Relevance of the uncertainty in evapotranspiration inferences for surface water balance projections in mountainous catchments

FREDDY SORIA
Agua Sustentable, Water Resources Section, Irpavi entre calles 12 y 11D, La Paz, Bolivia
soriakaigan@gmail.com

Abstract The paper draws attention to the relevance of the predictive uncertainty in potential evapotranspiration (PET) calculations, towards improved surface water balance calculations in remote high-elevation catchments. The study is in two Andean catchments in Bolivia; the first is in the headers of the Amazon basin and the second is in the headers of the Uyuni basin. The common feature at both sites is the high altitudinal gradient. A semi-distributed water balance model and a Monte Carlo-based sensitivity analysis are employed in the study. In general, for a given modelling condition, results show that the sensitivity of the water balance to an imperfect measuring network is likely to induce uncertainty ranges as high as 53 L^{-1} km^{2}. In addition, results have shown that the water balance in Andean mountainous systems under arid conditions is likely to be more sensitive to variations in the PET than their humid counterparts.

Key words tropical Andes; sensitivity analysis; surface water balance

INTRODUCTION

An important advance in surface and subsurface hydrological research is the growing interest on assessing the final user's degree of belief in the modelling products (Montanari *et al*., 2009), as a form to bridge the gap between research and practice. Following such a path, our concern is to draw attention to the practical relevance of the predictive uncertainty range in potential evapotranspiration calculations, towards improved surface water balance studies in remote high elevation mountainous catchments. In this paper, such practical relevance is measured in terms of catchment discharge, which is calculated using a semi-distributed surface water balance model. The methods involve the analysis of model outcomes produced by computational experiments designed and carried out under a Monte Carlo approach. The result from the analysis is the investigation of the model predictive uncertainty through the investigation of the global sensitivity of the model.

STUDY AREA AND DATA

The computational experiments are conducted for two remote poorly-gauged basins in the tropical Bolivian Andes, which develop along a spatially heterogeneous and high elevation region. The first catchment is located in the headwaters of the Amazon basin, whereas the second catchment is located in the headwaters of the Uyuni basin. The distance between both basins is approximately 240 km; a common feature in both sites is the large difference in altitude between the headwaters and the catchment outlet. The former catchment was studied in Soria & Kazama (2011), whereas the latter uses unpublished data collected at a remote site where no comparable research has been carried before the current initiative.

The former catchment has an area of 1471 km^2, with the headwaters located in the remote highlands of the Cordillera Real (15.8S to 16.3S), in a sub-basin of the Amazon River basin. The high elevation of the Cordillera determines variations in altitude of 5500 m a.m.s.l. within a horizontal distance of 50 km on average. Along the catchment, such high altitudinal gradient defines an equally enormous spatial heterogeneity, along landscapes with small glacier caps in the headwaters and tropical forests in the downstream area. Regarding the hydroclimatic network, eight stations situated between 4800 and 1196 m a.m.s.l. record the precipitation on a daily basis. Along the network, only two climatic stations record climatic variables for the calculation of evapotranspiration. No pan evaporation information is available. Two gauging stations record discharge on a daily basis. The precipitation is monitored by the local hydropower generation company, COBEE-Bolivia. Climatic data is monitored by the National Meteorological and Hydrological Service, SENAMHI-Bolivia. In general, this basin is not under arid or semi-arid conditions (i.e. the annual average precipitation of 400 mm surpasses the annual average evapotranspiration of 141 mm).

To the south of the Cordillera Real is the second study site, downstream of the volcanic Andean Western Cordillera. In this mountain range and partially within the study catchment, resides the Sajama Volcano, the highest peak of the Andes in Bolivia (6542 m a.m.s.l.). This study site is located in Sajama National Park, on the headwaters of the Uyuni basin. The basin has an area of 568 km^2. It is located 240 km to the south of the former study site, in a region where on an annual basis the climatic conditions are in general semi-arid, with an overall annual average precipitation of 350 mm and higher annual potential evapotranspiration values. The precipitation, as well as the climatic data for the calculation of the potential evapotranspiration, are recorded daily at the Sajama weather station (SENAMHI-Bolivia) at an elevation of 4255 m a.m.s.l., and at the Chungará station (Chile) at an elevation of 4563 m a.m.s.l. Given the remote conditions of the study site, for the estimation of altitudinal lapse rates, additional data available for a single hydrologic year (2011–2012) from the Comisario weather station (4340 m a.m.s.l.) was used, which is monitored by Agua Sustentable. No pan evaporation information is available. For the current and the former basins, topographic information is obtained from the SRTM DEM (Rodriguez *et al*., 2005). The land use and vegetation data are obtained from the Vegetation Map of Bolivia at scale 1:250 000 (Navarro & Ferreira, 2007).

TOOLS AND METHODS

Response surfaces (Beven, 2004) are employed to evaluate the outcomes of a semi-distributed surface water balance model. Such outcomes are generated through computational experiments designed under a Monte Carlo-based approach. The main features of the tools and methods are described below.

Monthly semi-distributed water balance model

The model is semi-distributed in 10 horizontal buckets for the site in the headers of the Amazon basin and four horizontal buckets for the site in the headers of the Uyuni basin. For both study sites the boundaries of the horizontal buckets are in general defined by contour levels at every 500 m. The vertical buckets are composed by a surface layer which quantifies the response of the surface runoff, and two subsurface layers which quantify the response of the subsurface runoff; the sum of both contributions define the total monthly discharge at the outlet of the basin.

A saturation-excess runoff response is the basis of the perceptual model. The conceptual model is described in equation (1) (Collick *et al*., 2009), where S (L) is the soil water storage volume, t is time, Δt is time step, P (L/T) is the measured monthly rainfall intensity, Rse (L/T) is saturation excess runoff rate, $Perc$ (L/T) is percolation, and Ea (L/T) is the actual monthly evapotranspiration. The Ea is the monthly potential evapotranspiration PET multiplied by the number of rain days observed (*raindays* (days)). The PET is calculated from measured climatic variables under the FAO Penmann-Monteith method (Allen *et al*., 1998). The calculation of Rse and $Perc$ is explained below.

$$S_t = S_{t-\Delta t} + (P - Rse - Ea - Perc)_t \, \Delta t \qquad (1)$$

In the surface water balance model calculations, when P is higher in value than Ea, S_E (L) which becomes either Rse or $Perc$, is calculated with equation (2) (Collick *et al*., 2009), where Csc (non-dimensional) is the threshold when surface runoff occurs. Csc is calculated as the difference between the maximum soil storage S_{Tmax} (L) and the soil storage at wilting point S_{wilt} (L). Cse (non-dimensional) is a calibrated value (it is the only calibratable value) that decides the proportion of water that is converted into $Perc$ (equation (3)) or Rse (equation (4)). When P is lower in value than Ea, the soil water depth above saturation S_E is assumed to become zero. After Soria &

Kazama (2011), given the low sensitivity of the model to variations in *Csc*, that model factor is set to 0.2. The S_{wilt} is inferred from the information provided in Navarro & Ferreira (2007); the S_{Tmax} is assumed to be equal to the potential maximum retention after runoff begins, whose value is calculated through the SCS method for abstractions. The SCS method is explained elsewhere (e.g. Chow *et al.*, 1994). The initial conditions for *S* is calculated from the product of S_{Tmax} (L) by *Csc*, under the consideration that the effect of the initial conditions are likely to affect only the response at the initial time steps of the calculation.

$$S_{E,t} = S_{t-\Delta t} + (P - Ea)_t \Delta t - Csc(S_{T\,max} - S_{wilt}) \tag{2}$$

$$Perc = Cse * S_E / \Delta t \tag{3}$$

$$Rse = (1 - Cse) * S_E / \Delta t \tag{4}$$

The contribution of *Perc* to the groundwater storage (S_{GW} (L)) is calculated with equation (5). The contribution to groundwater flow over a unit of surface area (R_{GW} (L/T)) is calculated with a linear reservoir model (equation (6)), where *k* (non-dimensional) is the recession constant calculated from a series of observed discharge.

$$S_{GW,t} = S_{GW,t-\Delta t} + (Perc_t - R_{GW,t-\Delta t}) \cdot \Delta t \tag{5}$$

$$R_{GW,t} = S_{GW,t-\Delta t}(1 - e^{-k_t}) / \Delta t \tag{6}$$

Numerical experiments, uncertain model parameters and uncertainty analysis

The computational experiments are carried out under a Monte Carlo approach (Saltelli, 2000). Within that framework, first, the sample set of sensitive model factors is generated. The size of the sample is *n* and in our case the sample is generated under a Sobol scheme (Saltelli, 2000). Each set of model factors is fed into the water balance model, a number of *n* model runs are carried out and an equal number of model outcomes are generated. Before the latter mentioned step, the surface water balance model is calibrated using discharge measurements at the outlet of the two basins. In order to reduce the number of computational experiments, the surface water balance is calculated in two horizontal buckets for the two catchments studied. Finally, after having obtained the *n*, model outcomes are constructed as 2-D response surfaces of specific discharge (i.e. the response variable) *vs* model factor (i.e. the explanatory variable), for each model factor considered in the numerical experiments. For the interpretation of the response surfaces, the recommendation of Beven (2004) is adopted; he points out that a model parameter that dominates the response of the system, is likely to have plots whose trends follow identifiable patterns. Even though the sample set for the Monte Carlo experiments is generated under the Sobol sampling scheme for first and second order sensitivity indices calculations (Saltelli, 2000), the investigation presented here only analyses the model outcomes through response surfaces; the analysis through sensitivity indices will be carried out in a future publication. The aspects considered in the computational experiments at each study site are detailed below.

For the catchment in the headwaters of the Amazon basin, Soria & Kazama (2011) carried out Monte Carlo experiments for the period from September 1981 to August 1982 (n = 2048). The model parameters considered to be uncertain were *Csc*, S_{Tmax}, S_{wilt} and *Cse*. In addition, in order to assess the predictive performance of the water balance model under an imperfect measuring network, the observed *P*, *PET*, and *raindays* were also assumed to be uncertain. The assumption of input data as uncertain information may serve in the assessment of the impacts of changes in climatic conditions; however, considering the complexity of such an affirmation, the results were not strictly analysed from such a perspective. In reference to the probabilistic distribution considered, ignorance was assumed on the model predictive response. As a consequence the behaviour of the uncertain variables was explained by uniform probabilistic distribution functions. Having a heterogeneous study basin, the assessment was carried on two representative buckets: Z1 (rainforest) and Z9 (sparse vegetation, shallow soil depths; downstream of the glacierized bucket).

For the numerical experiments, uncertainty bounds were assumed to fall within a range ±20% in reference to calibrated and observed values. In the current paper, given the experience gained under the usage of the water balance model, the number of relevant (i.e. sensitive) factors are reduced to three: *Cse*, *P*, *PET*, and the results initially presented in Soria & Kazama (2011) are enhanced under the same scheme by increasing the number of replications to $n = 4096$.

For the design of the numerical experiments at the catchment in the headwaters of the Uyuni basin, the same considerations mentioned in the paragraph above are adopted. The number of sensitive factors are also three: *Cse*, *P* and *PET*; $n = 4096$; the uncertainty range is ±20%. Model factors are also assumed to be uniformly distributed. The Monte Carlo experiments are carried out over the period March 2005–January 2006, which is when the highest precipitation rates were measured within the entire recording period. Also, with the objective to increase and contrast the water balance model responses, the assessment on this catchment is carried out for the downstream most horizontal bucket R1 (a non-arid bucket surrounded by Andean wetlands) and the high elevation horizontal bucket R3 (downstream of the glacierized bucket).

At both study sites, the investigation of the model predictive uncertainty is carried out for the wettest observations (January 1982 and January 2006). The wettest month is assumed to be a relevant uncertainty indicator of the model performance, because during such wet periods strong relationships between model response and hydroclimatic conditions are likely to occur (Yapo *et al.*, 1996). Table 1 summarizes the calibrated and observed values considered in the experiments.

Table 1 Calibrated and observed values for the model parameters (wettest observation). Z1 and Z9 are horizontal buckets that belong to the catchment in the headers of the Amazon basin; R1 and R3 are in the headers of the Uyuni basin. S_{wilt}, *Csc*, $S_{Tmáx}$, *Cse* are model factors; *P*, *PET*, *raindays* are model inputs.

Altitudinal range of the horizontal bucket (m a.m.s.l.)	Calibrated values				Observed values		
Catchment 1 (In the headers of the Amazon basin)	S_{wilt} (mm)	*Csc*	$S_{Tmáx}$ (mm)	*Cse*	*P* (mm)	*PET* (mm)	*raindays* (days)
Z1: 500–1000	10.0	0.2	100.0	0.2	610.0	141.0	30.0
Z9: 4001–4500	5.0	0.2	40.0	0.8	247.0	141.0	10.0
Catchment 2 (In the headers of the Uyuni basin)	S_{wilt} (mm)	*Csc*	$S_{Tmáx}$ (mm)	*Cse*	*P* (mm)	*PET* (mm)	*raindays* (days)
R1: 4120–4340	41.0	0.2	51.0	0.2	211.5	235.9	23.0
R3: 4500–5000	30.0	0.2	149.0	0.8	211.5	146.8	23.0

RESULTS

According to the outcomes of the numerical experiments summarized in the response surfaces of Fig. 1, the uncertainty range originated in an imperfect measuring network varies according to the system being modelled. In addition to this aspect, which was expected as we carried out a water balance study in high elevation-mountainous catchments, the outcomes from the Monte Carlo computational experiments provide an insight to the imperfect knowledge we have regarding the relevance of the uncertainty contained in the estimations of model inputs, towards the assessment of the water balance predictive uncertainty range.

When comparing the response surfaces of the uppermost buckets modelled, results have shown that their trends and tendencies are similar (R3 and Z9 in Fig. 1), with patterns of the factor *P* that appear to be as well defined (identifiable) as the patterns drawn by the plots of *PET* and *Cse*; however, the differences arise when their respective uncertainty ranges are compared, which are shorter for *P* compared to the respective ranges calculated for *PET* and the model factor driving the model response *Cse*. In practice, our assumed ignorance of the system response, implicitly represented through the uncertainty range, originated in an imperfect inference of the factor *PET*, suggests an implied uncertainty range of 37 L/s/km^2 on average, measured in terms of catchment specific discharge in the two basins investigated. In addition, results suggest that an imperfect knowledge of factor *P* implies an uncertainty range from 32 to 20 L/s/km^2 in terms of specific

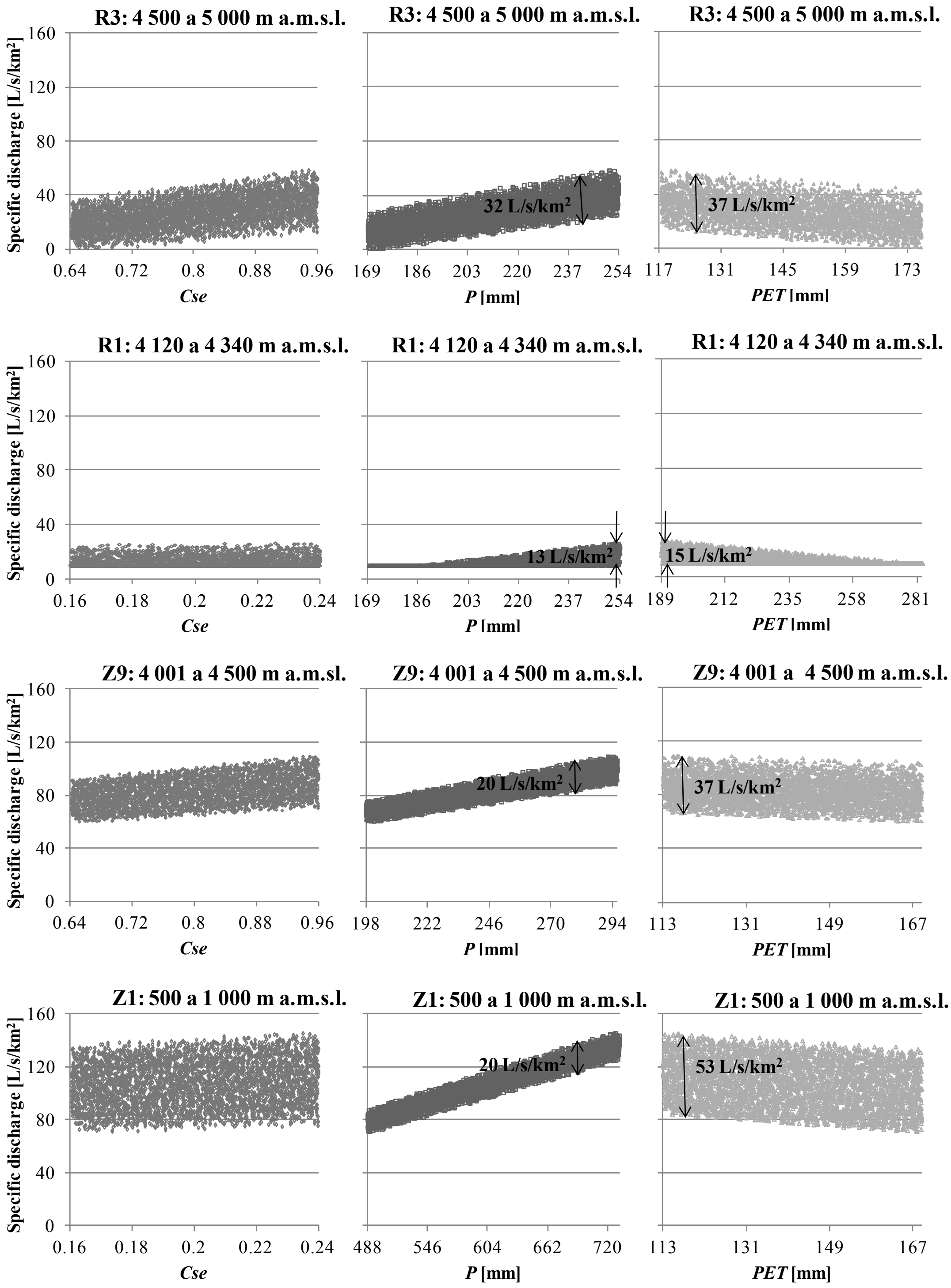

Fig. 1 Comparison between response surfaces in horizontal buckets for the catchment situated in the headers of the Uyuni basin (R3 and R1) and the catchment situated in the headers of the Amazon basin (Z9 and Z1). The sample size is n = 4096. The horizontal axis shows the uncertainty range for the explanatory variables; the response variable is represented in the vertical axis. The graphs are ordered in a sequence that emphasizes the variability of the model responses with the altitude. The graph aims to demonstrate that in general, the more humid the conditions (Z1), the lower the relevancy of the *PET* (i.e. the shape of the *PET* patterns would draw a trend that is less defined, which means that the parameter is unlikely to become a dominant factor of the system). In contrast, it is also aimed at demonstrating that the more arid the conditions (Z9, R1, and eventually R3), the highest the identifiability and adequacy of *PET* to predict the behaviour of the response variable in the water balance model.

calculated discharge in buckets R3 and the Z9, respectively. Such a comparison suggests that given the identifiable patterns and the small uncertainty range found in the *P* response surface, the model structure proposed to represent the surface water balance is adequately described by the response of factor *P*.

As the catchments develop in the downstream direction and the humidity of the system increases, the patterns tend to differ. In the site in the headwaters of the Amazon basin, the identifiability of *PET* and *Cse* decreases notably, suggesting that its adequacy to describe the response variable also decreases; simultaneously, the identifiability of *P* remains high, suggesting that its relevance on the description of the response variable remains as good as the one observed in buckets R3 and Z9. In the case of the latter mentioned variable, the width of the uncertainty range remains relatively short (20 L/s/km^2). In contrast, in the bucket of the basin in the headwaters of the Uyuni basin the aridity increases (i.e. the mean values of the *PET* surpass the mean values of *P*, Fig. 1), and the identifiability of the *PET* appears to be as equally relevant as the identifiability in *P*, suggesting an equally high adequacy of both factors for the explanation of the response variable.

CONCLUSIONS

After comparing the water balance results obtained for the horizontal buckets studied, our computational experiments suggest that: (i) in general, for a given modelling condition, the sensitivity of the water balance to an imperfect measuring network in mountainous Andean catchments with high altitudinal gradients is likely to induce uncertainty ranges as high as 53 L/s/km^2 (Fig. 1, bucket Z1). Our results also suggest that (ii) Andean mountainous systems under arid conditions are likely to be more sensitive to variations in the *PET* than their humid counterparts. The conclusions are accomplished after comparing the response surfaces of the buckets Z9 and Z1, and can be confirmed after comparing the results from the computational experiments carried out in buckets R3 and R1; notice that in Z1 there is no deficit in the surface water balance and the component that dominates the response of the system is *P*, but as the altitude increases, in Z9 the system gradually reaches an state where *P* decreases and the *PET* rates are similar in magnitude to the former. Among the recommendations, our results suggest to water balance modellers that they pay particular attention to the establishment of a measuring network that collects data at different altitudes (including gauging stations), in order to reduce the resulting predictive uncertainty range.

Acknowledgements The research is carried out by Agua Sustentable, supported by The Nordic Environment Finance Corporation and Diakonia-Sweden. The data for the basin in the upper Amazon basin is provided by COBEE-Bolivia and SENAMHI-Bolivia. The data for the Sajama National Park is provided by Agua Sustentable. Thanks are also due to Magalí García for the analysis of the climatic data, and to field technicians in Agua Sustentable and to Mathieu Beaulieu (CUSO-Canada).

REFERENCES

Allen, R., Pereira, L., Raes, D. & Smith, M. (1998) Crop evapotranspiration – Guidelines for computing crop water requirements. *FAO Irrigation and drainage paper 56*. FAO – Food and Agriculture Organization of the United Nations, Rome.

Chow, V., Maidment, D. & Mays, L. (1994) *Applied Hydrology*. McGraw-Hill, New York.

Collick, A., Easton, Z., Ashagrie, T., Biruk, B., Tilahun, S., Adgo, E., Awulachew, S., Zeleke, G. & Steenhuis, T. (2009) A simple semi-distributed water balance model for the Ethiopian highlands. *Hydrol. Processes* 23, 3718–3727.

Beven, K. (2004) *Rainfall Runoff Modelling – The Primer*. John Wiley, Chichester.

Montanari, A., Shoemaker, C. A. & van de Giesen, N. (2009) Introduction to special section on uncertainty assessment in surface and subsurface hydrology: An overview of issues and challenges. *Water Resour. Res.* 45, W00B00.

Navarro, S. & Ferreira, W. (2007) Mapa de vegetación de Bolivia a escala 1:250000, The Nature Conservancy: La Paz.

Rodríguez, E., Morris, C., Belz, J., Chapin, E., Martin, J., Daffer, W. & Hensley, S. (2005) *An assessment of the SRTM topographic products*. Technical Report JPL D-31639, Jet Propulsion Laboratory, California.

Saltelli, A. (2000) What is a sensitivity analysis? In: *Sensitivity Analysis* (ed. by A. Saltelli, K. Chan & E. Scott), 174–195. John Wiley, Chichester.

Soria, F. & Kazama, S. (2011) Uncertainty analysis of water balance downstream the Cordillera Real in Bolivia. In: *Tohoku Shibu Regional Conference Proceedings – Japanese Society of Civil Engineering* (JSCE). JSCE, Sendai.

Yapo, P., Gupta, H. & Sorooshian, S. (1996) Automatic calibration of conceptual rainfall-runoff models: sensitivity to calibration data. *J. Hydrol.* 181, 23–48.