Regularized calibration of a distributed hydrological model using available information about watershed properties and signature measures

PRAFULLA POKHREL & HOSHIN V. GUPTA

Department of Hydrology & Water Resources, The University of Arizona, Tucson, Arizona 85721, USA

pokhrel@hwr.arizona.edu

Abstract Physically-based distributed models are increasingly being used to predict the behaviour of hydrological processes in data-sparse regions. However, a model is a simplified representation of the real system and some form of calibration cannot be avoided. Because distributed models have large numbers of parameters to be specified, the resulting parameter estimation problem becomes ill conditioned. In this study we investigate a calibration approach that uses: (a) a simple form of spatial regularization (using scalar multipliers) to reduce the dimension of the calibration problem, and (b) signature measures targeting specific behavioural response of a watershed system to guide the parameter search towards a more "hydrologically consistent" set of parameters. Signature measures are applied as "regularization constraints", in an approach that is functionally similar to "Tikhonov regularization", and which results in a better-conditioned optimization problem compared to the benchmark case. The approach is demonstrated for the Blue River Basin in Oklahoma, USA.

Key words regularization; multicriteria optimization; parameter estimation; distributed hydrological model

BACKGROUND

While spatial discretization of the domain in a distributed hydrological model can provide more useful information about the spatial heterogeneity of the watershed, it also increases the complexity of the parameter estimation problem by increasing the number of unknown parameters. One way to tackle the problem of high dimensionality is to recognize that the spatially distributed elements of the model parameter fields are not independent entities that can take on arbitrary values in the parameter space. Instead, their values are somehow related to the spatial distributions of various hydrologically-relevant watershed characteristics including, for example, geology, soil type, vegetation, and topography (Grayson & Blöschl, 2000). The nature of these dependencies, if well understood, can facilitate both: (a) the development of prior estimates for the parameter fields (Koren *et al*., 2000) and (b) the implementation of regularization relationships that constrain the dimensionality of the parameter estimation problem (Pokhrel *et al*., 2008). The other way, to tackle high dimensionality, is to develop enhanced strategies for extracting information about the parameter fields from the historical time series. For example, Yilmaz *et al.* (2008) demonstrated that a set of signature measures, properly selected from the historical time series, can be used to constrain the model performance in a hydrologically meaningful way.

Spatial regularization

Problems associated with high dimensionality of the parameter search space during calibration can be treated by inclusion of additional (prior) information about the parameters, through a process known as regularization. In its broadest sense, regularization involves the use of additional information to help stabilize ill posed problems (Doherty & Skahill, 2005). The use of scalar multipliers to reduce dimensionality when dealing with spatially distributed parameter fields is a common form of regularization, that both reduces the dimension of the optimization problem, while simultaneously improving its conditioning. In Pokhrel *et al*. (2008), we used information embedded in the prior parameter estimates of 11 spatially distributed parameters of the Sacramento Soil Moisture Accounting Model (SACSMA; Burnash *et al*., 1973), to introduce regularization constraints on parameter variability and facilitate a stable solution to the calibration problem. The prior estimates were derived from information about antecedent soil moisture, soil sand-silt-clay fractions, depth of soil horizon, vegetation type and land use, using the procedure reported by

Koren *et al*. (2000). To infer the form of the regularization constraints, we assumed that the Koren *a priori* parameter estimates (KAP) are representative of the actual spatial relationships among the parameters, and therefore contain information about the dominant patterns of spatial correlation to be preserved during calibration. We used a regression approach to derive empirical equations that relate each KAP estimate to one or more observable (or inferable) watershed characteristics, resulting in a set of nonlinear regularization equations that were valid over the spatial domain (one for each model parameter field). This resulted in substantial reduction in the dimension and a stable solution to the calibration problem. By optimizing on the coefficients of the regularization equations, a better simulation of the input-state-output behaviour of the watershed was achieved.

Response signatures

Recently, efforts have been made at formulating "diagnostic" criteria that aid in understanding the causes of poor model performance (Gupta *et al*., 2008). For example, Yilmaz *et al*. (2008) used properties of the flow duration curve (FDC) to develop a set of "signature measures" that target three specific behavioural responses of a watershed system: the overall water balance, and redistribution of water to slow and fast components. The FDC was divided into: (1) the high flow segment which characterizes watershed response to large precipitation events, (2) the mid-flow segment which characterizes watershed response to moderate size precipitation events as well as the medium-term baseflow response of the watershed, and (3) the low flow segment which characterizes the long-term sustainability of flow and the influence of riparian evapotranspiration during extended dry periods. The response signatures developed were: (a) BRR which quantifies the % volume bias in the overall runoff ratio, (b) BFHV which quantifies % volume bias in the highest 5% of the flows, (c) BFMS which quantifies the % difference in the mid-segment slope of the FDC, (d) BFLV which quantifies the % volume bias in the lowest 30% of the flows, and (e) BFMM which quantifies the % difference in the median flow. For details, see Yilmaz *et al*. (2008).

In Pokhrel *et al*. (2009) we used spatial regularization (Pokhrel *et al*., 2008) and response signatures to estimate a set of hydrologically consistent parameters for a distributed version of the SACSMA model. In this paper we extend the methodology using an approach that is functionally similar to Tikhonov regularization (Tikhonov & Arsenin, 1977). The approach is demonstrated using data for the Blue River Basin in Oklahoma, USA.

WATERSHEDS, THE MODEL AND DATA

The Blue River basin in southern Oklahoma is a narrow and elongated gently sloping river valley, with an area of approximately 476 square miles (1232 km^2). The average annual rainfall in the basin is 1036 mm and the average annual flow at the outlet is 302 mm (Smith *et al.*, 2004). The Distributed Hydrologic Model University of Arizona (DHMUA) (Pokhrel *et al*., 2008) consists of a vertical water balance component based on the SACSMA applied to each cell of the HRAP (Hydrologic Rainfall Analysis Project) grid covering the study area, and a horizontal channel routing component based on the Muskingum method (Gill, 1978). Inputs to the model are spatially distributed precipitation derived from Next Generation Weather Radar coverage (NEXRAD; 4×4 km^2, 1-hour) and potential evaporation estimates based on annual free water surface evaporation maps and mean monthly station data (Smith *et al.*, 2004). Six years of data were used for the study (1 October 1999 to 30 September 2005) including a model spin-up period (WY 99-00), a calibration period (WY 00-02), and an evaluation period (WY 02-05).

METHODS USED IN THE STUDY

Regularization method

To reduce the dimension of the calibration problem, this study employs a simple spatial regularization method using scalar multipliers (Bandaragoda *et al*., 2004) to adjust the magnitudes of KAP estimates. The assumption here is that KAP estimates contain information representative of the dominant patterns of spatial correlation that should be preserved during calibration. However, their magnitudes need to be adjusted to match the output response of the watershed. Of the 16 SACSMA parameters, we adjust the magnitudes of 11 KAP fields and treat five as uniform and fixed (not adjusted). This reduces the dimension of the calibration problem from 860 (78 grid cell × 11 parameters in each cell + 2 lumped routing parameter) to 13 (11 multipliers for each KAP field + 2 routing parameters).

Conditioning using signature measures

We use the five signature measures defined earlier to help condition the optimization problem and to guide the calibration towards a hydrologically consistent set of parameters. For each signature measure, a pre-specified "acceptance" threshold was selected and deviations beyond this threshold were penalized. Optimization was implemented by solving the following minimization problem,

$$L(\phi,\alpha) = F_k(X_i \cdot \phi_i) \cdot \alpha \tag{1}$$

$$\gamma_n = \max\left\{1, \left|\frac{B_n}{T_n}\right|\right\} \tag{2}$$

$$\alpha = \max(\gamma_1, \gamma_2, \ldots \gamma_n) \tag{3}$$

where, X_i is the KAP estimate ϕ_i is the scalar multiplier to the *i*th parameter field (i = 1 ... NP are the parameters to be optimized), T_n is the threshold value specified for the *n*th signature measure, and B_n is the computed value (n = 1 ... NS are the multiple signature measures to be satisfied). In this formulation, F_k is the criteria measuring model performance against the data (k = 1 ... NC are the multiple criteria to be optimized; in our case, k = 2 and the criteria are mean squared error [MSE] and mean squared error of the log flows [MSEL]), and γ_n measures deviations of the signature measure values from the acceptable threshold values. Therefore, α represents the overall penalty applied whenever deviations to satisfying one or more of the regularization constraint occur.

Calibration

For optimization we use the Multiple Objective Shuffled Complex Evolution Metropolis (MOSCEM) stochastic sampling scheme (Vrugt *et al*., 2003) with 20 complexes terminated after 65 loops. Each optimization run involved a population size of 540 and approximately 36 000 function evaluations, requiring approximately 24 hours on a Macintosh 2.66GHz, 4GB RAM machine. As a benchmark, we calibrated the model without using the signature measures (case B-MC), to match the observed flows at the basin outlet. A compromise solution was selected from the final set of non-dominated solutions (Pareto frontier) by further constraining the solutions to the one having minimum value of the modified correlation coefficient (ModR; see Smith *et al*., 2004). For calibration using the signature measures (case SI-MC), the thresholds (T_n) were initially specified as the minimum values obtained by the final set of non-dominated solutions for the B-MC case. The thresholds were then subjectively adjusted (reduced if possible, or increased if the threshold was so tight that it became difficult to find feasible solutions) and a re-calibration performed. Final values selected for the thresholds were T_{BRR} = 10%; T_{BFMS} = 20%; T_{BFHV} = 10%; T_{BFLV} = 25% and T_{BFMM} = 10%.

RESULTS

Pareto Frontier obtained after the calibration

Figure 1 shows the objective function space obtained by the B-MC and SI-MC calibrations, respectively. The black dots indicate the non-dominated solutions (Pareto frontier) and the grey dots indicate the dominated solutions. The B-MC calibration (Fig 1(a)) clearly shows at least two

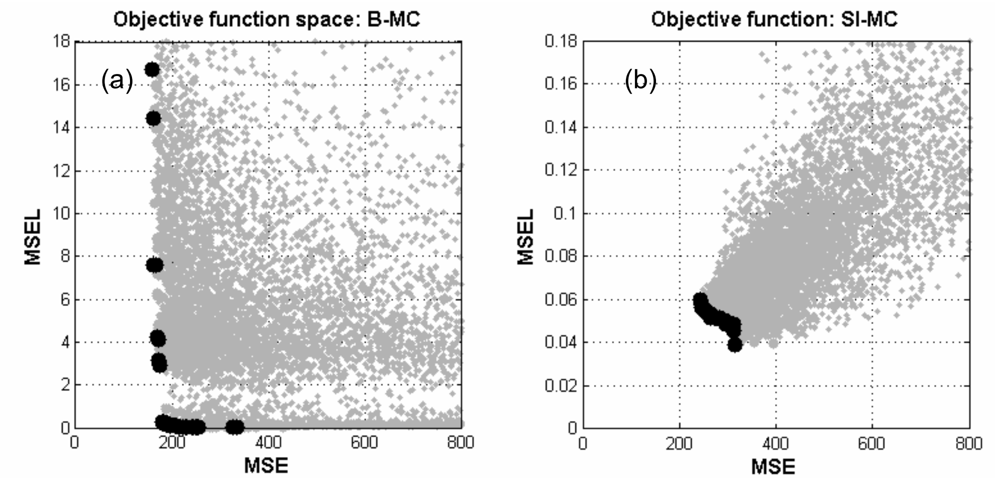

Fig. 1 Objective function space after B-MC (1a.) and SI-MC (1b.) calibrations.

regions of attraction, indicated by the gap in the grey dots around 1 < MSEL < 2. Note that the scale of MSEL in the SI-MC plot is one order of magnitude smaller than in the B-MC plot. This occurs because the SI-MC calibration problem is better conditioned by the use of signature measure, resulting in a "smoother" response surface and smaller feasible region.

Signature measures

Figure 2 shows box and whisker plots of the distributions of the signature measures at the Pareto frontier (corresponding to black dots in Fig. 1), and the modified correlation coefficient (converted to %), indicating the lower, median and upper quartiles. The whiskers extend up to 1.5 times the

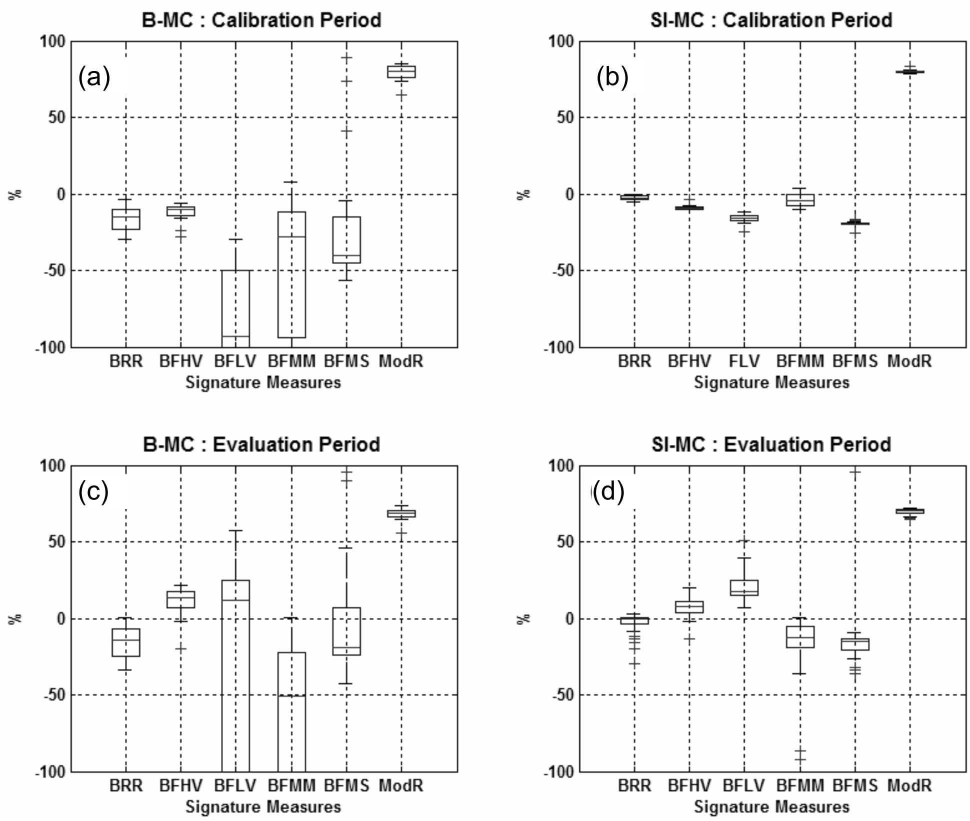

Fig. 2 Signature measures after B-MC (a) and (c), and SI-MC (b) and (d) calibrations.

inter-quartile range and the extreme values are represented by a "+" sign. In the B-MC calibration we observe a large % bias in the BFLV and BFMM signature measures during both calibration and evaluation. During calibration the median BFLV is close to 100% and the lower quartile extends beyond the limits of the plot. During evaluation the median BFLV improves, but the lower quartile for both BFLV and BFMM still extend beyond the plot limits. In the SI-MC calibrations this problem is corrected, with very small spread in the values of the biases and all signature measures having deviations less than 25% during calibration and 50% during evaluation. In both cases ModR is similar except for some outlier values.

Flow duration curves

Figure 3 shows flow duration curves for the compromise solutions selected from the B-MC and SI-MC non-dominated solutions. The SI-MC calibration result improves the simulation of mid level flows during both calibration and evaluation. However, we also observe a slight deterioration in simulation of low flows during the evaluation period.

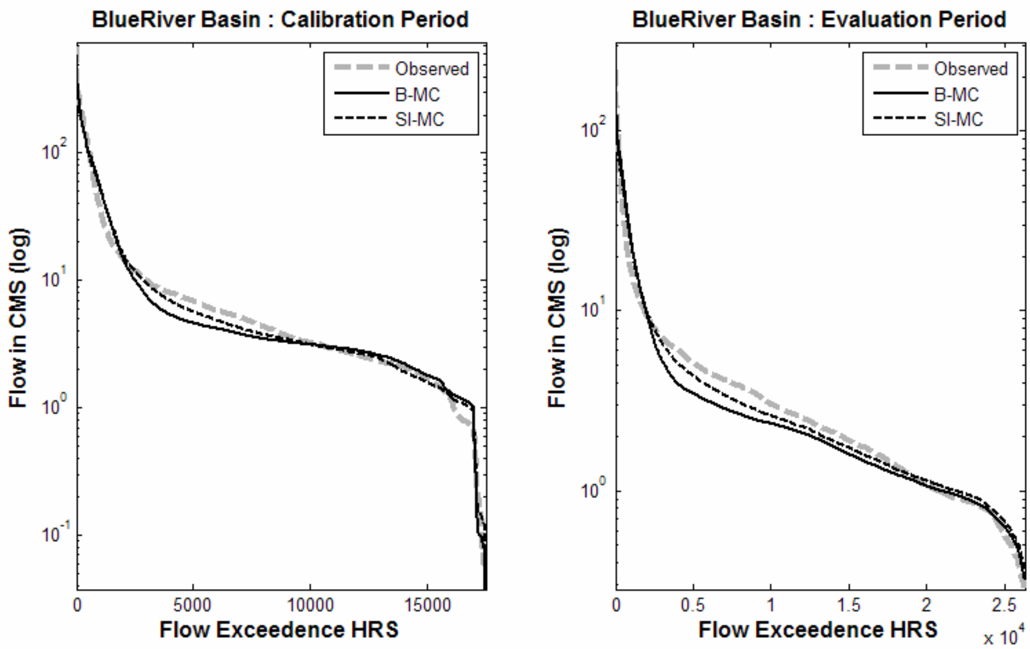

Fig. 3 Simulated and observed flow duration curves comparing B-MC and SI-MC.

CONCLUSIONS

This study has investigated a calibration approach using simple spatial regularization (via scalar multipliers) to reduce the dimension of the calibration problem, and five signature measures to improve conditioning while guiding the search towards more hydrologically-consistent values for the parameters. The approach is demonstrated for the Blue River Basin in Oklahoma. When signature measures were not used, the search space contained at least two regions of attractions (local minima) causing some solutions to converge to high values of MSEL with poor simulation of low flows. This resulted in the river going almost dry, as can be inferred by the very high values of BFLV. This problem was corrected by the use of signature measure constraints, reducing BFLV to be less than 25%, resulting in a hydrologically more consistent result. However, use of the modified correlation coefficient to select a compromise solution resulted in only small improvements by the SI-MC approach over B-MC. In summary, use of signature measures as regularization constraints improves the conditioning of the optimization problem, but further testing will be needed to establish whether the approach consistently provides hydrologically consistent parameter estimates.

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