

## Numerical simulation of non-equilibrium sediment transport in a flume

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**Abstract** Simulations of non-uniform bed load sediment transport processes under non-equilibrium conditions require characterization of alluvial system processes to immediately overcome variations of sediment boundary conditions. Due to the introduction of man-made sediment barriers, the amount of sediment load entering in the downstream river reach is different from that going out and river reaches adjust to a new equilibrium condition. In supply limited rivers, bed degradation can occur, while in transport limited rivers bed aggradation can occur. When alluvial streams are unable to adjust to variation of sediment boundary conditions, spatial lags or adaptation lengths are required to reach the equilibrium transport capacity. Accordingly, quantitative estimates of sediment transport in river-control engineering and water management projects are essential to evaluate the changes in riverbeds. This study focuses on erosion processes caused by decreasing sediment load imposed at the upstream boundary of a straight flume and a 1-D numerical approach is used to simulate the process.

**Key words** rivers; erosion; sediment transport; hydraulic structure; numerical simulation

### INTRODUCTION

The response of a river-bed reach to transient phenomena may produce several practical consequences and quantitative estimation of sediment transport and bed profile variations is important in river-control engineering. Many predictive mobile-bed one-dimensional (1-D) models have been developed and can be grouped into two categories: (1) decoupled models, such as HEC-6 (Thomas & Prashum, 1977), IALLUVIAL (Karim & Kennedy, 1982), FLUVIAL-12 (Chang & Hill, 1976) and others (van Niekerk *et al.*, 1992), where the equations for the water-flow and for sediment are decoupled at each time step, and (2) coupled models (Borah *et al.*, 1982; Lyn & Godwin, 1987; Rael *et al.*, 1989; Holly & Rael, 1990; Bhallamudi & Chaudhry, 1991; Saiedi, 1994; Cui *et al.*, 1996; Hu & Cao 2009), where the equations for water and sediment are solved simultaneously.

There is an ongoing debate on which mathematical approach is the most suitable to properly simulate fluvial processes. Cao *et al.* (2007) suggest that the relative time scale of bed deformation (i.e. the ratio between the time scale of bed deformation and the time scale of flow depth) is the most appropriate parameter to define the applicability regions of decoupled and coupled approaches. They state that, for shallow flows with high sediment concentrations, fully coupled models are normally required; for deep flows at low concentrations, decoupled models are mostly justified. However, decoupled models have been criticized (Saiedi, 1997; Cao *et al.*, 2002). On the basis of numerical tests, they indicate that decoupled models are numerically unstable and incapable of handling rapidly changing boundary conditions. Furthermore, the decoupled solution of governing equations is often based on total sediment load, and thus is unable to estimate mutual interactions among suspended-load and bed load within each time step, especially in non-equilibrium situations (Wu *et al.*, 2004). The coupled solution is more stable but the implementation of a coupled model for non-uniform sediment transport is rather complicated (Wu *et al.*, 2004). Although coupled models consider the transport of non-uniform sediments and the sorting process, they do not explicitly account for the complex interplay of all factors producing the bed roughness changes, in time and in space. Consequently, effort has been directed towards verifying the numerical stability and extending the applicability of decoupled models. Kassem & Chaudhry (1998) show that decoupled models often give results comparable to those of coupled models. Cui *et al.* (1996) analysed the performance of a fully coupled model and of a decoupled model in simulating aggradation and downstream fining. The performance of both models was identical even for Froude numbers near unity and when boundary conditions varied considerably over time.

Decoupled approaches are still preferred (Saiedi, 1997; Wu *et al.*, 2004; Vasquez *et al.*, 2005) and their use is justified when detailed information on rules of sediment transport and/or of the hydraulic resistance are not available; the sediment transport and the hydraulic resistance are described by empirical relations with various degrees of approximation influencing the accuracy of results from any decoupled or coupled model.

In fact, due to the complexity of the physical processes in rivers and the difficulty in collecting detailed data, one of the most typical difficulties in numerical modelling is related to the interpretation of sediment transport in non-equilibrium conditions (Barkdoll & Duan, 2008). Few of the proposed models consider movable-bed conditions and, often, simplified treatments of the sediment transport processes are applied. Some models ignore the effect of sediment transport and bed exchange on flow (Fraccarollo & Armanini, 1998), while other models do not consider the effect of sediment size gradation or simulate only suspended-load transport (Capart & Yang, 1998; Cao *et al.*, 2002). Finally, other models do not consider explicitly mutual interactions among suspended-load and bed-load and their effect on hydraulic resistance (Wu & Wang, 2008).

In this study, transient sediment transport phenomena created under steady flow conditions are examined when the upstream sediment transport rate changes with time. As an example, if the amount of sediment load is lower than the flow is capable of transporting, bed degradation occurs. The alluvial stream is unable to immediately overcome the variation of sediment boundary conditions and a spatial distance (spatial lag or adaptation length) is required to reach the equilibrium transport capacity (Armanini & di Silvio, 1988; Philip & Sutherland, 1989; Ruel *et al.*, 1989; Wu *et al.*, 2000). The spatial lag effect is particularly important just downstream of the fixed bed of the structure itself, where a scour hole develops (Nagakawa & Tsujimoto, 1980). But, because of the aforementioned uncertainty in the interpretation of transient sediment transport phenomena, only some coupled or semi-coupled 1-D models (Ruel *et al.*, 1989; Holly & Ruel, 1990; Wu *et al.*, 2004; Wu & Wang, 2008) include the spatial lag effect. The majority of experimental studies have analysed the case of the presence of a rigid-bed and zero sediment inflow at upstream boundary of a mobile-bed channel. Various aspects of local scouring downstream of the rigid-bed have been examined and relationships to predict the shape and the maximum scour depth have been also proposed (among others Mossa, 1998; Gaudio *et al.*, 2000), but very few works (Bell & Sutherland, 1983; Philip & Sutherland, 1989; Adduce *et al.*, 2004) have been conducted with the aim of determining the transient sediment transport rates.

The objectives of this study are: (1) to gain some insight into the interpretation of the transient bed-load sediment transport in numerical modelling; and (2) to verify the applicability of a 1-D decoupled approach (Termini, 2003, 2011a), which includes the formulations deduced through the experimental analysis (Termini, 2011b), to simulate the transient bed profiles in non-equilibrium situations. The advantages of the model's formulation over previous decoupled models are the separate treatment of bed-load and suspended load and the explicit inclusion of a friction factor. This allows user to take adequately into account: (1) the effect of different bed roughness conditions and of their variations in time and in space; (2) the effect of sediment properties in the hydraulic resistance formulation. Details on model formulation and numerical procedures can be also found in other works (see in Tucciarelli & Termini, 2000 for the hydraulic routing module; and Termini, 2003, 2011a for the sediment routing module). Experimental data, both appositely collected and found in the literature, have been used to assess the model's capability in transient bed profile computation. The sensitivity of model's results to the adaptation coefficient has been also analysed. It should be noted that, for the specific aims of the present work, due to the limited availability of scouring data, only laboratory experimental study cases have been used to test the model.

## GOVERNING EQUATIONS

In this paper, attention is restricted to a wide rectangular straight channel. The free surface is taken to be a horizontal line across the section and the pressure distribution is hydrostatic along the vertical. No specification of lateral distribution of sediment across the section is considered. The

numerical model is based on the depth-averaged equations and includes two decoupled modules for the hydraulic and the sediment routing.

### Equations for hydraulic routing

In unsteady flow conditions the governing equations are:

Continuity equation:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q^2}{h} \right) + gh \frac{\partial h}{\partial x} + gh \frac{\partial z_b}{\partial x} + g \frac{n^2 q^2}{h^{7/3}} = 0 \quad (2)$$

where  $h$  is the water depth,  $q$  is the flow rate per unit width,  $z_b$  is the bed level by considering an horizontal reference plane,  $t$  is time,  $x$  is the distance in flow direction,  $g$  is the gravitational acceleration,  $n$  is the Manning coefficient (that is related to by:  $n = \frac{h^{1/6}}{c\sqrt{g}}$  with  $c$  = dimensionless

Chèzy resistance factor).

### Equations for sediment routing

The bed is divided into two layers: (1) the surface active layer (thickness  $\delta_a$ ), where the exchange between the bed and the stream takes place, and (2) the subsurface layer that provides sediments when the upper layer tends to be completely eroded (Fig. 1). The surface layer and the subsurface layer contain sediments of all size fractions, but the fractions of coarser particles increase in the surface layer. In the stream ( $h$ ), the sediments are transported in suspension and the sediment motion is essentially due to turbulence and to particle fall velocity. The sediment size distribution is considered in  $N$  discrete  $k$ th size classes identified by the mean diameter,  $d_k$ , of the two limiting diameters of each class. The fractional representation of size class  $d_k$  in the active layer at the known time level is  $F_k$ .

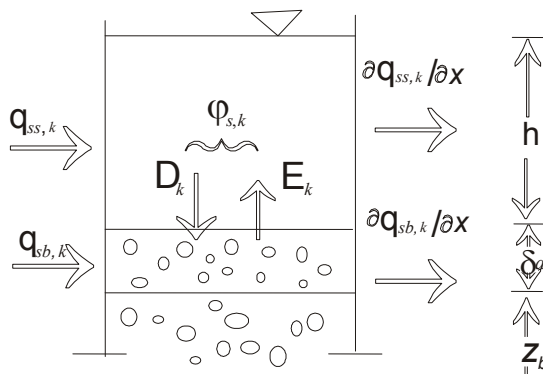


Fig. 1 Schematic representation of sediment routing.

The sediment routing consists of the following equations:

Conservation of sediments of  $k$ th size class transported as bed load:

$$\frac{\partial q_{sb,k}}{\partial x} + \phi_{s,k} = -(1 - \lambda) F_k \frac{\partial (z_b + \delta_a)}{\partial t} \quad (3)$$

Conservation of sediments of  $k$ -th size class transported in suspension:

$$\frac{\partial(C_k h)}{\partial t} + \frac{\partial(C_k q)}{\partial x} = \frac{\partial}{\partial x} \left( h K_x \frac{\partial C_k}{\partial x} \right) + \varphi_{s,k} \quad (4)$$

Exchange between bed load and suspended load for each  $k$ -th size class:

$$\varphi_{s,k} = (E_k - D_k) \quad (5)$$

Conservation of the total grain size distribution:

$$\sum_{k=1}^N F_k = 1 \quad (6)$$

where  $\lambda$  is the sediment porosity,  $q_{sb,k}$  is the actual specific (per unit width) volumetric bed-load sediment transport rate for size class  $k$ ,  $C_k$  is the vertically averaged concentration of suspended sediment of size class  $k$ ,  $K_x$  is the longitudinal dispersion coefficient for suspended sediment,  $D_k$  and  $E_k$  are, respectively, the sediment deposition and resuspension rates. Summing equations (3), (4) and (6), the global sediment conservation for all the size classes of sediment is:

$$(1 - \lambda) \frac{\partial(z_b + \delta_a)}{\partial t} + \frac{\partial \sum_{k=1}^N q_{sb,k}}{\partial x} + \frac{\partial \left( q \sum_{k=1}^N C_k \right)}{\partial x} + \frac{\partial \left( h \sum_{k=1}^N C_k \right)}{\partial t} - \frac{\partial}{\partial x} \left( h K_x \frac{\partial C_k}{\partial x} \right) = 0 \quad (7)$$

### Closure equations

The sediment routing module includes the following closure equations:

To take into account the effects of both the size of sediments and of the bed forms, the total resistance factor,  $c$ , is determined as:

$$\frac{1}{c^2} = \frac{1}{\bar{c}^2} + \frac{1}{c_A^2} \quad (8)$$

where  $\bar{c} = 2.5 \ln \left[ 11 \frac{h}{k_s} \right]$  ( $k_s = 2d_{50} =$  equivalent bed roughness;  $d_{50} =$  median sediment diameter),

$c_A$  denotes the friction coefficient related to the bed-form resistance factor. Because the energy

losses are additive,  $c_A$  is estimated as (Yalin & da Silva, 2001):  $\frac{1}{c_A^2} = \frac{1}{2} \left[ \sum_j \delta_j^2 \frac{A_j}{h} \right]$  ( $A_j =$  length of

the  $j$ -th bed form,  $\delta_j =$  steepness of the  $j$ -th bed form;  $A_j$  and  $\delta_j$  are determined using the relationships proposed by Yalin (1992)).

The value of  $K_x$  is the longitudinal dispersion coefficient for suspended sediment, determined as  $K_x = \beta 5.9 \frac{uh}{c}$  (where  $\beta$  is the coefficient of proportionality assumed equal to unity, in accordance with recent studies conducted by Graf & Cellino (2002) on suspension flows over movable beds with bed forms).

The deposition rate,  $D_k$ , is estimated as the product of the fall velocity,  $w_k$ , and the near-bed sediment concentration,  $C_{b,k}$ , of size class  $d_k$  (Celik & Rodi, 1988). Thus, it is:  $D_k =$

$w_k C_{b,k} = w_k [3.25 + 0.55 \ln(Rou)] C_k$  (with  $Rou = \frac{w_k}{u_* \kappa} = \frac{c w_k}{u \kappa} =$  Rouse parameter;  $\kappa =$  Von

Karman's constant).

The term  $E_k$  is estimated as:

$$E_k = w_k F_k P_{s,k} C_{be,k} \quad (9)$$

where  $C_{be,k}$  represents the near-bed equilibrium sediment concentration of size class  $d_k$  and  $P_{s,k}$  is

the probability to suspension for size class  $d_k$ . The term  $C_{be,k}$  is related to friction coefficient  $\bar{c}$  through the expression suggested by Garcia & Parker (1991):

$$C_{be,k} = \frac{0.39 Z_u^5}{3000000.0 + 1.3 Z_u^5} \quad ; \quad Z_u = \frac{q}{c w_k h} \left( \sqrt{(\rho_s/\rho) g d_k} d_k / \nu \right)^{0.6} \quad (10)$$

where  $\rho_s$  and  $\rho$  are density of sediment and water, respectively.

To estimate the term  $P_{s,k}$ , it is taken into account that a sediment particle is suspended by the turbulent flow when the vertical flow velocity fluctuation,  $u'$ , exceeds the fall velocity,  $w_k$ . The distribution of  $u'$  near the bed surface follows the Gaussian distribution. Thus  $P_{s,k}$  is determined as:

$$P_{s,k} = \frac{1}{\sigma_s \sqrt{2\pi}} \int_{w_k}^{\infty} \exp\left(-\frac{u'^2}{2\sigma_s^2}\right) du' \quad (11)$$

where  $\sigma_s$  is the root square error of the vertical velocity fluctuation.

The fractions of sediments left on the bed at the unknown time level  $n + 1$  are calculated as:  $F_k^{n+1} = (F_k) P_k$ , with  $P_k$  being the probability of size class  $d_k$  staying on the bed. The probability  $P_k$  coincides with the probability that the total bed shear stress,  $\tau$ , exceeds the critical shear stress for size class  $k$ ,  $\tau_{cr,k}$ . The dimensionless bed shear stress,  $\eta_k = \tau / \tau_{cr,k}$ , can be described by a Gaussian distribution (Cheng & Law, 2003) and  $P_k$  is determined as:

$$P_k = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X_k} \exp\left(-\frac{X^2}{2}\right) dX \quad \text{with} \quad X_k = \frac{\zeta_k - 1}{\sigma} \quad (12)$$

where  $\sigma$  is the standard deviation of the distribution,  $X$  is the variable of integration, and  $\zeta_k = \left(\frac{d_k}{d_{50}}\right)^{0.85}$  is the ‘‘corrective factor’’ that takes into account the hiding effect.

### Sediment transport capacity and non-equilibrium adaptation length

The estimation of the transient bed profile by equation (7) needs the determination of the spatial delay required by the alluvial stream to reach the equilibrium transport capacity.

Bell & Sutherland (1983) established that the spatial response of the local sediment transport rate is a function both of the transport rate deficit and of the spatial rate of change of the local equilibrium capacity. Thus, in accordance with Bell & Sutherland (1983), for each  $k$ -th size class, the spatial variation of local bed-load sediment transport rate could be determined through the following relationship:

$$\frac{\partial q_{sb,k}}{\partial x} = \phi_k (q_{sb,k}^* - q_{sb,k}) + \alpha_k \frac{\partial q_{sb,k}^*}{\partial x} \quad (13)$$

where  $\phi_k$  is the so-called adaptation coefficient, and  $q_{sb,k}^*$  is the equilibrium specific volumetric bed-load sediment transport rate for sediment of size class  $k$  and  $\alpha_k = q_{sb,k} / q_{sb,k}^*$ .

Different expressions for the adaptation coefficient can be found in the literature (see review in Wu *et al.*, 2004). According to such expressions,  $\phi_k$  is inversely proportional to a quantity  $L$ , which characterizes the non-equilibrium adaptation length. Bell & Sutherland (1983) related  $L$  to the magnitude of the scour hole and, thus, to the time  $t$  (i.e.  $L = t$  or  $L = 1 + 0.5t$  with  $t$  in hours). Other authors assume  $L$  to be a function of the average grain step length, which depends on the sediment size and on some measure of the excess bed shear stress over the critical value (Philip & Sutherland, 1989; Holly & Ruel, 1990; Wu *et al.*, 2000). Philip & Sutherland (1989), on the basis

of the comparisons between the different existing relations expressing the average grain step length, identified the relation  $L = \alpha_L (\theta - \theta_c) d_{50}$  (with  $\alpha_L = 4000$ ,  $\theta$  = grain Shield's parameter and  $\theta_c$  = critical grain Shield's parameter) as that allowing the best approximation to experimental data. Other authors related  $L$  to the dimension (length) of dominant bed form (Wu *et al.*, 2000) and, thus, proportional to water depth (if dunes are the dominant bed forms), i.e.  $L = 6h$  (Yalin, 1992), or to channel width  $B$  (if alternate bars are the dominant bed forms), i.e.  $L = 6B$  (Yalin, 1992).

In the present work, the spatial variation of local bed-load transport rate, with a new expression of the adaptation coefficient, has been defined on the basis of experimental data appositely collected in a straight laboratory channel under condition of zero sediment inflow.

The equilibrium specific volumetric bed-load rate  $q_{sb,k}^*$  is estimated as:

$$q_{sb,k}^* = s_k q_{sb,ku}^* \quad (14)$$

where  $q_{sb,ku}^*$  indicates the equilibrium specific bed-load transport rate for uniform sediment of size  $k$  and  $s_k = F_k P_k \zeta_k$  is a weighting factor of the size class  $k$  in the active layer. The term  $q_{sb,ku}^*$  is determined through Bagnold's formula for undulated bed (see Yalin & da Silva, 2001, p. 60):

$$q_{sb,ku}^* = \frac{bu (\lambda_c^2 \tau - \tau_{cr,k})}{\gamma_s c} \quad (\text{where } \gamma_s \text{ is the specific weight of grains in the fluid, } b \text{ is a function of the}$$

physical characteristics of flow and sediment,  $\lambda_c = c/\bar{c}$  expresses the reduction of  $c$  factor due to the presence of bed forms).

## EXPERIMENTAL RESULTS

### Experimental conditions and observations

Experiments were conducted in a rectangular, straight laboratory flume constructed at the Dipartimento di Ingegneria Civile, Ambientale e Aerospaziale, University of Palermo (Italy). The channel is 7 m long and the channel width is 0.40 m. The channel banks were rigid and realized with strips of Plexiglas 0.25 m high. The bed was of quartz sand ( $d_{50} = 0.65$  mm, geometric standard deviation  $\sigma_g = 1.334$ ); a reach (1 m long) of roughened fixed bed was realized upstream of the mobile-bed channel reach. The initial longitudinal bed slope was 0.45%. The thickness of the sand at the downstream end of the mobile-bed channel was maintained constant by a bed sill of height 0.105 m. Two runs, characterized by different values of the water discharge, were carried out. Table 1 reports the hydraulic conditions for each run:  $Q$  is the water discharge  $h_o$  is the initial overall water depth,  $Re^*$  is the shear Reynolds number,  $Fr$  is the Froude number. For both runs, the initial ( $t = 0$ ) sediment supply at the upstream boundary of the channel was set equal to zero. The experiments were conducted under steady flow conditions: the outlet and the inlet structures of the flume were connected to a hydraulic circuit allowing a continuous recirculation of both the stable discharge and the sand transported by flow. The bed profiles were measured during each run, along five longitudinal axes (axes symmetrical with respect to the channel axis) at different time steps (initial time step of 3 minutes), using a profile indicator PV09 by Delft Hydraulics (precision of 0.1 mm). The PV09 is designed to establish a constant distance of almost 1 mm between the probe and the bed, maintaining a constant electric capacity. The instrument is able to monitor the variation of the bed profile, sampling a value per second. The instrument moved along the longitudinal direction, through a carriage running over slides parallel to the channel banks, and along the transverse direction, by means of a one-dimensional Motion Control System by MICOS s.r.l. During each run, the process evolution was also monitored through a video-camera.

The analysis of measured bed profiles highlighted the formation of an evident scour hole downstream of the rigid-bed channel reach. Downstream of the scour hole, bed forms (alternate bars) formed on the bed. Each experiment was stopped after the "equilibrium condition" was

reached. Such a condition was established to exist when the maximum scour depths assumed stable values. The reaching of the equilibrium condition was checked through the measured bed profiles, as explained in a previous work. The duration of run 1 was 8 hours and the duration of run 2 was 7 hours.

Through the analysis of measured bed profiles, peculiar geometrical features of the scour hole were examined. At equilibrium, the scour length,  $L_s$ , varied in the range  $0.3Lc < L_s < 0.4Lc$  ( $Lc$  = length of the mobile-bed channel reach).

**Table 1** Hydraulic data of experimental runs.

Run	$Q$ ( $\text{m}^3 \text{s}^{-1}$ )	$h_o$ (m)	$Re^*$	$Fr$
1	0.007	0.038	58.43	0.46
2	0.013	0.060	70.35	0.54

### Spatial variation of bed-load sediment transport

Using the measured bed levels data, the instantaneous volumes of sand eroded and, thus, the instantaneous values of sediment bed-load transport were estimated at various sections within the developing scour hole. By estimating (for both runs) the local differences between the instantaneous specific volumetric bed-load transport rate and the corresponding equilibrium value, the following equation was developed to express spatial variation of local bed-load sediment transport rate:

$$\frac{\partial q_{sb,k}}{\partial x} = Ca_k(t)(q_{sb,k}^* - q_{sb,k}) + \frac{\partial q_{sb,k}^*}{\partial x} \quad \text{with} \quad Ca_k(t) = \frac{a(t)|_x}{(q_{sb,k})_{x=L_s}} \quad (15)$$

Equation (15) is similar in form to equation (13). The coefficient  $Ca_k(t)$  (called the adaptation coefficient) is a function of the local median sediment diameter and the sediment transport rate downstream of the scour hole.

## APPLICATIONS AND COMPARISON WITH EXPERIMENTAL DATA

The model's ability to simulate transient sediment transport processes using equation (15) was tested by applying it to different data sets and it was then applied to DIIAA's laboratory flume. In this case study, the model's sensitivity to the adaptation coefficient was analysed and then data from Suryanarayana (1969) were used to compare the computed bed and water surface profiles to measured ones.

### DIIAA flume study and model sensitivity to adaptation coefficient

Experimental conditions of runs performed in the DIIAA's (Dipartimento di Ingegneria Idraulica ed Applicazioni Ambientali, University of Palermo, Italy) laboratory flume are described in "Experimental conditions and procedures". The numerical simulations were conducted by assuming zero sediment inflow at the upstream boundary. The initial Manning roughness coefficient was set at  $0.019 \text{ m}^{-1/3} \text{ s}$ . This value was previously calibrated by using data collected during preliminary experimental runs carried out in the same laboratory flume (Termini, 2011a). In Fig. 2, for both runs, the temporal evolutions of the measured maximum depths of the scour hole,  $\Delta z_b$ , are compared with computed ones. Figure 2 shows that the simulated temporal evolutions of the maximum scour depths follow the general trend of the measured ones fairly well.

Figure 3 compares the instantaneous bed profiles (at  $t \cong 60$  min for run 1 and at  $t \cong 65$  min for run 2) measured at the channel axis with calculated values, and agreement between predicted and measured bed profiles is relatively good. For run 1, the scour hole downstream of the rigid-bed

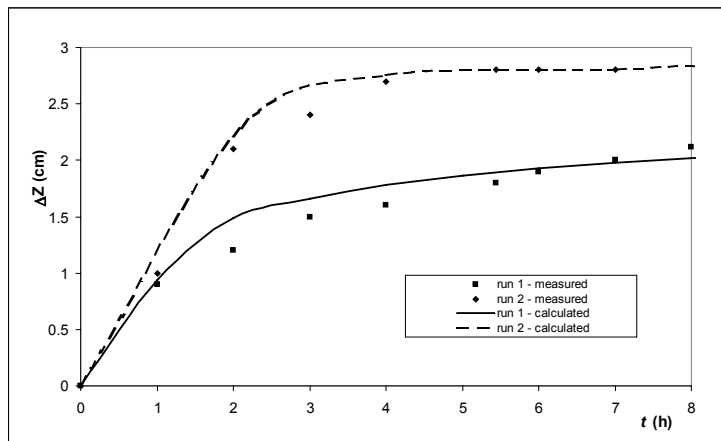


Fig. 2 Measured and computed maximum scour depths – DIIAA flume study.

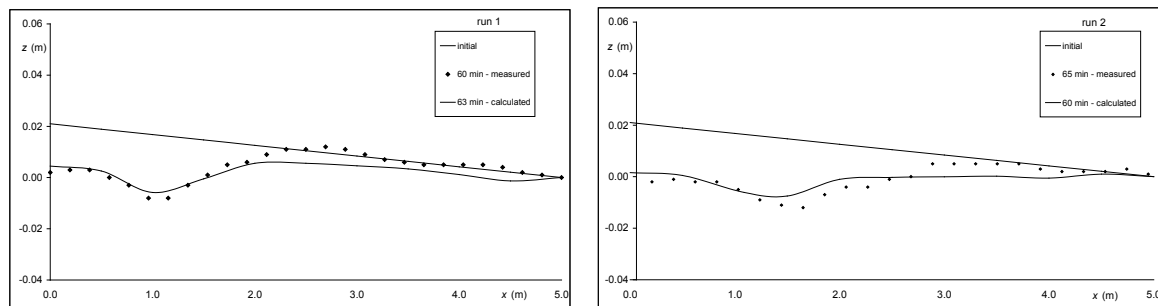


Fig. 3 Measured and computed bed profiles – DIIAA flume study.

reach was correctly predicted. The difference between the experimental data and the model results at the downstream end of the scour hole is probably due to the presence of the bed forms. Using equation (8), the model simulates bed form development but it does not predict their localization over the mean bed profile.

Then, a sensitivity analysis was performed to investigate the influence of the adaptation coefficient on the simulation results. For this purpose, several functions found in the literature, briefly mentioned above, ( $L = t$ ;  $L = 1 + 0.5t$ ;  $L = 4000(\theta - \theta_c)d_{50}$ ;  $L = 6.3B$ ), were tested, assuming  $Ca(t) = 1/L$  in equation (17). Figure 4 compares measured and calculated changes in maximum scour depth and the expression of  $Ca(t)$  proposed in this work allows the best result for the bed scouring evolution. The results obtained using  $Ca(t)$  are similar to that reported in the literature by Wu *et al.* (2004). They conducted a sensitivity analysis of numerical results to the non-equilibrium adaptation length, by using a 1-D semi-coupled model; testing the expressions  $L = t$ ;  $L = 1 + 0.5t$ ;  $L = 7.3h$ , Wu *et al.* (2004) verified that the calculated scour depths were not very sensitive to the adaptation length. Figure 4 also shows that the trend of scouring determined using  $Ca(t)$  of equation (15) differs from those estimated using expressions of literature especially in regard to the second evolution phase (weak scour). This behaviour confirms that the adaptation coefficient could exert an important rule in numerical simulation. It should be noted that, using the expression  $L = 4000(\theta - \theta_c)d_{50}$ , the adaptation coefficient varies inversely with the median sediment diameter and the excess bed shear stress over the critical value. This last term tends to decrease with time and, consequently, the adaptation coefficient tends to increase with time until reaching the equilibrium. All other tested expressions do not take into account the non-uniformity of the sediment transport process along  $x$ . In contrast, the adaptation coefficient of (15) depends



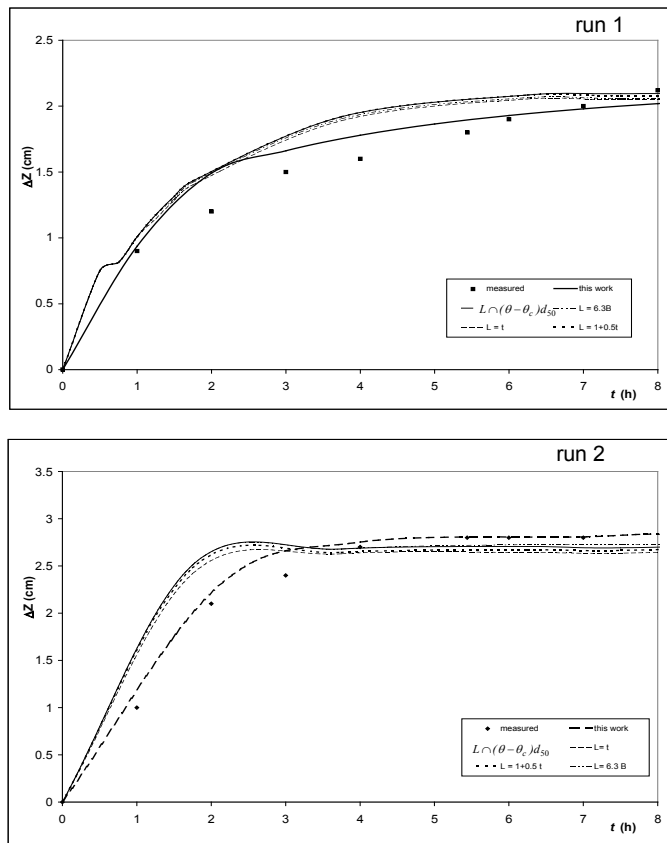


Fig. 4 Computed temporal evolution of maximum scour depths for different adaptation coefficients.

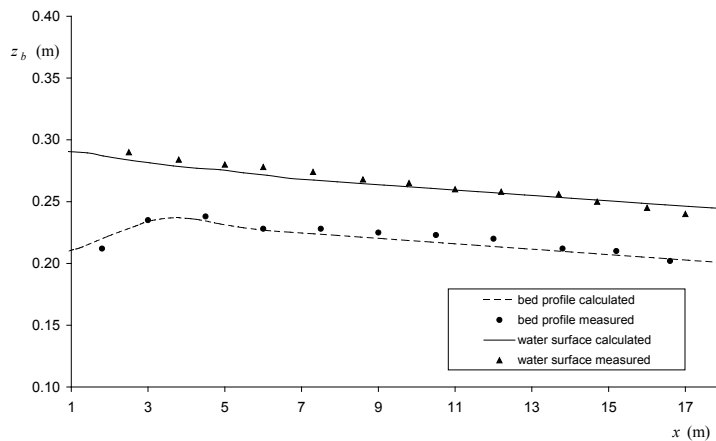


Fig. 5 Measured and computed bed and water surface profiles – Suryanarayana’s (1969) data.

both on the median sediment diameter and on the term  $(q_{sb,k})_{x=L_s}$ ; the former varies with the time  $t$  and with the space  $x$ ; the latter plays the role of a scale factor of scouring within the scour hole at each time step. Thus, the adaptation coefficient of (15) takes the non-uniform spatial variation of the bed-load transport rate into account, in accordance with the experimental results. Such coefficient is used for the following applications.

### Transient water and bed profiles test case

Experiments on bed degradation due to the shut-off of upstream sediment supply were performed by Surayanatayana (1969). Data from run 24 (data found in Kassem & Chaudry, 1998; Vasquez *et al.*, 2005) have been used in the present work. The flume was 18.29 m long and 0.61 m wide. The bed was of uniform sand with median sediment diameter of 0.45 mm. The flow discharge was  $Q = 0.0119 \text{ m}^3 \text{ s}^{-1}$ , the initial longitudinal channel slope was 0.007 and the initial water depth was 0.034 m. Results are given for 10 hours of test time. The numerical simulation was conducted for a spatial step of  $\Delta x = 0.61 \text{ m}$  and a time step  $\Delta t = 200 \text{ s}$ . Zero sediment inflow and a constant water discharge have been imposed as upstream boundary conditions. Figure 5 shows the comparison between computed bed and water profiles at  $t = 10 \text{ h}$ . From Fig. 5 reasonable agreement between measured and computed bed and water profiles can be observed.

### CONCLUSION

Transient sediment transport phenomena created in a flume under steady flow conditions and changing sediment transport rates were examined.

A new equation for the spatial delay of the bed-load transport rate, which includes a new expression of the so-called “adaptation coefficient” previously formulated (Termini, 2011b) was introduced into a 1-D numerical decoupled model to improve the transient bed profile simulation. The proposed expression of the adaptation coefficient allows the best estimation of non-uniform spatial variation of the bed-load transport rate. The results confirm that the adaptation coefficient could be an important parameter in numerical simulation.

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