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Towards the development of a consistent uncertainty framework for hydrological predictions in South Africa

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Abstract South Africa has a long history of using hydrological models to solve practical water resources management problems. Despite recent international advances in uncertainty analysis, uncertainty has yet to be explicitly included as part of standard modelling practice in the country. This paper reports on the initial development of a model independent framework for ensemble streamflow predictions in gauged and ungauged basins in data-poor regions. The proposed framework is largely based on existing methods and data and includes *a priori* parameter estimation, a Monte Carlo framework and constraining model ensembles in ungauged basins through regional signatures of the catchment runoff response. Progress to date includes the modification of an existing *a priori* parameter estimation procedure that includes Monte Carlo sampling from probability distribution functions and the generation of model output ensembles. Two regional signatures have been developed, one based on the Budyko relationship and the other on the slope of the flow duration curve. A test application demonstrated that all the *a priori* ensembles produced behavioural flow duration curves, while only approximately 50% fell within the flow volume constraint. While the overall conclusion is that the framework is both theoretically sound as well as practical to implement, future work will focus on the development of additional regional catchment signatures and the use of the constrained ensembles in other water resources management tools, such as system yield models.

Key words modelling; South Africa; uncertainty; regionalization; regional signatures; model evaluation; Budyko; parameter estimation

INTRODUCTION

Effective and sustainable management of water resources demand reliable quantification of freshwater amount, distribution and quality. However, a severe lack of observations regarding freshwater resources renders many basins throughout the world, and especially in developing countries, as effectively ungauged. Hydrological models have therefore emerged as practical tools to provide information on water availability, as well as being used to simulate the impacts of present day and future human development or climate change scenarios. It is impossible to accurately represent all hydrological processes in a model and the information that is available to establish a model for a specific basin (i.e. climate and basin physical property data such as topography, soils, vegetation, geology, etc.) is typically less than perfect. It is therefore necessary to acknowledge the different sources of uncertainty that exist in the use of imprecise representations of reality. Major sources of uncertainty in water resources estimation include input data, model structural and parameter estimation errors (Ratto *et al.*, 2007). An understanding and quantification of these uncertainties are expected to contribute to improved decision making and thus improved management practices. Uncertainty assessment of model simulations has risen to prominence in the last few years (Pappenberger & Beven, 2006; Refsgaard *et al.*, 2007) and uncertainty reduction is the focus of a 10-year initiative on Predictions in Ungauged Basins (PUB) (Sivapalan *et al.*, 2003).

The most common approach to continuous hydrological predictions in ungauged basins has been the extrapolation of information on model parameters from gauged basins in a process commonly known as regionalization (Nathan & McMahon, 1990). The basic tenet in regionalization is that, if there exists a relationship between model parameters and basin properties which holds for a gauged basin, then flow simulations can be achieved in an ungauged basin which has similar physical attributes. However, the transition from the identification of local models at gauged basins to the establishment of relationships for regional models suitable for ungauged sites has some significant shortcomings related to the uncertainties associated with the local models and how these are affected by data errors and their own parameter uncertainties (Wagener *et al.*, 2004; Wagener & Wheater, 2006). An alternative approach, called "regional calibration", simultaneously optimizes both the model parameter calibration and the regional relationships (Fernandez *et al.*, 2000).

Yadav *et al.* (2007) therefore proposed an alternative strategy in which catchment characteristics streamflow signatures are regionalized, rather than model parameters. This strategy can be seen as part of an alternative approach to hydrological modelling that is offered by Gupta *et al.* (2008) and involves the use of a signatures-based, diagnostic process of model evaluation. This approach incorporates modelling uncertainty analysis and deviates from traditional practice in that it does not just use statistically based objective functions to measure model performance. The reasoning is that these traditional approaches ignore hydrological understanding regarding how the model represents the functional behaviour of a catchment. The process of model evaluation makes use of catchment signature indices of dynamic system behaviour to constrain and condition continuous flow simulations at gauged and ungauged sites (Fig. 1). Wagener *et al.* (2007) and Yadav *et al.* (2007) define a signature as an index or a time series of the response behaviour of a catchment at a given time-scale, which is reflective of a catchment's functional behaviour and can be regionalised. Since these constraints arise out of the theoretical basis for hydrological modelling it should be possible to test them against observed data (Gupta *et al.*, 2008). Depending on the model, a range of constraints could be used and common ones include yield-storage curves, flow duration curve gradients, runoff ratio (runoff/precipitation or P/Q), aridity indices (precipitation/evapotranspiration or P/PE) and measures of discharge timing (Shamir *et al.*, 2005). Yadav *et al.* (2007) showed that such signatures can often be regionalized very well since they derive directly from observed streamflow, rather than from a noisy calibration process as in the case of model parameters. If the regionalization process includes estimates of uncertainty, then these regional signatures can be used as constraints on the behaviour of local hydrological models (Fig. 1). Another approach to the link between "input-state-output data" and "static basin data" represents the regionalization process in which regional signatures of catchment response behaviour are used to constrain model outputs. This approach uses direct measures (from observed information) of the

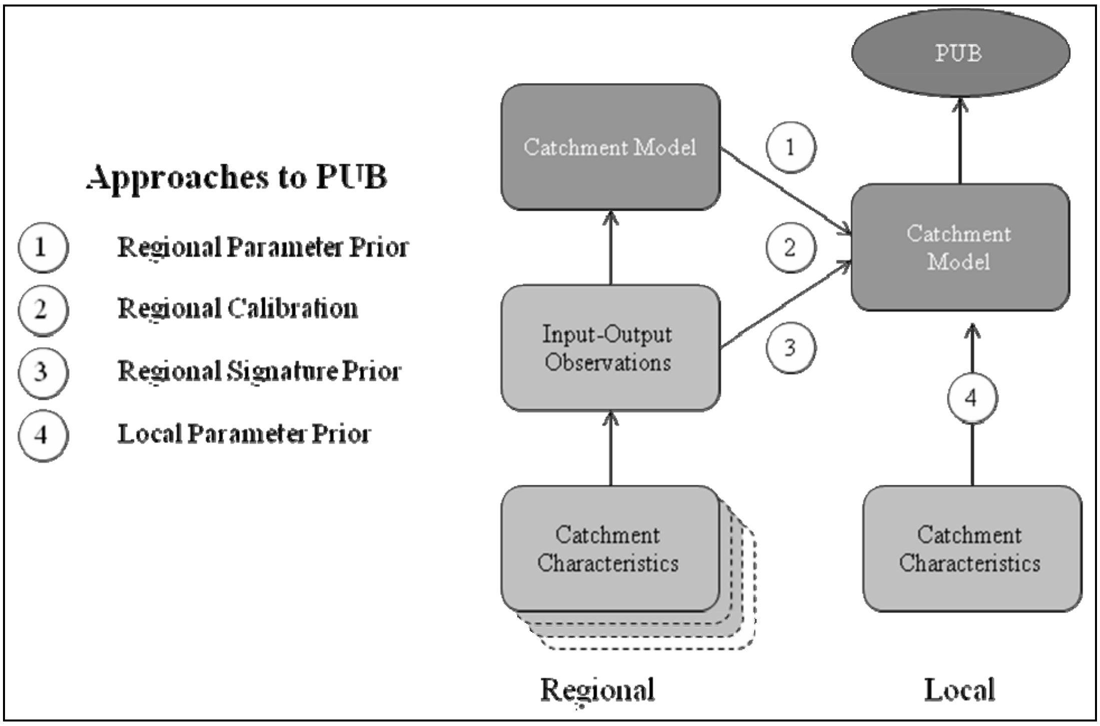

Fig. 1 The approach used to constrain and evaluate model application.

catchment behaviour to determine whether model outputs are "acceptable" or behavioural and has been tested in some United Kingdom catchments by Yadav *et al.* (2007). The catchment indices are regionalized through the use of simple regression relationships with the confidence limits used to define the distribution of possible "behaviours" for each index. For any given set of initial parameter values (defined either as equally likely values within a range, or as some type of distribution function), the model can be run for all possible parameter combinations to generate an ensemble of outputs. Predicted values of indices are then calculated from the model outputs and compared with the regional values to determine acceptable outputs from the output ensembles (Yadav *et al.*, 2007; Gupta *et al.*, 2008). These regional signatures can thus be seen as regional priors on the expected catchment streamflow behaviour. Additional information can be included if local priors on the model parameters are derived from static basin characteristics such as soil or topographic data. Such a framework therefore allows for the use of both local and regional priors and for testing their relative value.

South Africa has a long history of the use of hydrological models for practical water resources problem solving (Hughes, 2004a) and while it has always been recognized that the model outputs are uncertain, this uncertainty has never been explicitly quantified. There is therefore an urgent requirement to incorporate uncertainty assessment into model applications and this paper proposes a framework that incorporates the principles of the diagnostic model evaluation process outlined above (Fig. 1) together with some existing modelling practices used within South Africa. While the focus is on a recently revised version of the widely used monthly time-step Pitman model (Hughes, 2004b) the principles of the framework should be applicable to any hydrological model. Some of the details of the framework are still being developed and this paper concentrates on the initial research work that has been completed to date.

THE PROPOSED UNCERTAINTY FRAMEWORK

Figure 2 summarises the basic structure of the proposed framework and includes three main components:

- An approach for generating prior parameter distributions, i.e. local priors on the model parameters.
- An approach for sampling from these distributions and generating model output streamflow ensembles.
- An approach for developing regional signature constraints that define behavioural hydrological responses against which the model output ensembles are compared, i.e. regional priors on the expected catchment response.

The Pitman model

The recently revised version of the Pitman model (Hughes, 2004b) is a monthly time-step conceptual type rainfall–runoff model with parameters representing the main storages and fluxes of the natural water balance of catchments. The version that has been used here explicitly represents ground and surface water interactions and also includes components that allow artificial impacts, such as small farm dams, larger dams, abstractions and return flows, to be included in the modelling scheme. The model is relatively parameter intensive, typically requiring values for at least 12 parameters to be estimated either through calibration in gauged catchments or some form of *a priori* estimation approach in ungauged catchments.

Prior parameter distributions

The prior parameter distributions are based on existing parameter value estimation methods presented in Kapangaziwiri & Hughes (2008) coupled with the definition of probability distribution functions (pdfs) and Monte Carlo sampling. The estimation methods for the main runoff generation parameters make use of physical basin property data from the AGIS (2007) database that defines surface slope, soil depth and texture characteristics, as well as underlying

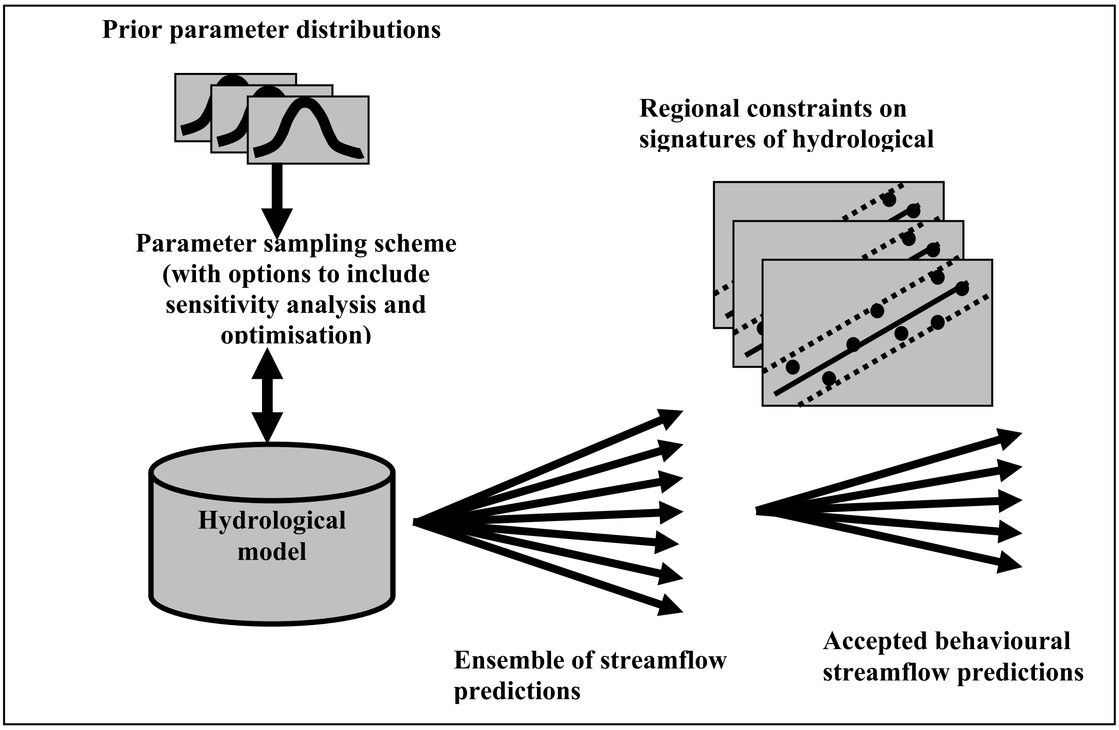

Fig. 2 Schematic of a model-independent framework for incorporating uncertainty in the generation of model outputs.

geology for different topographic units within a basin. It has been necessary to translate these estimates into probability distribution functions (pdfs) that can be used with random sampling to estimate the pdfs of secondary properties (e.g. soil depth, porosity, permeability and infiltration rate at the sub-basin scale). The parameter estimation equations use either the raw data or the calculated secondary property data and the same sampling procedure is used to estimate their pdfs. For example, soil depth information is provided as a range within each soil type/topographic unit combination (AGIS, 2007) and it has been assumed that the mean of the range represents the mean of a normal distribution, while the lower and upper values of the range represent the 5th and 95th percentiles of the cumulative pdf. The shape and type (normal, log-normal, etc.) of the secondary property or parameter pdfs are therefore partly determined by the spatial distribution of the raw data. If large parts of the sub-basin have similar soil depth ranges, while a small part has much deeper soils, it is possible for a secondary property or a parameter to be log-normally distributed.

The approach that has been adopted is believed to account for those uncertainties associated with the raw physical property data, those associated with the appropriateness of the parameter estimation equations, as well as those associated with up-scaling the raw data (based on topographic units within the sub-basin) to the scale of the sub-basin. While Kapangaziwiri & Hughes (2008) focused on the main surface runoff generation parameters (related to soil depth and texture, slope and other topographic properties), further work is currently in progress to define similar estimation equations for the interception, evapotranspiration and groundwater parameters of the model. Until these estimation equations have been finalised their pdfs are simply estimated based on likely parameter value ranges derived from previous experience of running the model. All the main model parameters can therefore be included in the uncertainty analysis, despite the fact that more realistic prior constraints need to be developed for some of the parameters.

Model output ensemble generation

A Monte Carlo sampling procedure (using the parameter pdfs as discussed above) is currently used to generate 5000 model output time series (ensembles) from which any signatures can be calculated and compared with the regional constraints discussed below. Future research will concentrate on more computationally efficient approaches which include some form of optimisation that ensures that the ensemble members are behavioural given the set of regional constraints.

Development of regional constraints

This is the part of the framework which requires the most development, and the assumption is that the regional constraints should be based on existing knowledge that can be used to characterise hydrological response within the South African region. It is prudent to note here that for the monthly Pitman model a number of functional catchment characteristics can be investigated. For instance, the water balance as a constraint on how basin precipitation is partitioned into actual evaporation and runoff, or the flow regime through the flow duration curve (FDC). At this stage of the framework development, two constraint relationships have been considered, the first based on the Budyko (1974) curve concept and the second on the slope of the monthly flow duration curve (FDC).

The first step in developing regional P/PE *versus* Q/P relationships (based on Budyko, 1974) was to use the mean annual runoff (Q) from the simulated 70 year (1920–1990) runoff time series and the estimated mean annual rainfall (P) and potential evaporation (PE) for all 1946 catchment units (known in South Africa as “quaternary” catchments and the scale at which local water resources management is undertaken) used in the WR90 database (Midgley *et al.*, 1994) covering the whole of South Africa, Swaziland and Lesotho. The runoff data used were the incremental flows, generated only within each quaternary catchment. Plotting all these data suggested a series of log–log relationships that converge at low values of both P/PE and Q/P. An iterative process was followed to define four regional relationships. The relationship for Region 1 was first established by identifying a regression equation that had a high R^2 value and for which the residuals were approximately equally divided between negative and positive values. Once the points to be included in Region 1 were finalised, the same process was followed to identify the Region 2 points, and so on. All of the points and the resulting regression relationships are shown in Fig. 3, while Table 1 lists the equations and R^2 values. Figure 4 indicates that the regions are generally spatially contiguous, although there are some areas that are not clearly defined as a single region. This may be due to localised variations in runoff response, as well as artefacts related to errors in the data used.

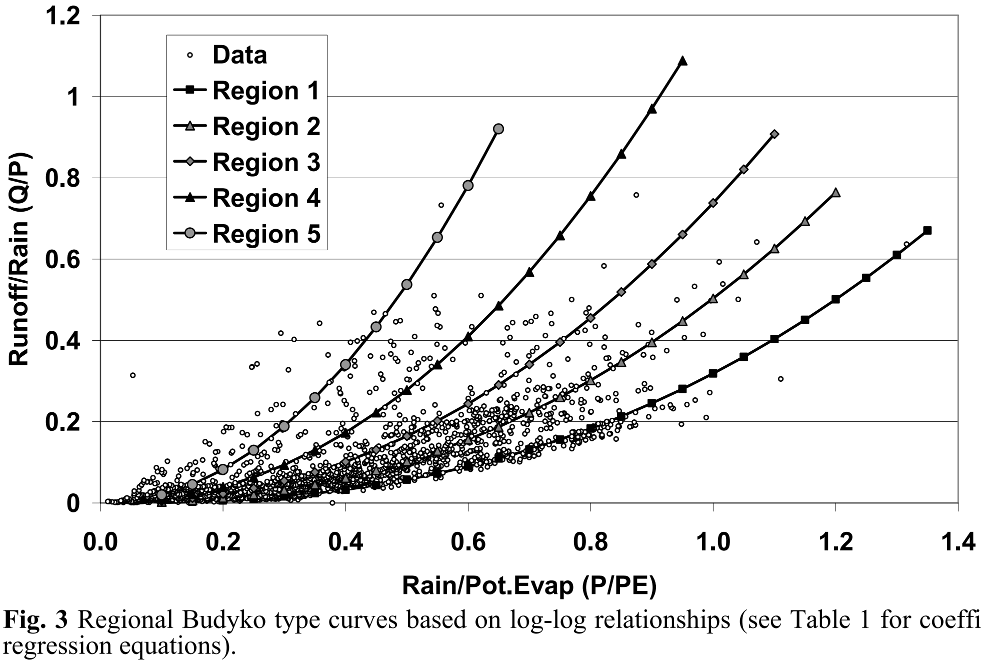

Fig. 3 Regional Budyko type curves based on log-log relationships (see Table 1 for coefficients of the regression equations).

Table 1 Regional Budyko type relationships.

	Regions: 1	2	3	4	5
Based on simulated data					
No. of Quats.	397	702	317	202	325
Area (km^2) range	59–8647	43–18108	72–10274	72–3913	89–8037
Slope (A)	2.527	2.293	2.168	2.126	1.770
Intercept (B)	–1.113	–0.687	–0.304	0.194	0.478
R^2	0.927	0.968	0.984	0.990	0.866
Based on observed flow data					
No. of gauges	40	135	45	23	27
Area (km^2) range	86–1887	81–1668	106–1691	84–873	101–1889
Slope (A)	2.322	2.154	2.171	2.406	1.351
Intercept (B)	–1.079	–0.741	–0.338	0.475	0.173
R^2	0.932	0.905	0.890	0.917	0.820

Note: equations are of the form $\ln(Q/P) = A \times \ln(P/PE) + B$

It would not be strictly good practice to develop the regional constraint relationships using simulated data, although it is considered here to be acceptable to use these data to define the regions. These are the only data that have a reasonable national coverage and are deemed a good starting point for a first order definition of regions before these are refined using other coarser data, such as observed or naturalised flows, for the definition of the relationships. Therefore, the second step involved the use of the naturalised time series (also given in Midgley *et al*., 1994) for all available streamflow gauges. Gauges were initially rejected if they had less than 10 years of observations, if their drainage areas included quaternary catchments that fell into more than a single region, or if the amount of missing (and in-filled) data was excessive. Some very small gauged sub-basins were also rejected. For each of the regions identified during the first step, Table 1 lists the number of gauges included in the analysis, the range of catchment areas, the coefficients of the final estimation equations and the R^2 value. It is apparent that the final equations are very similar to the initial equations (Fig. 3 and top part of Table 1 and based on the simulated data) for regions 1 to 3, but that there are quite large differences for regions 4 and 5. The derived Budyko curves are regionally consistent, implying some underlying physical basis despite being empirically derived. It could be instructive to further examine the physical basis for the curves following the approach of Yang *et al.* (2008).

In a region such as South Africa with very diverse flow regime characteristics, the shape of the flow duration curve (FDC) can be a very useful indicator of hydrological response characteristics. The shapes of FDCs are also important in determining potential levels of sustainable abstraction, the need for artificial storage and are relevant to determining environmental flow requirements (Hughes & Hannart, 2003). FDC shape is therefore highly relevant to water resources management. As with the Budyko relationships, the starting point for the analysis was to use the simulated flow time series for all 1946 quaternary catchments to try and identify regional relationships. For largely perennial river systems the FDC slope values were calculated as the difference between the Q10 and Q90 values divided by 80 (i.e. 90–10). For those sub-basins with periods of zero flow, the Q90 value was replaced with the first non-zero FDC percentage point value and the difference in flows divided by the appropriate % differences.

Various readily available predictor variables (or combinations thereof) expected to influence FDC shapes were used to try and find suitable estimation equations, either for the whole country or for different regions. It was found to be very difficult to find suitable variables and there were no obvious regional patterns in the data. While further analyses are still being done to improve the development of a constraint relationship, Fig. 5 illustrates an interim solution. The estimation equation is based on an index that combines a measure of aridity (P/PE) and a measure of sub-basin slope (relative relief). The R^2 value of the relationship is 0.63. The relationship illustrated in

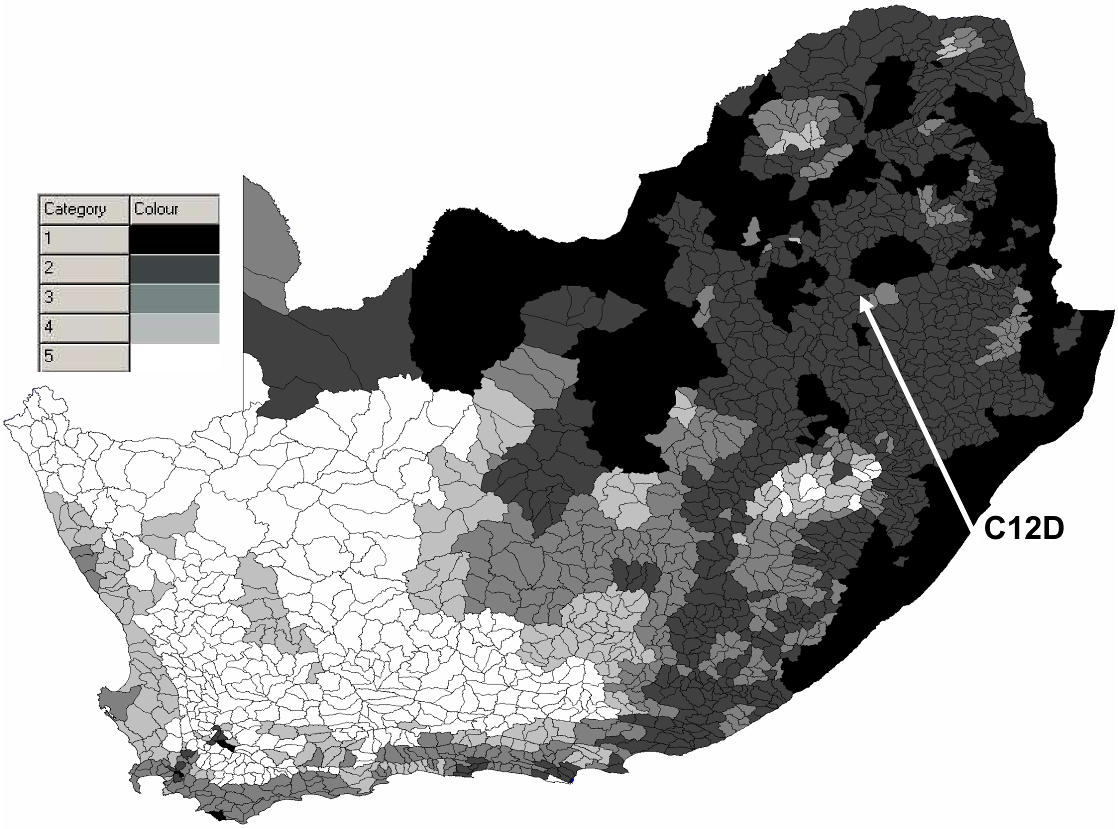

Fig. 4 Regions based on Budyko type relationships between P/PE and Q/P.

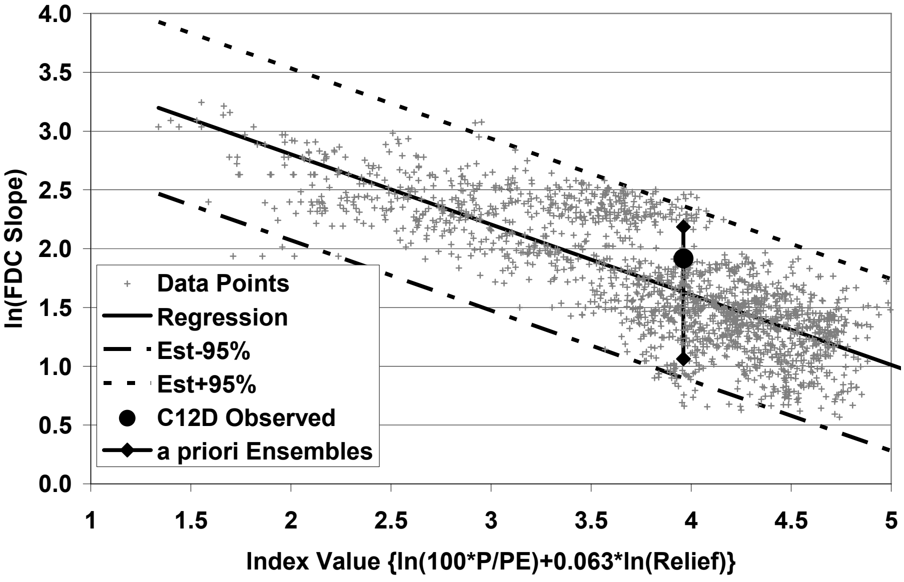

Fig. 5 Relationships between an index of aridity (P/PE) and sub-basin slope (Relief) and the slope of flow duration curves based on simulated WR90 data.

Fig. 5 excludes a number of sub-basins in the country that are strongly influenced by dolomitic geology and a region in the northeast of South Africa that appears to be anomalous based on the simulated flow data. Some of the scatter in the relationship as well as the existence of anomalies could be artefacts associated with the use of simulated data. The next phase in the study is to repeat the analysis using the available observed data.

AN EXAMPLE CASE STUDY

Hughes *et al.* (2008) presented an initial attempt to include uncertainty into outputs from the Pitman model using a gauged sub-basin (C12D) of the Vaal River. The *a priori* model parameter values were estimated using the approach of Kapangaziwiri & Hughes (2008) but the possible ranges of values were determined subjectively without any attempt to sample based on probability distribution functions of the physical basin attributes data. One of the conclusions reached by Hughes *et al.* (2008) was that the parameter sets that produced the higher runoff simulations were not behavioural when the results were compared with the available observed data.

The simulations have been repeated using more formal sampling procedures to generate 5000 ensembles based on varying the values of the six main runoff generation parameters of the model (the same parameters formed part of the earlier study by Hughes *et al.*, 2008). All of the ensembles use the same hydro-climate inputs. The outputs from the 5000 model runs have been added to Fig. 5 (FDC slope) and Fig. 6 (Budyko type relationships), which also indicate the position of the observed data (after naturalisation to account for some irrigation and return flow impacts). Figure 5 suggests that all of the ensembles are acceptable from the point of view of the shape of the FDC, while Fig. 6 suggests that many of the ensembles (approximately 49%) are not acceptable in terms of overall volume of runoff. This is consistent with the original study observations that considered the simulations based on the upper parameter bounds to be non-behavioural.

These results seem to suggest that the local (parameter) priors are more effective in constraining the basin flow regimes from model simulations, whereas the regional (signature) priors would perform better on the water balance restrictions. However, this might change if the FDC would be regional rather than national.

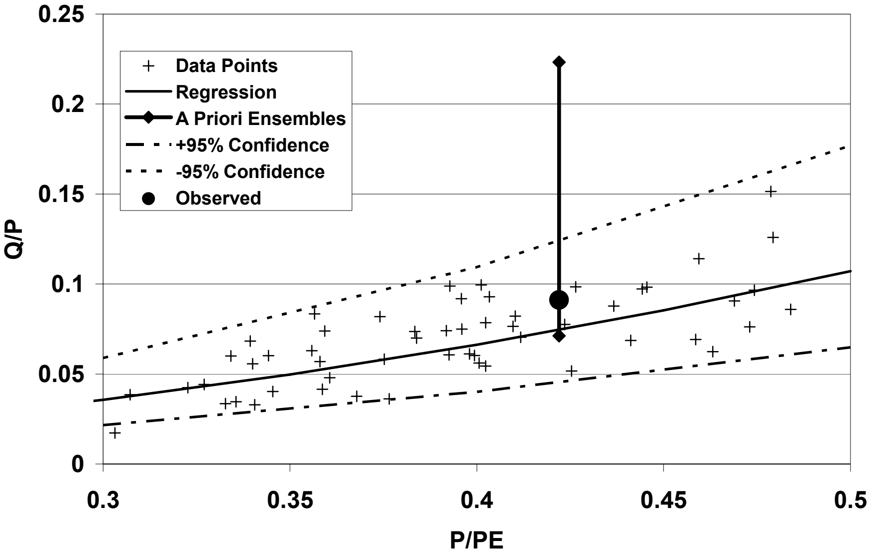

Fig. 6 The final regression relationship for region 2 (using the Budyko type relationships based on gauged data), 95% confidence limits and the location of all 5000 ensemble results for sub-basin C12D.

DISCUSSION AND CONCLUSIONS

A framework for the application of models in a South African context has been proposed. This framework is based on a diagnostic model evaluation process and uses regional indices of catchment characteristics to constrain simulated catchment behaviour. Catchment scale signatures of the hydro-climate response (Budyko relationships) and the slope of flow duration curves have been selected as initial constraints and estimation equations have been developed for most of South Africa from existing information. The regional Budyko relationships (Table 1) have been developed from observed data and are generally very good (narrow confidence bands) with high

R^2 values. The FDC curve constraints require further refinement and are currently based on simulated data that may include some unknown model artefacts. The highest priority for the further development of the framework is to refine the existing constraints and identify additional relationships that can be used to further constrain the model output ensembles.

The single test example demonstrates the merits of the framework and illustrates that different constraints may lead to different ensemble members being accepted. In this case all the simulations could be considered behavioural based on the properties of the FDCs, while only approximately 50% of the ensembles were within the simulated volume (Q/P) constraint, making it a strong regional constraint but rather weak on flow regime. It is possible that future work could involve examining the parameter sets that generated acceptable results to provide feedback to the *a priori* parameter estimation approach using physical basin property data. This work could also involve an examination of the use of these basin physical data for the derivation of the Budyko curves (Yang *et al.*, 2008).

During this phase of the framework development only parameter uncertainty has been considered. However, there is no reason why additional uncertainties cannot be considered using the same framework. For example, a series of different rainfall inputs could be generated (using stochastic rainfall models, different approaches to sampling gauge data or the use of different rainfall data products such as radar and satellite) and used to extend the number of output ensembles. The preliminary conclusion is that the development of the basics of the framework has been successful from both theoretical and practical points of view, while it clearly requires further refinement and extension. The required software only involved straightforward modifications to the existing model code and the time taken to generate the parameter distributions and model output ensembles is not restrictive for practical model applications.

This paper has not addressed the issue of how the accepted ensembles are used in other components of water resources development decision making that may involve additional models (system yield models, environmental flow determinations, resource economics models, etc.). The fact that such methods are currently based on a single input to represent the time series of the natural resource availability suggests that they could require substantial modifications to deal with the hydrological model ensembles. These modifications may prove to be more of a challenge than the development of the framework discussed in this paper. However, the long-term benefits of making decisions with an improved understanding of the uncertainties of predictions should more than compensate for the efforts required to achieve these modifications.

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REFERENCES

AGIS (2007) Agricultural Geo-Referenced Information System, accessed from http://www.agis.agric.za during March 2008.

Budyko, M. I. (1974) *Climate and Life*. Academic Press, New York, USA.

Gupta, H. V., Wagener, T. & Liu, Y. (2008) Reconciling theory with observations: elements of a diagnostic approach to model evaluation. *Hydrol. Processes* **22**, 3802–3813.

Fernandez, W., Vogel, R. M. & Sankarasubramanian, A. (2000) Regional calibration of a watershed model. *Hydrol. Sci. J.* **45**(5), 675–688.

Hughes, D. A. (2004a) Three decades of hydrological modeling research in South Africa. Special Rhodes University Centenary Edition of *South African Journal of Science* **100**, 638–642.

Hughes, D. A. (2004b) Incorporating ground water recharge and discharge functions into an existing monthly rainfall–runoff model. *Hydrol. Sci. J.* **49**(2), 297–311.

Hughes, D. A. & Hannart, P. (2003) A desktop model used to provide an initial estimate of the ecological instream flow requirements of rivers in South Africa. *J. Hydrol.* **270**(3-4), 167–181.

Hughes D. A., Sawunyama, T. & Kapangaziwiri, E. (2008) Incorporating estimation uncertainty into water resource development planning in ungauged basins in southern Africa. In: *Sustainable Hydrology for the 21st Century*. Proc. 10th BHS National Hydrology Symposium, Exeter, Sept. 2008.

Kapangaziwiri, E. & Hughes, D. A. (2008) Revised physically-based parameter estimation methods for the Pitman monthly rainfall-runoff model. *Water SA* **32**(2), 183–191.

Midgley, D. C., Pitman, W. V. & Middleton, B. J. (1994) *Surface Water Resources of South Africa 1990*, vol I to IV. WRC Report No. 298/1.1/94 to 298/6.2/94. Water Research Commission, Pretoria, South Africa.

Nathan, R. J. & McMahon, T. A. (1990) Identification of homogeneous regions for the purposes of regionalization. *J. Hydrol.* **121**, 217–238.

Pappenberger, F. & Beven, K. (2006) Ignorance is bliss: Or seven reasons not to use uncertainty analysis. *Water Resour. Res.* **42**, W05302, doi:10.1029/2005WR004820.

Ratto, M., Young, P. C., Romanowicz, R., Pappenberger, F., Saltelli, A. & Pagano, A. (2007) Uncertainty, sensitivity analysis and the role of data based mechanistic modeling in hydrology. *Hydrol. Earth Syst. Sci.* **11**, 1249–1266.

Refsgaard, J. C., Jeroen P. van der Sluijs, J. P., Anker Lajer Højberg, A. L., Peter A. & Vanrolleghem, P. A. (2007) Uncertainty in the environmental modelling process – A framework and guidance. *Environmental Modelling & Software* **22**, 1543–1556

Sivapalan, M., Takeuchi, K., Franks, S. W., Gupta, V. K., Karambiri, H., Lakshmi, V., Liang, X., McDonnell, J. J., Mendiondo, E. M., O'Connell, P. E., Oki, T., Pomeroy, J. W., Schertzer, D., Uhlenbrook, S. & Zehe, E. (2003) IAHS decade on Prediction in Ungauged Basins (PUB), 2003–2012: shaping an exciting future for the hydrological sciences. *Hydrol. Sci. J.* **48**(6), 857–880.

Shamir, E., Imam B., Morin, E., Gupta, H. V. & Sorooshian, S. (2005) Application of temporal streamflow descriptors in hydrologic model parameter estimation. *Water Res. Res.* **41**, W06021. Doi:10.1029/2004WR003409.

Wagener, T. & Wheater, H. S. (2006) Parameter estimation and regionalization for continuous rainfall-runoff models including uncertainty. *J. Hydrol.* **320**(1-2), 132–154.

Wagener, T., Sivapalan, M., Troch, P. & Woods, R. (2007) Catchment classification and hydrologic similarity. *Geography Compass* **1**, 10.1111/j_1749-8198.2007.00039.x.

Wagener, T., Wheater, H. S. & Gupta, H.V. (2004) *Rainfall-Runoff Modelling in Gauged and Ungauged Catchments*. Imperial College Press, London, UK. ISBN 1-860944-66-3.

Yadav, M., Wagener, T., & Gupta, H. V. (2007) Regionalisation of constraints on expected watershed response behaviour. *Adv. Water Resour.* **30**, 1756–1774.

Yang, H., Yang, D., Lei, Z. & Sun, F. (2008) New analytical derivation of the mean annual water-energy balance equation. *Water Resour. Res.* **44**, W03410, doi:10.1029/2007WR006135.