbol{YW} + \boldsymbol{F}\Theta \tag{10}$$

if suppose $\boldsymbol{M} = [\boldsymbol{Y}, \boldsymbol{F}]$, $\Omega = [\boldsymbol{W}, \Theta]^T$, equation (11) can be obtained:

$$\boldsymbol{X}_t = \boldsymbol{M}\Omega \tag{11}$$

After the coefficient matrix Ω is calculated with the least square method (equation (12)), simulation or forecasting can be conducted.

$$\Omega = (\boldsymbol{M}^T \boldsymbol{M})^{-1} \boldsymbol{M}^T \boldsymbol{X}_t \tag{12}$$

$\hat{x}^1(t)$ represents the fitting or predicting values by accumulating at a time; its reducing value $\hat{x}_0(t)$ can be obtained by equation (13):

$$\hat{x}^0(t) = \hat{x}^1(t) - \hat{x}^1(t-1) \tag{13}$$

in which $t = 1, 2, \ldots, N$ and $\hat{x}^0(0) = 0$.

BP NEURAL NETWORK

An artificial neural net is a flexible mathematical model, which can simulate the large-scale, continuous and complicated system with high nonlinearity. The BP network falls into the feedforward type. Because BP arithmetic is easy to operate, has the merit of little computational complexity and fine parallelism, it is one of the most adopted, the widest to be applied, and the most developed arithmetic in neural networks at present. Its essence is to solve the minimum of error function. The structure of a tri-layer BP network model is set up in Fig. 1.

It has been theoretically proved (Xu, 2002) that a tri-layer BP network model could realize the discretional continuous mapping (Kitahara *et al.*, 1992). Because learning of BP network is carried out, based on the error reverse spread arithmetic of gradient descent (Xu *et al.*, 2002), network training speed is often slow, and it is easy to arrive at local minima during network training. Therefore, some improved algorithm with high speed could be used to solve the problem.

In the practical application, the design of BP neural networks is conducted in virtue of the neural network tool box in the Matlab software bag. In order to overcome the defect of general BP learning algorithm, the "trainbfg" function of Quasi-Newton optimization numerical algorithm in Matlab toolbox is used. This function is suitable to fitting (Xu *et al.*, 2002), and its convergence rate is fast.

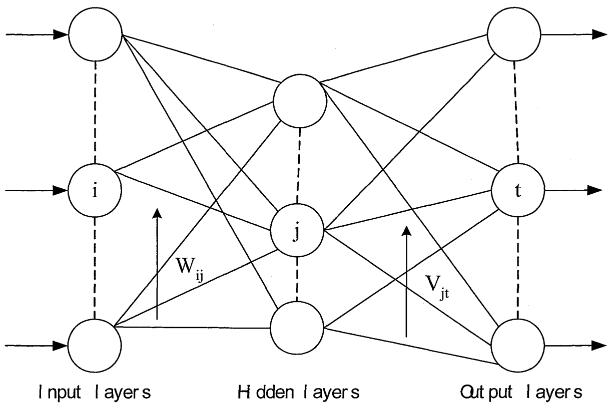

Fig. 1 Structure of a tri-layer BP neural network.

APPLICATION

Weihe River is the biggest tributary of the Yellow River, which is the longest river in Guanzhong Plain. Linjia village hydrometric station is the first station which the Weihe River flows into in Guanzhong. Runoff in the middle reaches of Weihe River is analysed and predicted to be significant for water resources utilization and economic development in Guanzhong Plain.

According to above modelling theories, the grey self-memory neural network model is set up. The observed data from 1956 to 1995 is used for parameter calibration, and from 1996 to 2000 for the model test. For the sake of the comparison, the grey self-memory model is also established.

According to grey theory, the differential equation is set up as follows:

$$F = -0.0081x^1 + 0.6158 \tag{14}$$

A self-memory equation is set up whose backdate order is 3. Its self-memory coefficient is calculated with the least square method for the discretization form of the self-memory equation. At last, the forecasting equation is achieved as follows:

$$x_t = \sum_{i=0}^{3} \alpha_i y_i + \sum_{i=0}^{3} \theta_i F(x,i) \tag{15}$$

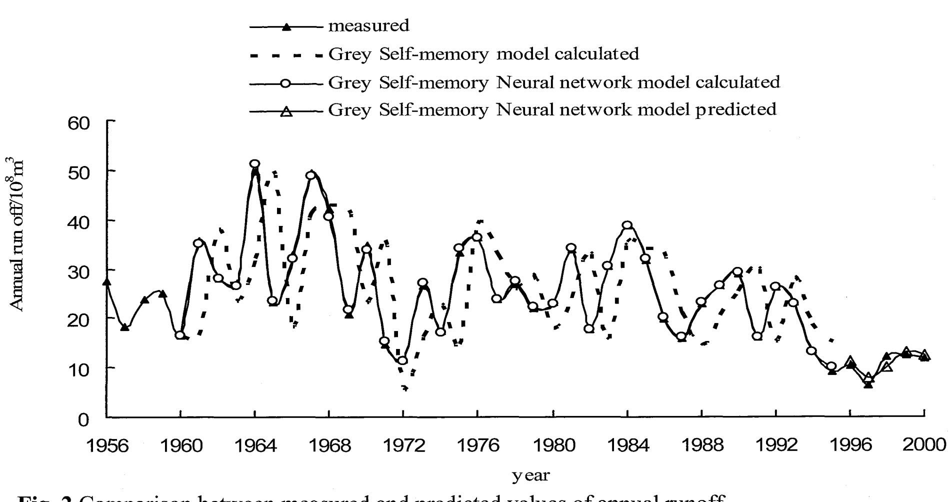

Fig. 2 Comparison between measured and predicted values of annual runoff.

Self-memory coefficients are as follows:

$$\Omega = [\alpha_0, \alpha_1, \alpha_2, \alpha_3, \theta_0, \theta_1, \theta_2, \theta_3]^T \tag{16}$$

$$= [-1.83, 3.17, -2.53, 2.18, 0.25, 0.17, 0.34, 0.09]^T .$$

Error series of runoff fitting is trained and fitted by tri-layer BP neural network. Annual runoff is predicted with grey self-memory neural network model. Model fitting and predicting results are shown in Fig. 2.

From Fig. 1, it can be shown that the grey self-memory model can reveal the long-term trend and fluctuation change by virtue of the grey model and self-memory theory so that it can effectively reflect the extreme trend of runoff series, but there is the phenomenon of phase lag between forecasting values and observed data. Based on the idea of integrative forecasting, the grey self-memory neural network model is set up. The error series from the grey self-memory model is nonlinear; it can be estimated with neural network model to accord with the changing characteristics of runoff. So the model can be used to predict the annual runoff series whose degree of accuracy is much higher than that of the grey self-memory model.

CONCLUSIONS

The runoff process is a complicated, nonlinear, dynamical system; runoff forecasting is still at the probing stage. Improving the accuracy of runoff forecasting depends on making full use of the information of runoff series and extracting what can reflect the change in law. Based on the combination predicting idea, grey theory, self-memory and neural network methods are combined to set up a grey self-memory neural network model. From the sample, it is available to predict the annual runoff.

REFERENCES

Cao, H. (1995) Self-memorization equation and self-memory model. *Meteorological Monthly* **21**(1), 9–13.

Cao, H. (2002) *The Self-Memory Theory About Dynamic System – Forecasting and Computational Application.* Geological Publishing House, Beijing, China.

Guo, Q., Guo, X. & Li, G. (2003) *The Theory and Method about the System Modelling.* University of National Defence Science and Technology Press, Changsha, China.

Li, R., Shen, B. & Zhang, J. (2006) Self-memory hydrologic forecasting model based on phase-space reconstitution. *J. Hydraul. Engng* **37**(5), 583–587.

Li, Z., Xu, J. & Zou, X. (2005) Application of grey system theory in agricultural water demand forecasting. *China Rural Water & Hydropower* **2**(11), 24–26.

Shang, J., Wang, B. & Cui, H. (2005) Application of the grey system to the analysis and forecasting of concrete cracks. *J. Xi'an Univ. Architecture & Technology (Natural Science Edition)* **37**(1), 100–104.

Shen, B., Li, R. & Huang, L. (2005) Predicting annual runoff with grey self-memory model. *J. Northwest A & F Univ. (Natural Science Edition)* **33**(4), 132–134.

Xu, J. (2002) *Mathematical Methods in Contemporary Geography.* High Education Press, Beijing, China.

Xu, D. & Wu, Z. (2002) *System Analysis and Design Based on the MATLAB 6.x Neural Network.* Xidian University Press, Xi'an, China.

Zhang, D., Zhang, S. & Shi, K. (2002) Theoretical defect of grey forecasting formula and its improvement. *Systems Engineering-Theory & Practice* **8**, 140–142.

Zhang, M., Liao, S. & Gu, Z. (2005) A mixed stochastic model for runoff process and its application. *J. Hydraul. Engng* **24**(3), 1–5.