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Hydrograph transposition between basins through a geomorphology-based deconvolution–reconvolution approach

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Abstract The aim of this study is to consider couples of basins and their respective non-calibrated robust geomorphology-based transfer functions. In the frame of discharge transposition, the two basins are respectively considered as the provider and the receiver. A discharge series of the provider basin is deconvoluted, through the inversion of its transfer function, to assess the net rainfall series. Assuming, as a first step, homogeneity between the two basins, the assessed net rainfall series is considered to be relevant for the receiver basin and convoluted with its own transfer function to simulate the discharge series at its outlet. Optimistically, the homogeneity between basins could be sufficient for nested, neighbouring and similar basins to make this approach promising when the receiver basin is ungauged. The approach is tested with simulated events for a set of four Tunisian basins (192, 180, 18.1 and 3.16 km^2). Transposition performs correctly in terms of the timing, volumes and shapes of hydrographs.

Key words geomorphology-based transfer function; deconvolution; net rainfall; transposition; PUB

INTRODUCTION

The geomorphological structure of hydrological paths can be observed for any basin, from information – of various kinds and qualities – about relief and watercourses. Moreover, it can be transformed into basin-level transfer functions, through more or less complex conceptualisations, according to the available data and knowledge. In data-sparse contexts, including ungauged basins, the use of geomorphology-based transfer functions is a strong approach, provided that the net rainfall coupling variable at the hillslope–river network interface is assessed.

The aim of this study is to consider couples of basins and their respective non-calibrated robust geomorphology-based transfer functions. In the framework of discharge transposition, the two basins are respectively considered as the provider and the receiver. A discharge series of the provider basin is deconvoluted through the inversion of its transfer function, to assess the net rainfall series. Assuming, as a first step, homogeneity between the two basins, the assessed net rainfall series is considered to be relevant for the receiver basin and convoluted with its own transfer function to simulate the discharge series at its outlet. Optimistically, the homogeneity between basins could be sufficient for nested, neighbouring and similar basins to make this approach promising when the receiver basin is ungauged.

DIRECT AND INVERSE MODELLING

Observing the basin geomorphology allows one to identify and synthesize water paths within a given basin down to the outlet, and a hydrological conceptualisation further allows deduction of a synthesis of water travel times. Among various approaches (for broader reviews see Cudennec *et al.*, 2004; Rodriguez *et al.*, 2005; Cudennec, 2007), rainfall–runoff modelling was proposed based on the coupling of two functions, respectively modelling the hillslope and the river network processes (Wang *et al.*, 1981; Gupta & Mesa, 1988; Robinson *et al.*, 1995; Woods & Sivapalan, 1999; Sivapalan *et al.*, 2002; Sivapalan, 2003). Furthermore, assuming the transfer function through the river network to be linear was demonstrated and considered to be acceptable (Naden, 1992; Beven & Wood, 1993; Blöschl & Sivapalan, 1995; Robinson *et al.*, 1995; Yang *et al.*, 2002; Giannoni *et al.*, 2003); this is a very valuable simplifying assumption in data-sparse basins.

In accordance with these, and for application to un- or poorly-gauged basins, we use the following geomorphology-based approach (Cudennec *et al.*, 2005; Boudhraâ *et al.*, 2006). With a dedicated GIS tool, the water path is identified successively through the hillslope and through the river network, down to the outlet for any location within the basin, as well as the length of the latter L. The probability density function (pdf) of L is assessed at the basin level. The simplifying assumption of a linear transfer function through the river network, based on an average velocity $\bar{v}$, leads to the pdf of water travel time through the river network t, i.e. the transfer function TF through the river network. Discharge at the outlet Q is obtained by convolution along the time t between the transfer function TF and an assessment of net rainfall R_n at the interface between hillslopes and streams (Fig. 1):

$$Q(t) = S.R_n(t) * TF(t) \quad (1)$$

where S is the basin surface. In a comprehensive rainfall–runoff model, R_n has to be simulated by a production function PF encompassing all the hillslope processes (Fig. 1(a)). In the transposition approach it is identified from deconvolution within the provider basin (Fig. 1(b)), which allows us to shortcut the modelling of highly contingent and diverse hillslope processes, and to benefit from neighbouring available data.

The deconvolution aims at determining the net rainfall vector series $\boldsymbol{R_n}$ which best reconstitutes the observed outflows vector series $\boldsymbol{Q_{mes}}$ according to the model given by equation (1). It is an inverse problem (Tarantola & Valette, 1982; Menke, 1989) which consists in minimizing the following:

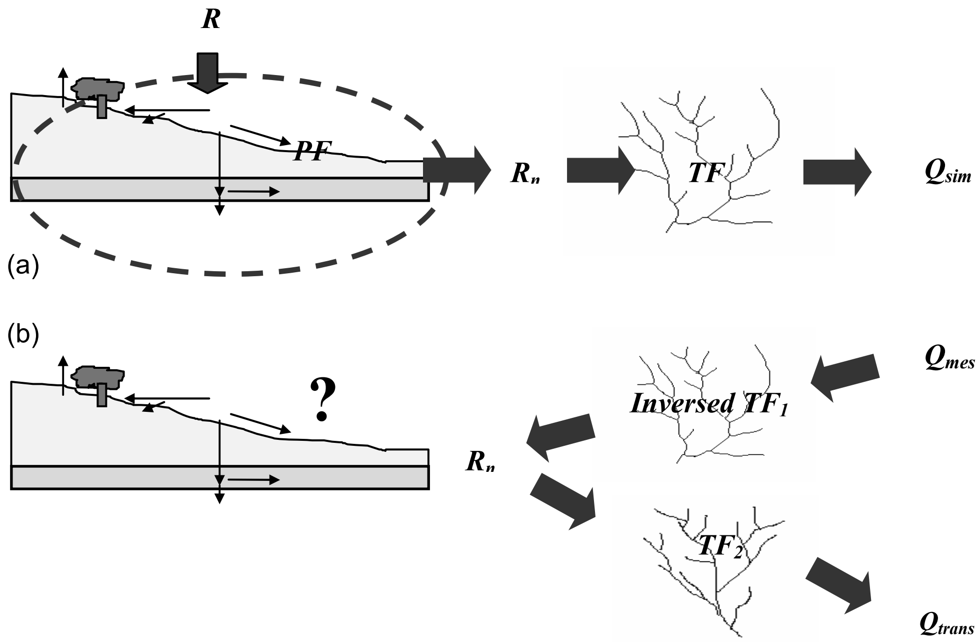

Fig. 1 Geomorphology-based modelling framework: (a) the whole basin-wide rainfall–runoff process modelling is interpreted as the coupling of a production function PF encompassing hillslope processes, and of a transfer function TF through the river network. The coupling variable is net rainfall R_n. (b) The discharge transposition consists in assessing net rainfall through the deconvolution of measured discharge at the provider outlet with the inversed corresponding transfer function TF_1, and simulating transposed discharge at the receiver outlet through the convolution of assessed R_n with the corresponding transfer function TF_2.

$$(\boldsymbol{Q} - \boldsymbol{Q}_{\boldsymbol{mes}})^{\boldsymbol{T}} . \left(\boldsymbol{C}_{\boldsymbol{Q}}^{\boldsymbol{mes}}\right)^{\boldsymbol{-1}} . (\boldsymbol{Q} - \boldsymbol{Q}_{\boldsymbol{mes}}) + \left(R_n - R_n{}^{aprio}\right)^{\boldsymbol{T}} . \left(\boldsymbol{C}_{\boldsymbol{Rn}}^{\boldsymbol{aprio}}\right)^{\boldsymbol{-1}} . \left(\boldsymbol{R}_{\boldsymbol{n}} - \boldsymbol{R}_{\boldsymbol{n}}{}^{aprio}\right) \tag{2}$$

where $\boldsymbol{R}_{\boldsymbol{n}}{}^{aprio}$ is an initializing *a priori* information on the searched vector, $\boldsymbol{C}_{\boldsymbol{Rn}}^{\boldsymbol{aprio}}$ and $\boldsymbol{C}_{\boldsymbol{Q}}^{\boldsymbol{mes}}$ are covariance matrices on vectors $\boldsymbol{R}_{\boldsymbol{n}}{}^{aprio}$ and $\boldsymbol{Q}_{\boldsymbol{mes}}$, and superscript "*T*" means the matrix transpose.

As set up by Boudhraâ *et al.* (2006), according to the inverse problems theory (Tarantola & Valette, 1982; Menke, 1989), a likelihood maximum solution can be obtained as follows:

$$\boldsymbol{R}_{\boldsymbol{n}} = \boldsymbol{R}_{\boldsymbol{n}}{}^{\boldsymbol{aprio}} + \boldsymbol{C}_{\boldsymbol{Rn}}^{\boldsymbol{aprio}} . \boldsymbol{TF}^{T} . \left(\boldsymbol{TF} . \boldsymbol{C}_{\boldsymbol{Rn}}^{\boldsymbol{aprio}} . \boldsymbol{TF}^{T} + \boldsymbol{C}_{\boldsymbol{Q}}^{\boldsymbol{mes}}\right)^{-1} . \left(\boldsymbol{Q}_{\boldsymbol{mes}} - \boldsymbol{TF} . \boldsymbol{R}_{\boldsymbol{n}}{}^{\boldsymbol{aprio}}\right) \tag{3}$$

where $\boldsymbol{TF}$ is the matrix expression of the transfer function convolution (equation (1)).

Running this deconvolution is based: (a) on the assessment of errors related to data Q_{mes} and parameters R_n, with the hypothesis that errors are 0-centred Gauss-distributed; and (b) on an initialisation through the *a priori* assessment of searched parameters $\boldsymbol{R}_{\boldsymbol{n}}{}^{aprio}$. Further details are provided by Boudhraâ *et al.* (2006).

SIMULATION FRAMEWORK AND TRANSPOSITION RATIONALE

The approach is tested with a set of four neighbouring basins of semi-arid Central Tunisia: Skhira–192, Zebbes–180, El Gouazine–18.1 and Dekekira–3.16 km^2 (Fig. 2; Albergel *et al.*, 2004; Cudennec *et al.*, 2007; Lacombe *et al.*, 2008). Their transfer functions (Fig. 2(e)) are obtained from the observation of their geomorphology and a robust assessment of their respective average velocity $\bar{v}$ (Nasri *et al.*, 2004; Boudhraâ *et al.*, 2006; Cudennec *et al.*, 2005, 2006).

As a first step of a downward rationale, simulated reference – yet inspired by actual measurements – spatially homogeneous net rainfall series R_{ref} are considered. Further, reference hydrographs $Q_{ref}i$ are simulated for each of the four basins' outlets (i = 1 to 4 for Skhira, Zebbes, El Gouazine and Dekekira, respectively) through the convolution (equation (1)) with the

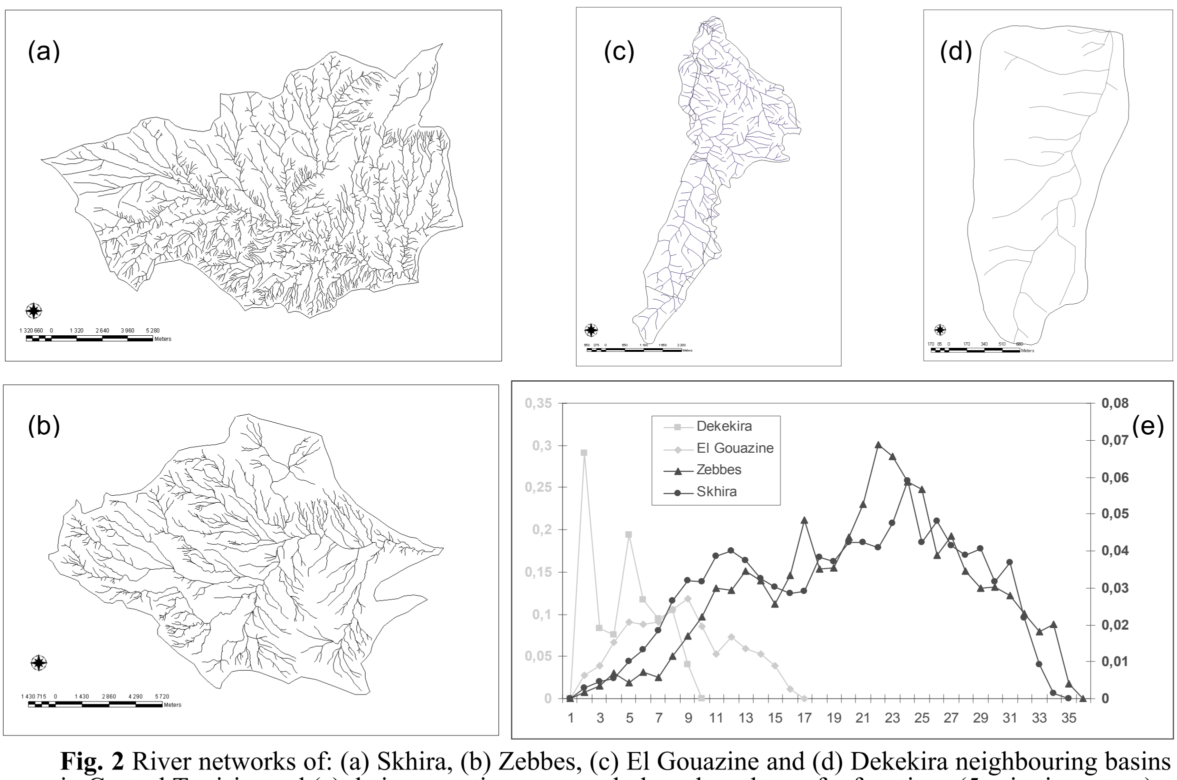

Fig. 2 River networks of: (a) Skhira, (b) Zebbes, (c) El Gouazine and (d) Dekekira neighbouring basins in Central Tunisia; and (e) their respective geomorphology-based transfer functions (5-min time steps).

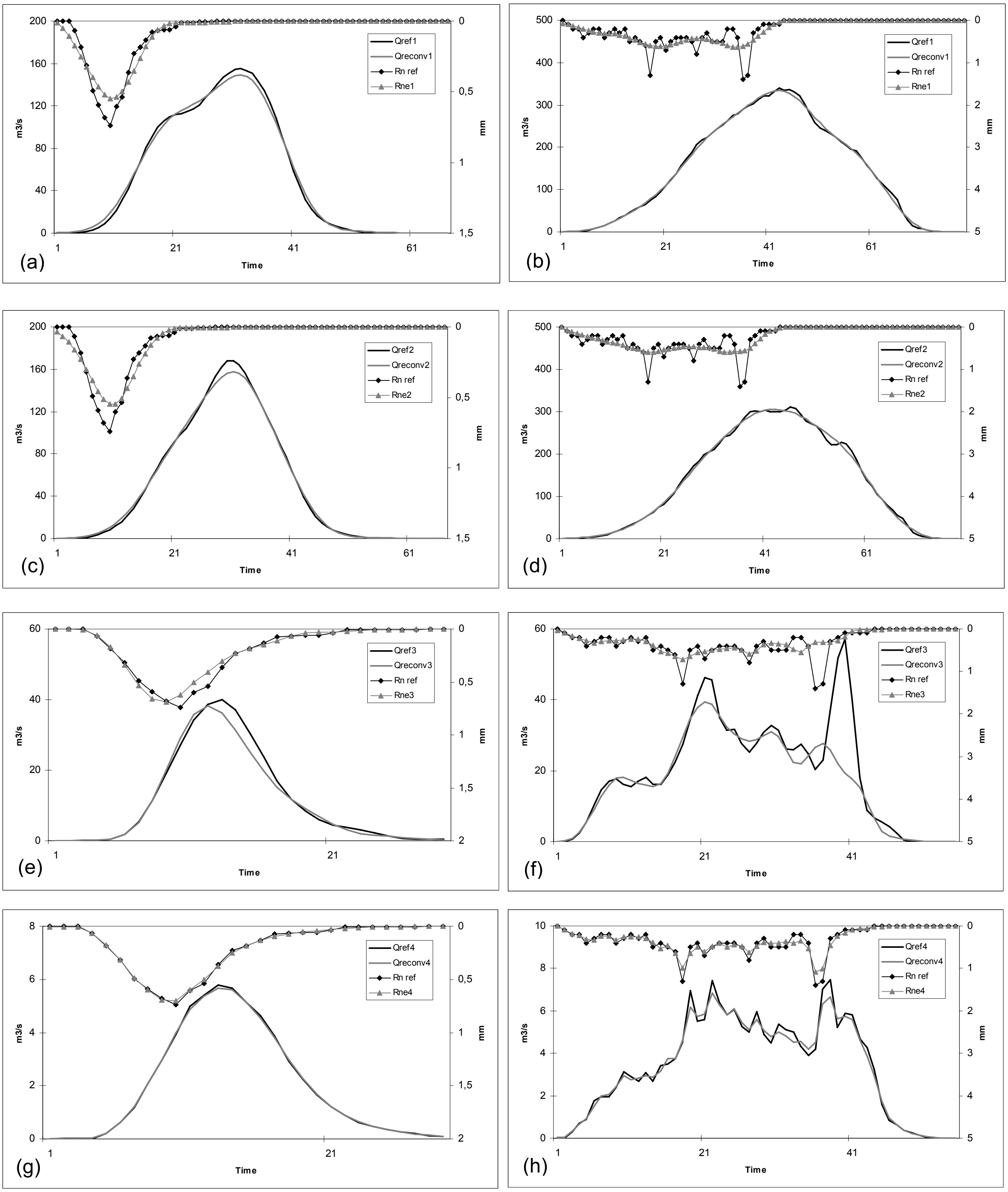

Fig. 3 Reference framework and modelling robustness: reference net rainfall, reference discharge, net rainfall assessed by deconvolution, and reconvoluted discharged for: (a) and (b) Skhira, (c) and (d), Zebbes, (e) and (f) El Gouazine, (g) and (h) Dekekira basins; respectively for events 1 and 2 (5-min time steps).

corresponding transfer functions TF_i. Two such events are built: event 1 and event 2 which are respectively uni- and multi-modal (Fig. 3).

Within this reference framework of simulated events, deconvolution (equation (2)) of hydrographs – initialised according to Boudhraâ *et al.* (2006) – provides an estimated net rainfall series for each of the four basins and for each event. These are necessarily different between themselves and from the reference net rainfall series R_{ref}, due to inversion contingencies (Boudhraâ *et al.*, 2006). The quality of each deconvolution i is evaluated through the comparison between the assessed and the reference net rainfall series, and through the comparison between the reconvoluted and the reference discharge series (Fig. 3). The NSE (Nash-Sutcliffe Efficiency criteria; Nash & Sutcliffe, 1970) between the reference Q_{ref} and the reconvoluted Q_{reconv} discharges equals 0.99 (resp. 0.99) for Skhira, 0.99 (resp. 0.99) for Zebbes, 0.97 (resp. 0.78) for El Gouazine, and 0.99 (resp. 0.98) for Dekekira basins for event 1 (resp. event 2).

Then the transposition is tested (Fig. 4); each basin is alternately considered as the receiver basin from all three others. The net rainfall series assessed through deconvolution for the provider basin i, $R_n i$, is – still under the initial hypothesis of spatial homogeneity – generalized over the three other basins j and convoluted with the respective transfer functions TF_j to obtain simulated transposed discharge series, $Q_{trans}i\_j$. The obtained hydrographs are compared to those of the reference framework, $Q_{ref}j$.

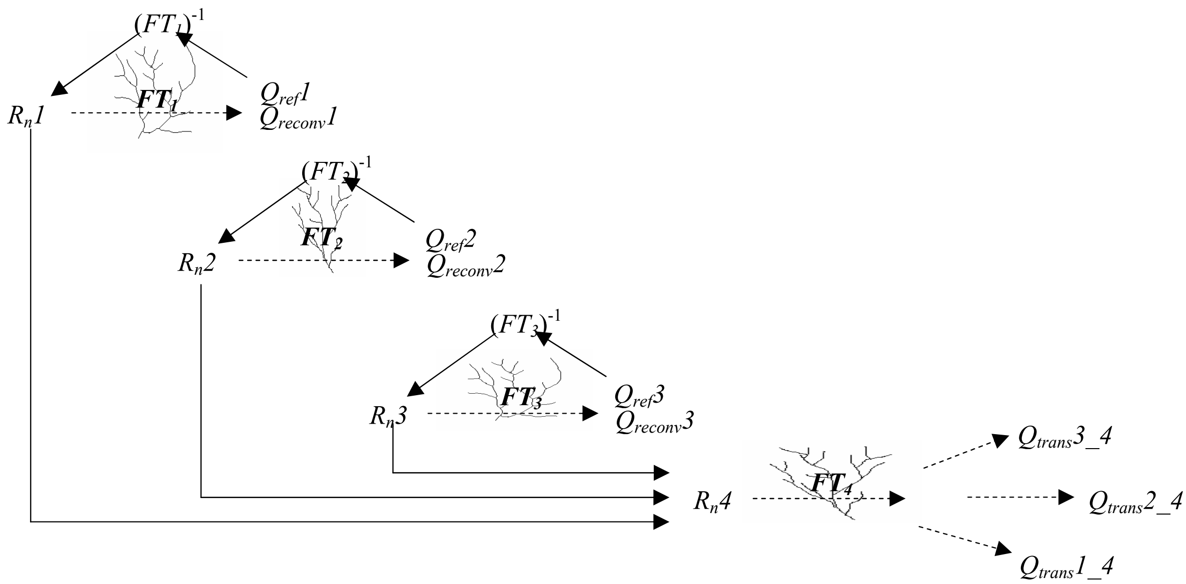

Fig. 4 Example of transposition from basins 1 (Skhira), 2 (Zebbes) and 3 (El Gouazine) towards basin 4 (Dekekira) within the reference framework. Discharge of a provider basin i, $Q_{ref}i$ is deconvoluted with the corresponding inversed transfer function $(TF_i)^{-1}$ to assess net rainfall $R_n i$, which is generalised to the receiver basin 4 as $R_n 4$ and convoluted with the corresponding transfer function FT_4 to simulate the transposed discharge $Q_{trans}i\_4$. For each deconvolution, the assessed net rainfall $R_n i$ can be reconvoluted with the corresponding transfer function TFi and reconvoluted discharge $Q_{reconv}i$ be compared to the reference discharge $Q_{ref}i$

RESULTS

Figure 5 presents the reference event together with the three transposed hydrographs $Q_{trans}i\_j$, obtained for each receiver basin j and both events 1 and 2. Table 1 displays the NSE criteria values between the transposed $Q_{trans}i\_j$ and the reference $Q_{ref}j$ hydrographs.

Transposed series appear to fit reference series very well, especially when the net rainfall event is unimodal. For the four considered basins, transposition performs correctly in terms of timing, volumes and shapes of hydrographs. Basin scale seems to be influent when series are transposed from a larger basin to a smaller one.

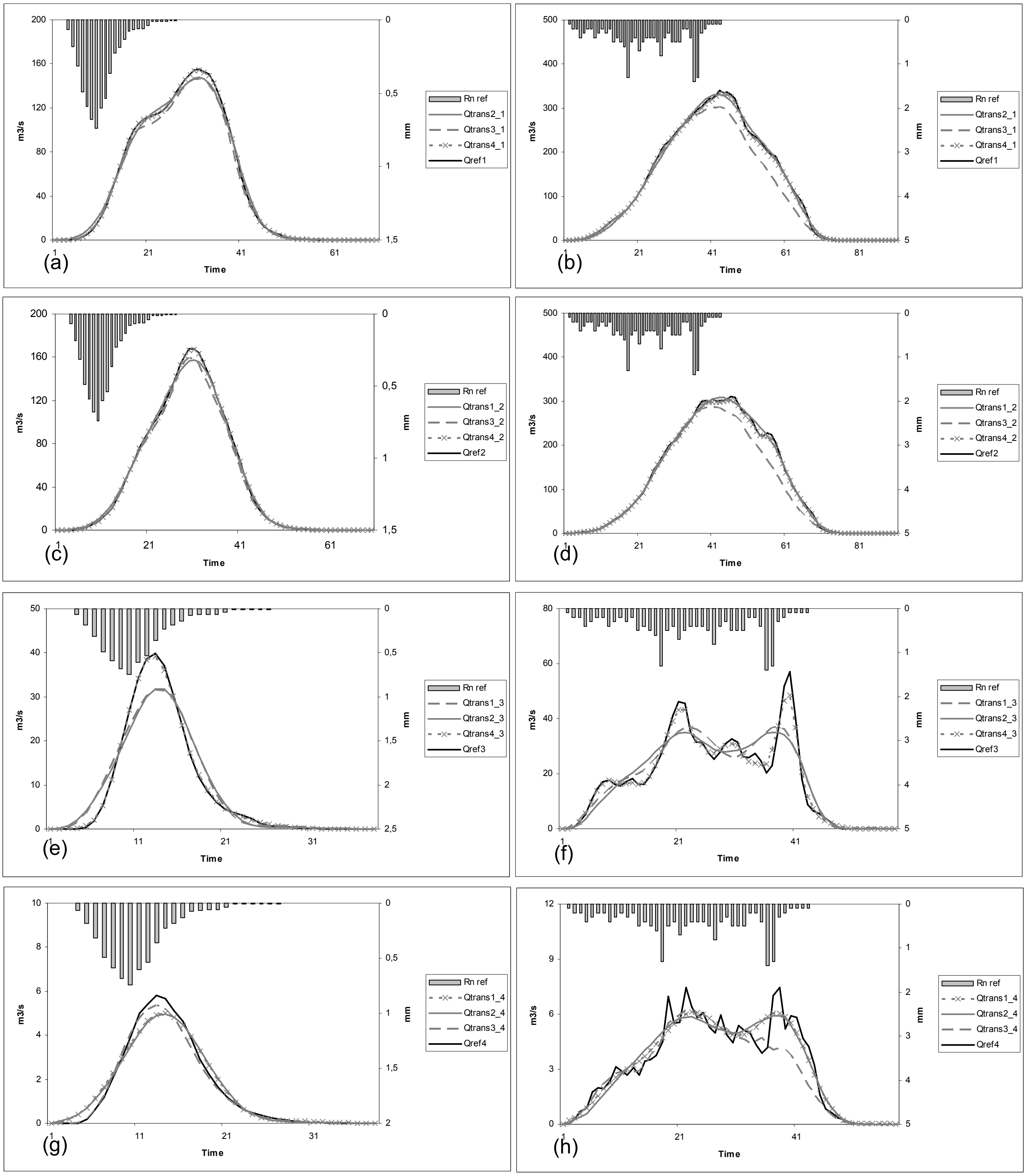

Fig. 5 Reference net rainfall and discharge, and transposed hydrographs from the three other basins considered as providers, towards: (a) and (b) Skhira, (c) and (d) Zebbes, (e) and (f) El Gouazine, and (g) and (h) Dekekira basins; respectively, for events 1 and 2 (5-min time steps).

Results within such a reference framework, under the assumption of net rainfall homogeneity, within and between the considered provider and receiver basins are encouraging. There are proposals to apply the approach to actual discharge data; and eventually to improve it as regards

Table 1 Nash-Sutcliffe Efficiency criteria values between the transposed $Q_{trans}i\_j$ and the reference $Q_{ref}j$ hydrographs for the four basins, alternately used as the provider *i* and the receiver *j* basins.

Receiver basin *j*	Provider basin *i* Skhira	Zebbes	El Gouazine	Dekekira
Event 1				
Skhira	-	0.996	0.994	0.999
Zebbes	0.997	-	0.994	0.999
El Gouazine	0.935	0.931	-	0.999
Dekekira	0.974	0.992	0.970	-
Event 2				
Skhira	-	0.996	0.929	0.999
Zebbes	0.997	-	0.930	0.999
El Gouazine	0.838	0.823	-	0.984
Dekekira	0.943	0.931	0.844	-

the modelling assumptions, especially thanks to the robust accounting for net rainfall space–time variability within convolution (Cudennec *et al.*, 2005; Chargui *et al.*, 2009). This is a major finding in the semi-arid context (Cudennec *et al.*, 2007; Slimani *et al.*, 2007) and more widely (Hung & Wang, 2005; McIntyre *et al.*, 2007), in relation to both rainfall event variability and river basin – structural and contingent – heterogeneity (Cudennec, 2007). Furthermore, in accordance with the downward rationale, this could help improving the understanding of relative causes of variability and nonlinearity in catchment hydrology (Sivapalan *et al.*, 2002; Sivapalan, 2003).

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