Inverting the hydrological cycle: when streamflow measurements help assess altitudinal precipitation gradients in mountain areas

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Abstract This paper presents an attempt to "invert" the hydrological cycle and to use streamflow measurements to improve our knowledge of precipitation input in data-sparse mountainous regions. We use two data sets of 31 Swiss and 94 Swedish catchments, and three simple long-term water balance formulas. By assuming a simple two-parameter correcting model to regionalize precipitation from the too-sparse precipitation gauging network, we show that it is possible to identify, without ambiguity, the altitudinal precipitation gradient from streamflow. Although the snow undercatch parameter remains more difficult to identify, its range seems coherent with values found in the literature.

Key words orographic precipitation gradient; water balance formulas; Budyko formula; Ol'dekop formula; Turc formula

INTRODUCTION

Catchments produce streamflow as a complex answer to energy and precipitation fluxes. Precipitations are extremely variable, both spatially and temporally, and the knowledge of (at least) its areal mean is a prerequisite to any serious water balance computation. But we sometimes find ourselves in a situation where we can only work with a rather rough estimate of the incoming precipitation flux, not only because of the uncertainties which arise from the too-limited spatial-sampling ability of precipitation gauging networks (Rodda, 1968), but also from instrument- and location-specific factors (Sevruk & Nespor, 2000). Sugawara (1993) was correct when he stressed that "*we cannot observe the mean areal precipitation itself*"; in fact, the catchment is the only system to *know* the mean areal precipitation it receives. This situation holds for any but the most densely gauged experimental catchments … and it is obviously worst in data-sparse mountainous regions.

Scope of the paper

For years, hydrologists have been striving to cope with precipitation input estimation errors, in order to be able to run their hydrological models. In this paper, our idea is to look the opposite way, in order to see how we can "invert" the hydrological cycle, by using streamflow measurements in order to infer the elevation–precipitation relationship in mountainous regions.

We work here with a large data set of 94 Swedish and 31 Swiss catchments, for which we have daily streamflow measurements, as well as point precipitation measurements. Due to the great spatial variability of precipitation in these mountainous areas, it is necessary to account for the elevation–precipitation dependency to get a reasonable catchment water balance. Our goal is to parameterize this relationship by "inverting the hydrological cycle", i.e. using the streamflow measurements in order to guess how much rain falls at the higher elevations where no observations are made.

Relevant literature

There is a large literature dealing with the impact of precipitation uncertainties on hydrological modelling (see e.g. the review by Andréassian *et al.*, 2001). However, much of this literature focuses on synthetic rainfall fields, and deals with the propagation of precipitation estimation errors within a hydrological model. Only a few hydrologists seem to have been interested to invert their hydrological model, i.e. to use streamflow measurements to try to improve the catchment precipitation estimate. Linsley & Crawford in 1965, and Hamlin in 1983, mentioned the question.

In 1993, Sugawara was more explicit, by advocating that when computing a weighted precipitation estimate, the weights of each precipitation stations should be determined taking into consideration their usefulness for discharge calculation. More recently, the BATEA approach of Kuczera *et al*. (2006) uses streamflow measurements to infer storm-dependent correcting parameters.

As far as the specificities of mountainous regions are concerned, there is also a large literature on altitudinal precipitation gradients (Barry, 1992; Sevruk, 1997; Sevruk & Mieglitz, 2002). To our knowledge, however, the only attempt to explicitly use streamflow data is that of Weingartner *et al.* (2005), who produced a spatialized precipitation map of Switzerland on the sole base of streamflow measurements and evapotranspiration estimates.

Our method differs from the above approach in that we do not choose streamflow against precipitation measurements or *vice versa*. We use both sources of information in a complementary way, with streamflow measurements being utilized in order to estimate the parameters of a correcting model, addressing: (i) the altitudinal precipitation gradient, and (ii) a snow undercatch correction factor.

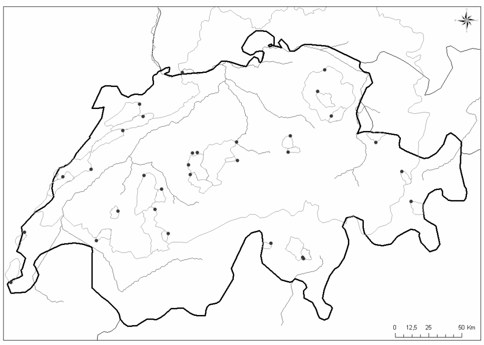
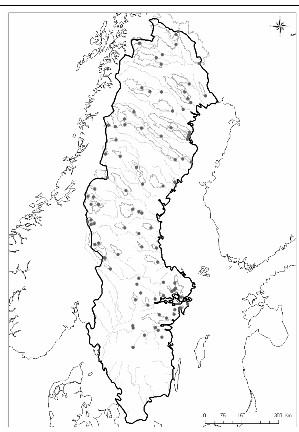
MATERIAL

This paper is based on the use of two large catchment sets, one in Switzerland, the other in Sweden. We use three simple, widely known water balance formulas.

Catchment set

We used a database of 94 Swedish catchments and 31 Swiss catchments (see Table 1). Data are available between 1995 and 2005.

Table 1 Summary of the data set characteristics.

Swiss data set	Swedish data set
	
31 catchments (1 to 35 000 km²) 30 air temperature gauges 166 precipitation gauges DEM grid : 250 m × 250 m	94 catchments (1 to 15 000 km²) 247 air temperature gauges 626 precipitation gauges DEM grid : 600 m × 600 m

Water balance formulas

We use here three classical water balance formulas, which relate long-term average catchment streamflow to long-term catchment precipitation and potential evapotranspiration: the Budyko (1974) formula, the Ol'dekop (1911) formula and the Turc (1954) formula as reformulated by Le Moine *et al*. (2007).

Table 2 List of the three water balance formula used in this study (Q – long-term average catchment streamflow in mm/year; P – long-term average catchment areal precipitation in mm/year; PE – long-term average catchment potential evapotranspiration in mm/year).

Name	Water balance formula	
Budyko	$Q = P * \left\{1 - \sqrt{\frac{PE}{P} \cdot \left(1 - \exp\left(-\frac{PE}{P}\right)\right) \cdot \tanh\left(\frac{P}{PE}\right)}\right\}$	Equation (1)
Ol'dekop	$Q = P * \left\{1 - \frac{PE}{P} \cdot \tanh\left(\frac{P}{PE}\right)\right\}$	Equation (2)
Turc- Le Moine	$Q = P * \left\{1 - \frac{1}{\left\{1 + \left(\frac{P}{0.9 * PE}\right)^{2.5}\right\}^{1/2.5}}\right\}$	Equation (3)

METHODOLOGY

Each of the water balance formulas in Table 2 expresses catchment discharge (Q) as a function of catchment areal precipitation (P) and potential evapotranspiration (PE). PE is estimated with a formula proposed by Oudin *et al.* (2005), which depends only on extra-terrestrial radiation (a function of the Julian day and latitude) and temperature. P is estimated by a classical inverse-distance weighted average of precipitation measured at point gauges, after applying two corrections: first a correction accounting for snow undercatch, second a correction accounting for the orographic gradient.

Correction for snow undercatch

For each precipitation gauge, a first correction is made to account for the undercatch of solid precipitation; it depends on a single parameter α, which needs to be estimated:

$$\text{if } T_k(j) < 0, \text{ then } P_k^{cor}(j) = \alpha \times P_k(j) \tag{4}$$

where $Tk(j)$ is temperature at precipitation gauge k on day j in °C; $Pk(j)$ is precipitation measured at gauge k on day j; $P_k^{cor}(j)$ is precipitation corrected for snow undercatch; α is the snow undercatch correction parameter (to be estimated).

Correction for orographic gradient

Then, we transfer the daily information on a grid covering each catchment. At each grid point, daily precipitation is estimated as a weighted sum of neighbouring precipitation gauges. A correction is applied to account for the elevation-precipitation gradient, which depends on a second parameter, β:

$$P(j) = \frac{1}{\sum_{k=1}^{n} w_k} \times \sum_{k=1}^{n} w_k \times P_k^{cor}(j) \times \exp[\beta(z - z_k)] \tag{5}$$

where $P(j)$ is precipitation at a grid point on day j; $P_k^{cor}(j)$ is corrected precipitation for gauge k on day j; β is orographic correction parameter (to be estimated); n is total number of neighbouring gauges; z is elevation of the grid point considered; z_k is elevation of gauge k and w_k is the weighting factor of gauge k (it is based on the inverse distance between precipitation gauges: $\frac{1}{d^\delta}$, where δ is a parameter to be calibrated by trial-error. δ equals 2.5 in Switzerland and 1 in Sweden).

Note that we adopted here a multiplicative correction model to avoid the threshold effect of a subtractive model, and that the exponential-based formulation is the simplest, allowing the symmetry of the transfer of information. The parameter β (formulated in m^{-1}) needs to be estimated, by optimizing a criterion measuring the proximity between the measured long-term Q_{obs} and the calculated long-term Q_{sim}.

Criterion used

We look for the optimal α and β values by trial and error: for each catchment, we average all $P(j)$ to obtain a long-term value, and we then use the three formulas in Table 1 to compute Q. The optimal couple (α,β) is the one that gives the smallest relative error for each of the data sets.

Mean Absolute Relative Error (*MARE*) is computed as:

$$MARE = \frac{1}{N} * \sum_{i=1}^{N} \left| \frac{Q_{obs}(i) - Q_{sim}(i)}{Q_{obs}(i)} \right| \tag{6}$$

where $Q_{obs}(i)$ is the observed long-term streamflow for catchment i (in mm); $Q_{sim}(i)$ is the computed long-term streamflow for catchment i (in mm); and N is the total number of catchments in the data set.

RESULTS

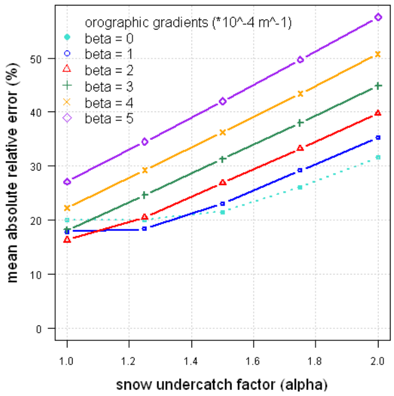
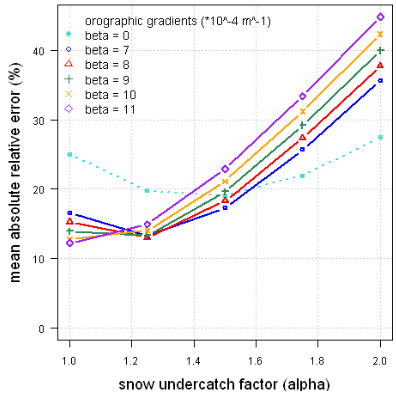
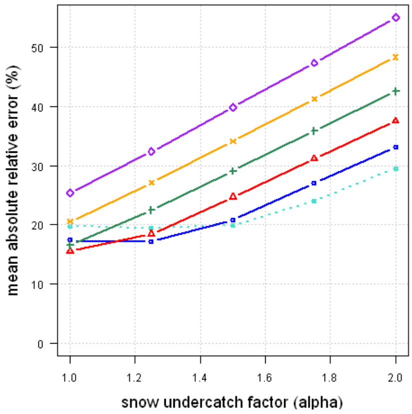
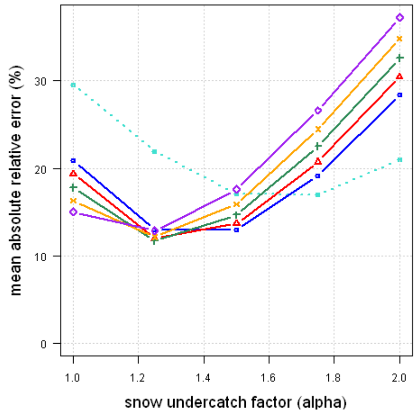
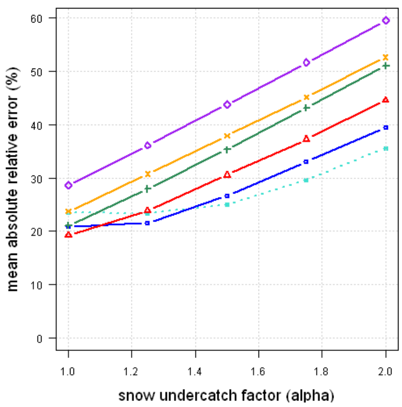
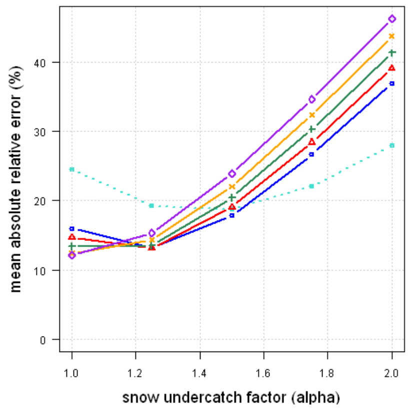
Results are summarized in Table 3. We can see that the results differ between Switzerland and Sweden, but that within a data set, the three water balance formulas give coherent results. The main findings are:

1. Correcting only snow undercatch (with β = 0 and α optimized) is clearly not enough: improving the areal precipitation estimation does require the introduction of an altitudinal precipitation gradient. Indeed, if we neglect the impact of the altitudinal precipitation gradient (β = 0), the performances of the three formulas drop considerably.
2. Correcting only the altitudinal precipitation gradient (with α = 1 and β optimized) can be enough, although an α of 25% does reduce the error a little with the Ol'dekop model. This value is consistent with those found in the literature (Sevruk & Nespor, 2000).
3. Optimal altitudinal precipitation gradients are very different in Switzerland and Sweden: 2×10^{-4} m^{-1} in Switzerland, and 9 to 11×10^{-4} m^{-1} in Sweden (depending on the formula).
4. One of the reasons why the snow undercatch factor, α, is so poorly identifiable is that both parameters interact strongly: indeed, with the interannual formulas used here, an underestimation of the altitudinal gradient β can be somewhat compensated by an overestimation of the snow undercatch factor α. Another reason is the site-specific nature of snow undercatch problems (Sevruk & Nespor, 2000): we are not able to differentiate between shielded and non-shielded gauges, as well as between gauges located in an open place and others.

CONCLUSION

The aim of this paper was to assess whether it is possible to "invert" the hydrological cycle and to use streamflow measurements to improve our knowledge of precipitation input in data-sparse mountainous regions. We used two data sets of 31 Swiss and 94 Swedish catchments and three simple long-term water balance formulas: Budyko's, Ol'dekop's and Turc-Le Moine's. We assumed a simple two-parameter correcting model to regionalize precipitation from the too sparse precipitation gauging network: the first parameter (α) aimed to correct snow undercatch by precipitation gauges, while the second parameter (β) targeted the precipitation–elevation relationship.

Table 3 Results presented for the three water balance models and the two catchment sets (note: the lower the *MARE* – mean average relative error – the better the model).

<table>
<tr><th>Model</th><th>Switzerland</th><th>Sweden</th></tr>
<tr><td>Budyko</td><td></td><td></td></tr>
<tr><td>Ol'dekop</td><td></td><td></td></tr>
<tr><td>Turc- Le Moine</td><td></td><td></td></tr>
</table>

Our results show that it is possible to identify the precipitation-elevation relationship (β) from streamflow, while the snow undercatch parameter (α) remains more difficult to identify.

Although these preliminary results are encouraging, we see several further issues which would be interesting to address. It would be instructive to compare this method with the more traditional approach which calibrates the precipitation-elevation relationship by trying to reconstitute point precipitation measurements. It would also be interesting to compare our estimates with those of Weingartner *et al.* (2005).

Last, we would also like in the future to use more complex hydrological models, working at finer time steps.

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