Importance of thermal dispersion in temperature plumes

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Abstract The objective of this study is to evaluate the influence of thermal dispersion on the simulation of temperature plumes that evolve from the application of vertical ground source heat pump (GSHP) systems in aquifers. Various hydrogeological scenarios are simulated with longitudinal dispersivity ranging between 0.5 and 2 m and a Darcy velocity between 10^{-8} m s^{-1} and 10^{-5} m s^{-1}. In addition, thermal dispersivity is assumed to be scale-dependent. Based on a field scale of 10 m, the study shows that the thermal dispersion is an important factor for the prediction of shape and extension of temperature plumes in medium-grained sand to gravel aquifers. From the perspective of environmental regulators, such assumptions might be crucial for licensing applications of neighbouring GSHP systems. In contrast, ignoring thermal dispersion provides appropriate predictions of the temperature plume length for hydrogeological conditions dominated by fine sands, silts and clays.

Key words thermal dispersivity; analytical solution; ground source heat pump system; temperature plume

INTRODUCTION

During the last two decades, the usage of shallow geothermal energy and, in particular, the application of vertical ground source heat pump (GSHP) systems was increasing continuously (Sanner *et al*., 2003; Rybach & Eugster, 2010; Lund *et al*., 2011). These systems are used for heating and/or cooling of buildings and facilities by making use of the heat available in the upper part of the subsurface. The application of such shallow geothermal systems results in temperature anomalies in the subsurface, which can extend to a significant size and prevail for a long time, depending on the subsurface conditions, size and mode of the system, i.e. cooling and/or heating (Hähnlein *et al*., 2010a; Hecht-Méndez *et al*., 2010). Temperature plumes that adversely affect adjacent and neighbouring geothermal systems, particularly in urban areas, have to be avoided (Butscher *et al*., 2011). Thus, they have to be well predicted to assure sustainable use. In a few countries minimum distances between two borehole heat exchangers (BHEs) are already defined. In Germany, for instance, a distance of 10 m between individual BHEs is often suggested by the regulators. In contrast, a distance of 4–8 m is typically recommended in Switzerland (Hähnlein *et al*., 2010b).

Analytical solutions can be used to calculate the spatial and temporal extension of a temperature plume, as long as simple geometries are used and homogeneous aquifers can be assumed. If groundwater flow is present, advection and thermal dispersion has to be considered. The latter is generated by microscale mixing of the pore-scale interstitial water (Bear, 1972) and by differential transport in macroscale geological heterogeneities (Sauty *et al*., 1982; Ferguson, 2007; Hidalgo *et al*., 2009). Various analytical solutions are available that simulate the effect of groundwater flow for an infinite line source (Sutton *et al*., 2003; Diao *et al*., 2004). They do not include thermal dispersion, though the mechanical mixing can affect the thermal spreading of the heat plumes in the subsurface (Ferguson, 2007; Hidalgo *et al*., 2009). Hence, there is a need to assess the importance of thermal dispersion in the regulation and monitoring of GSHP systems. In the current study, we therefore apply an existing two-dimensional analytical approach for transient conditions by Metzger *et al*. (2004), which also considers thermal dispersion.

GOVERNING EQUATIONS

Heat transport in the subsurface

The heat transport process in porous medium is traditionally characterized by the heat advection/conduction equation (Domenico & Schwartz, 1998), which can be expressed in a 2D form as follows:

$$\rho_m c_m \frac{\partial T}{\partial t} + q\rho_w c_w \frac{\partial T}{\partial x} - \lambda \frac{\partial^2 T}{\partial x^2} - \lambda \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

Here T denotes temperature, q is the Darcy velocity, λ is the thermal conductivity, and $\rho_m c_m$ and $\rho_w c_w$ are the volumetric heat capacity of the bulk porous medium and water, respectively. The partial differential equation (PDE) for heat transport in porous media in two dimensions, considering the mechanical thermal dispersion process, can be written as follows (de Marsily, 1986):

$$\rho_m c_m \frac{\partial T}{\partial t} + q\rho_w c_w \frac{\partial T}{\partial x} - (\lambda + \alpha_x q\rho_w c_w)\frac{\partial^2 T}{\partial x^2} - (\lambda + \alpha_y q\rho_w c_w)\frac{\partial^2 T}{\partial y^2} = 0 \tag{2}$$

where α_x and α_y are the longitudinal and transverse dispersivities, respectively. Rearranging equation (2) results in:

$$\frac{\partial T}{\partial t} + v_T \frac{\partial T}{\partial x} - D_x \frac{\partial^2 T}{\partial x^2} - D_y \frac{\partial^2 T}{\partial y^2} = 0 \tag{3}$$

in which $v_T = q\rho_w c_w / \rho_m c_m$ is the effective heat transport velocity and $D = \lambda / \rho_m c_m + \alpha v_T$ the effective thermal dispersion coefficient, which denotes hydrodynamic dispersion and sums up the effects from thermal diffusion and mechanical thermal dispersion.

Analytical solution

The solutions of a line source with infinite length along the z-direction with a continuous and constant heat flow rate per unit length, q_L, is given by (Metzger *et al.*, 2004):

$$\Delta T(x,y,t) = \frac{q_L}{4\pi\rho_m c_m \sqrt{D_x D_y}} \exp\left[\frac{v_T x}{2D_x}\right] \int_0^{v_T^2 t/4D_x} \exp\left[-\phi - \left(\frac{x^2}{D_x} + \frac{y^2}{D_y}\right)\frac{v_T^2}{16 D_x \phi}\right]\frac{d\phi}{\phi} \tag{4}$$

For steady state conditions, equation (4) reduces to the following form:

$$\Delta T_{MILS_S} = \frac{q_L}{2\pi\rho c \sqrt{D_x D_y}} \exp\left[\frac{v_T x}{2D_x}\right] K_0 \left[\frac{v_T}{2}\sqrt{\frac{D_y x^2 + D_x y^2}{D_x^2 D_y}}\right] \tag{5}$$

in which K_0 is the modified Bessel function of the second kind of order zero. For steady state conditions, an approximation can be made in order to calculate the length of the temperature plume. Solving equation (6) for the temperature plume length yields (Molina-Giraldo *et al.*, 2011):

$$L_p = \left(\frac{{q_L}^2}{8\pi(\rho c)^2 D_y v_T \Delta T^2}\right)\left(1 \pm \sqrt{1 - \frac{8\pi(\rho c)^2 D_x D_y \Delta T^2}{{q_L}^2}}\right) \tag{6}$$

where L_p is the temperature plume length and ΔT is evaluated in the line of symmetry along the x-axis with y = 0. This approximation, however, is valid only for $v_T L_p / 2D_x$ >> 1.

Model set up

Temperature profile simulations along the centreline of the plume (x: distance downgradient from the source with y = 0) for transient and steady state conditions are computed using equations (4) and (6). Thermal conductivity and heat capacity of the bulk porous media are set to 2.5 W m^{-1} K^{-1} and 2.8 × 10^{6} J m^{-3} K^{-1}, respectively. Hydraulic conductivity is varied from 10^{-5} m s^{-1} to 10^{-2} m s^{-1}, which results in a Darcy velocity (q) range of 10^{-8}–10^{-5} m s^{-1}, assuming a constant hydraulic gradient of 10^{-3}.

Thermal dispersivity is assumed to be comparable to solute dispersion and dependent on the travel distance (scale-dependent). For a distance of 10 m (recommended minimum distance between two BHEs in the state of Baden-Württemberg, Germany (Hähnlein *et al.*, 2010b)), longitudinal dispersivities vary from 0.5 to 2 m depending on empirical relationships, which relate the field scale to the solute longitudinal dispersivities (Neuman, 1990; Gelhar *et al.*, 1992; Xu &

Eckstein, 1995; Schulze-Makuch, 2005). Moreover, for the sake of simplicity, the following ratio is assumed, $\alpha_y = 0.1\alpha_x$ (Smith & Chapman, 1983; Molson *et al*., 1992; Hopmans *et al*., 2002).

Evaluation of the importance of thermal dispersion is based on the computed root mean square error (RMSE):

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(\Delta T_{o(i)} - \Delta T_{(i)}\right)^2}{n}} \qquad (7)$$

in which $\Delta T_{(i)}$ corresponds to the results considering thermal dispersion ("true" values), and $\Delta T_{o(i)}$ corresponds to the results with only diffusion. We consider that RMSE values > 0.1 K represent conditions in which the influence of thermal dispersion must be accounted for.

RESULTS AND DISCUSSION

Figure 1 shows the transient behaviour of the temperature plume length for different longitudinal thermal dispersivities. It can be seen that the length of the temperature plume increases over time. Thermal dispersivities have a different effect on the plume length, depending on the isotherm of interest. In Fig. 1(a), for a ΔT = 0.1K, we can see, for instance, that an increase in α_x results in a longer temperature plume. However, in Fig. 1(b), where the isotherm considered is 1K, an increase in α_x results in shorter temperature plumes. For the specific case shown in this figure (λ = 2.5 W m^{-1} K^{-1}, $q = 2 \times 10^{-6}$ m s^{-1}), the isotherm of 0.1K is still under transient condition, whereas the isotherm of 1K has already reached steady state conditions. For long time simulation, the influence of longitudinal dispersivity on the temperature plume simulation is minimal, while transverse dispersivity dominates. Therefore, under steady state conditions, scenarios considering thermal dispersion yield lower temperature changes close to the source, in comparison to scenarios considering only diffusion. As a result, neglecting thermal dispersion results in an overestimation of the temperature plume length under steady state conditions.

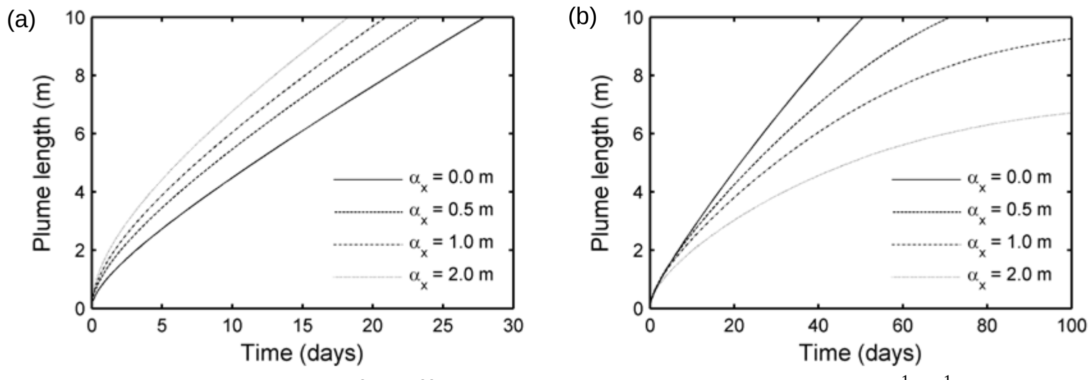

Fig. 1 Transient plume length for different transverse dispersivity (λ = 2.5 W m^{-1} K^{-1}, q_L = 60 W m^{-1}, $q = 2 \times 10^{-6}$ m s^{-1}); a) ΔT = 0.1K; b) ΔT = 1.0K.

In Fig. 2 the perspective is changed and we ask what energy extraction is required to result in a specific temperature change downgradient from the source? The larger the Darcy velocity, the larger the energy extraction needed to yield the same temperature change. This can be interpreted as a dissipation of energy by groundwater flow. In the same sense, the dispersion process also dissipates energy. The trend shown in Fig. 2 clearly indicates that for the specific case of x = 10 m (distance downgradient from the source), ΔT = 1K and flow velocities > 2×10^{-7} m s^{-1}, an increment of the thermal dispersivity leads to an increase in the energy needed to create the same temperature increment underground. For Darcy velocities < 10^{-7} m s^{-1}, dispersion only plays a minor role for the evaluation of dissipated energy.

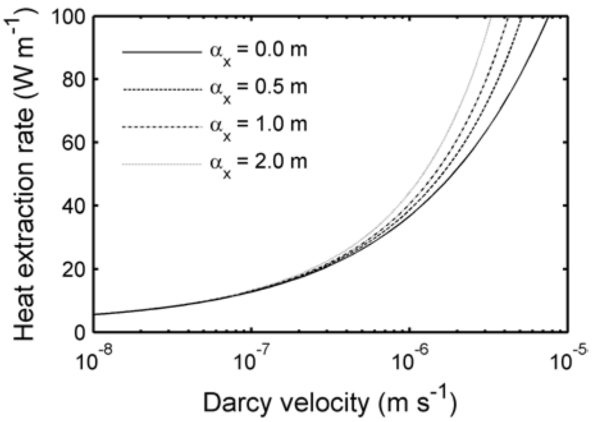

Fig. 2 Energy extraction needed to result in a temperature change of 1.0 K at 10 m downgradient from the source. The y-axis provides possible energy extraction values for one borehole heat exchanger.

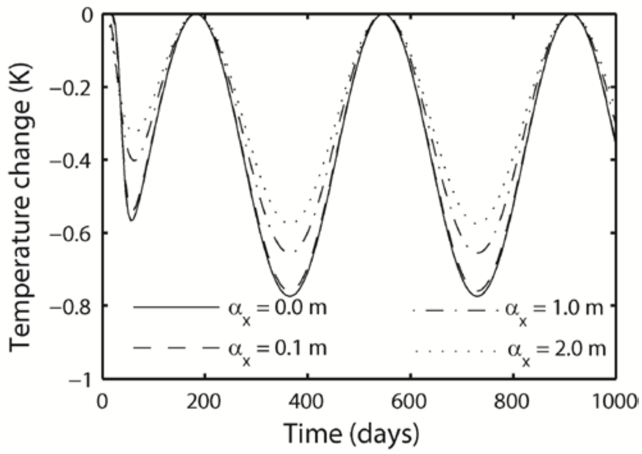

Fig. 3 Periodic temperature signal as function of time for difference thermal dispersivity values at 10 m distance downgradient from the source (λ = 2.5 W m^{-1} K^{-1}, $q = 2 \times 10^{-6}$ m s^{-1}).

Figure 3 shows the influence of dispersivity on the periodic signals. The larger the thermal dispersivity the stronger the dampening of the periodic temperature fluctuation. We can see that for this specific case (λ = 2.5 W m^{-1} K^{-1}, q = 2 × 10^{-6} m s^{-1}), at a distance of 10 m from the source, the amplitude of the temperature signal is 0.8 K when thermal dispersion is neglected, whereas the amplitude is reduced to 0.6 K when a thermal dispersivity of 2 m is considered for long time simulations.

For steady state conditions, the discrepancies of the temperature plume distribution are shown in Fig. 4 for different aquifer materials. Conditions possible for fine sand, medium sand, coarse sand and gravel aquifers are evaluated with a Darcy velocity of 10^{-8} m/s, 10^{-7} m/s, 10^{-6} m/s and 10^{-5} m/s, respectively. It can be seen that the RMSE varies up to 0.56 K. Only for Darcy velocities of 10^{-8} m/s, is the RMSE less than 0.1 K for almost the whole range of neglected longitudinal dispersivity. For Darcy velocities of 10^{-7} m/s, the RMSE only exceeds 0.1 K for longitudinal dispersivities larger than 0.5 m. For coarse sand and gravel aquifers (10^{-6} m/s, 10^{-5} m/s) the RMSE increases up to 0.56 K. These results, however, vary depending on the value of the thermal conductivity. An increase in thermal conductivity results in a lower effect of neglecting thermal dispersion.

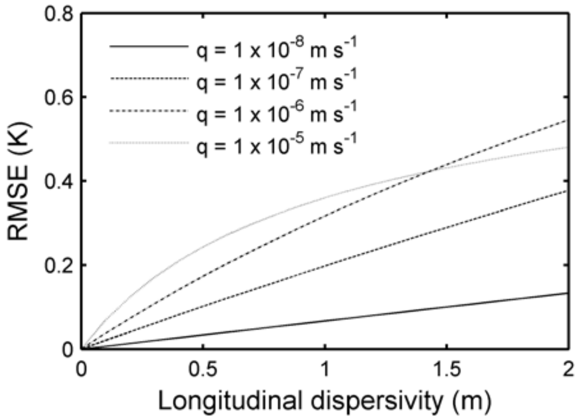

Fig. 4 RMSE as a function of Darcy velocity for steady state conditions with and without thermal dispersion. Fine sand, medium sand, coarse sand and gravel aquifers are simulated with a Darcy velocity of 10^{-8} m/s, 10^{-7} m/s, 10^{-6} m/s and 10^{-5} m/s, respectively (x = 0.1 to 10 m, λ = 2.5 W/m/K).

CONCLUSIONS

In the present study, the effect of neglecting thermal dispersion on the temperature plumes that evolve from GSHP systems under different groundwater flow and dispersion conditions are examined. The results show that if the length of the temperature plume in the subsurface is of concern, the consideration of thermal dispersion is an important factor for steady state conditions typical for medium-grained sand and gravel aquifers ($q = 10^{-8}$ m/s). In contrast, considering only diffusion provides appropriate predictions of the temperature plume length for flow conditions with $q < 10^{-8}$ m s^{-1} (typical for moderate to low conductive media such as fine sands, clays, and silts). Underestimation of plume length occurs with increasing thermal dispersion for steady state conditions. For transient conditions, however, the plume length might be overestimated with increasing thermal dispersion.

It has to be mentioned that in practice, the uncertainty commonly associated with representative hydraulic conductivity values might lead to higher errors in computing the length of temperature plumes than those associated with neglecting thermal dispersion. This has to be evaluated from case to case. Finally, the results presented here are based on a field scale of 10 m. For larger distances, thermal dispersion can have a greater influence on the simulated length of a temperature plume.

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