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Analysis of time-drawdown data from heterogeneous leaky aquifer systems

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Abstract The analysis of drawdown data from pumping tests is often performed using graphical techniques that are based on the assumption of aquifer homogeneity. However, natural subsurface formations are heterogeneous with complex patterns of spatial variability. In this paper, we describe a novel interpretation method that uses the time derivative of the drawdown to infer information about the spatial variability of the flow parameters in heterogeneous leaky aquifer systems. The method uses the observed drawdown and its time-derivative at a single point to estimate the hydraulic parameters. By applying the procedure to different portions of the time-drawdown data, variations in the flow parameters with radial distance from the pumping well are detected. The method can also be used as a tool to identify the type of aquifer system present. For demonstration the method is applied to pumping test data from an alluvial leaky aquifer in California, USA. Various data smoothing and differentiation techniques were evaluated for the estimation of the time-drawdown derivative. Because of the noise typically observed in field data, optimal estimates of the drawdown derivative were obtained by first fitting the drawdown data to high order polynomials and splines and then differentiating the fitted functions with respect to time. Results of the analyses show that the proposed methodology is a viable tool for the interpretation of pumping test data and that it may yield important information about the heterogeneity of the aquifer, which is generally ignored in conventional pumping test analysis techniques.

Key words well hydraulics; analysis of pumping test; groundwater flow modelling; heterogeneity; transmissivity; leaky aquifers

INTRODUCTION

Pumping tests are routinely used to infer subsurface flow parameters. Time-drawdown data are generally analysed using analytical or graphical methods, e.g. the Theis method (1935) for confined aquifers and the Walton method (1963) for leaky aquifers, that have been developed based on the assumption of homogeneity of the aquifer system. As such these methods yield single estimates of the aquifer parameters that are representative of the perturbed aquifer system as a whole. They are, however, unsuitable for identifying the spatial variability of the flow parameters at scales smaller than the perturbed aquifer radius.

Therefore, in recent years there has been renewed interest in evaluating what additional information can be inferred from pumping test data beyond the single estimates of the flow parameters that are typically obtained from conventional pumping test analysis methods (e.g. Meier *et al*., 1998; Sanchez-Vila *et al*., 1999; Neuman *et al*., 2004; Trinchero *et al*., 2008; Copty *et al*., 2011) The purpose of this study is to extend the recently developed method proposed by Copty *et al*. (2011) to leaky aquifer systems and to apply it to real pumping test data from a complex multilayer alluvial aquifer in California, USA. The method was previously tested with synthetic pumping tests (Copty *et al*., 2011) that do not exhibit the complexities and difficulties generally encountered in real aquifer systems.

The method is referred to as the continuous differentiation (CD) method because it relies on the time-derivative of the drawdown data. The time-derivative, or diagnostic curve, tends to be more sensitive to variations in the flow parameters compared to the drawdown (Renard *et al*., 2009).

The focus in this paper is on leaky aquifer systems. A classical example is that of alluvial multilayered aquifer-aquitard systems, which are present worldwide. The first mathematical analysis of well hydraulics in leaky aquifers was developed by Hantush & Jacob (1955). The authors presented the analytical solution for the transient drawdown due to a constant pumping rate in leaky aquifers based on a series of simplifying assumptions: vertical flow in the aquitard,

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horizontal flow in the aquifer, negligible storage in the aquitard, constant hydraulic head in the unpumped (recharging) aquifer, and a pumping well of infinitesimal radius that fully penetrates the pumped aquifer. Under such conditions, the drawdown becomes a function of the hydraulic parameters of the aquifer (transmissivity, *T*, and storativity, *S*) and the conductance of the aquitard, *C*, defined as the ratio of the vertical hydraulic conductivity over the thickness of the aquitard, (K'/b'). The solution of Hantush and Jacob formed the starting point in the development of other pumping test interpretation techniques such as the inflection point method (Hantush, 1956) and the type-curves method defined by Walton (1962).

METHODOLOGY

The main feature of the CD method is that it uses the drawdown data and its time derivative at one particular point in time to estimate the apparent flow parameters at that moment in time (Copty *et al.*, 2011). This is in contrast to conventional pumping test interpretation such as the Walton method (1963) which uses drawdown data observed at different times to estimate "representative" or lumped values of the flow parameters.

The transient drawdown due to pumping in a leaky aquifer is given by (Hantush & Jacob, 1955):

$$s = \frac{Q}{4\pi T}\int_{\frac{r^2S}{4tT}}^{\infty}\frac{1}{y}\exp\left(-y-\frac{r^2}{4B^2y}\right)dy = \frac{Q}{4\pi T}W(u,\rho) \tag{1}$$

where $\rho = r/B$ and $u = \frac{r^2S}{4tT}$

The drawdown derivative as a function of logarithm (base 10) of time is:

$$s' = \frac{\partial s}{\partial \log t} = 2.3t\frac{\partial s}{\partial t} = \frac{2.3Q}{4\pi T}\exp\left(-\frac{r^2S}{4tT}-\frac{tT}{B^2S}\right) = \frac{2.3Q}{4\pi T}\exp\left(-u-\frac{\rho^2}{4u}\right) \tag{2}$$

The ratio of the drawdown to its derivative observed at any particular time is:

$$\gamma_L = \frac{2.3s}{s'} = W(u,\rho)\exp\left(u+\frac{\rho^2}{4u}\right) \tag{3}$$

Figure 1 shows the plot of γ_L as a function of $1/u$ and r/B. The shape of γ_L can be used as a diagnostic tool to determine the aquifer characteristics such as whether it is confined or leaky, or whether the aquifer is bounded or infinite. It can also be used to produce estimates of the flow parameters at different times.

For the special case when $\rho = r/B$ approaches zero, the aquifer becomes essentially non-leaky. Under such conditions, Copty *et al.* (2011) show that $\gamma_L = \frac{2.3s}{s'}$ can be directly computed from the ratio of the drawdown and its derivative at any moment in time and used to estimate *u* at that particular time. The transmissivity is then estimated from:

$$T = \frac{Q}{4\pi s}W(u) \tag{4}$$

While the storativity is estimated from:

$$S = \frac{4tTu}{r^2} \tag{5}$$

The method is repetitively applied to all times in order to estimate the flow parameters as the cone of depression expands in time (and space), yielding a plot of *T* and *S* as a function of time since the start of pumping. Because of the interrelation between time and radial distance from the well for radially convergent flow, it can be shown that the *T vs t* relationship can be converted into a *T vs r* relationship where *r* is the radial distance from the pumping well defined as (Copty *et al*., 2011):

$$r = \sqrt{\frac{4tT}{1.65S}} \tag{6}$$

Copty *et al*. (2011) evaluated the above pumping test interpretation method using simulated pumping test data conducted in synthetically-generated heterogeneous multi-Gaussian transmissivity fields. The results show the interpreted transmissivity as a function of radial distance is a good estimate of the geometric mean of the transmissivity, $T_g(r)$, defined over an evolving radial distance from the well although the drawdown cone due to pumping from a heterogeneous aquifer is strictly not circular. This observation was based on multi-Gaussian ln-transmissivity distributions with variances as high as 2.

If the aquifer is leaky and an estimate of the leakance or aquitard vertical conductance is available (such as for example from other interpretation methods like the Walton method), then the above procedure can be applied to drawdown data from leaky aquifers using the appropriate *r/L* curve in Fig. 1. An important feature of the above method is that it relies on the time-derivative of the drawdown data. Because of the difficulty of estimating accurately the time-derivatives of the data, particularly from field data with a high level of noise, some smoothing techniques would be needed. The data differentiation techniques evaluated in this study includes the central differences, Bourdet *et al*. (1989) and Spane & Wurstner (1993) methods. Derivatives were also computed by first fitting the drawdown data to high order polynomials (order 6–9) or splines, followed by differentiation of the fitted functions with respect to time.

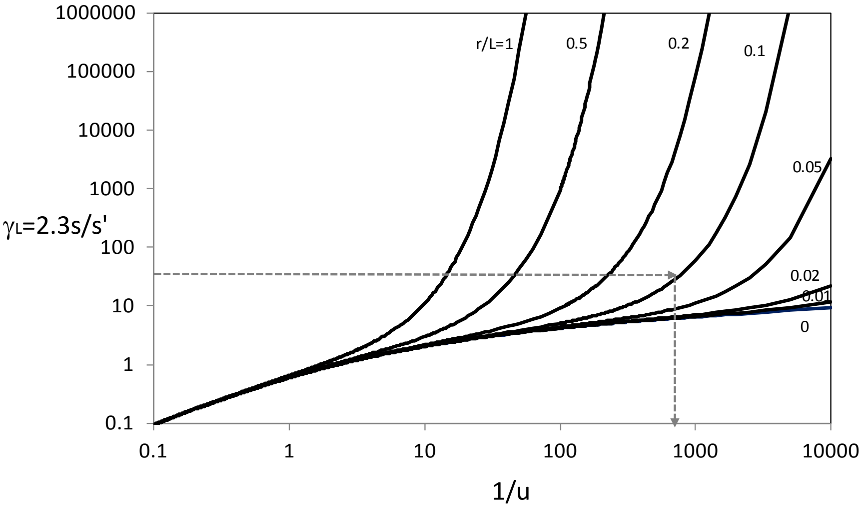

Fig. 1 Plot of $\gamma_L = \dfrac{2.3s}{s'}$ as a function of 1/*u* and *r/L* for leaky aquifers.

FIELD APPLICATION

The above method was applied to real pumping tests conducted within a large alluvial-filled groundwater basin in California, USA. The basin is filled with Holocene through Pliocene alluvial deposits consisting of clay, silt, sand, and gravel. These deposits were laid down as convergent alluvial fans, creating a complex and heterogeneous mix of sediments. The predominant water bearing zone is an approximately 6 m-thick sand and gravel layer at a depth of about 25 m. Over- and under-lying this zone are relatively continuous confining clay layers. Laterally extensive sand and gravel lenses are commonly found within the confining layers; however, they do not appear to be hydraulically connected to the main water bearing zone. Above the upper confining layer is a thin layer of sand and gravel. The clay zones separate the main water bearing zone from the sand and gravel lenses within the confining layers (and the sand/gravel layer above the upper confining layer), and as a result, exhibit the characteristics of a leaky aquifer.

Three constant-rate pumping tests were conducted in the main water-bearing zone. Each test was conducted for a pumping period of three days, with each test consisting of pre-pumping and post-pumping monitoring of water levels to enable removal of trends prior to analysis. The main water-bearing zone, along with the confining layers and sand/gravels within the confining layers were instrumented to record water level measurements throughout the monitoring period.

Prior to analysis, water level trends not related to pumping were removed from the data. Three trends were observed that required removal: long-term water level trends, occasional disturbance of transducer cable and barometric pressure influences. Calculation of aquifer parameters was done using conventional type-curve techniques (Walton, 1962) to determine the transmissivity and storage of the main water-bearing zone. These estimates are later compared with those obtained with the new method described in this paper.

Estimation of time derivative of the drawdown

The time-derivative of the drawdown data from the different pumping tests were initially estimated using the central difference, Bourdet *et al*. (1989) and Spane & Wurstner (1993) methods for different sampling intervals. These results show that the noise level tends to increase at the late time and for smaller time intervals. The noise is slightly less with the Spane & Wurstner and Bourdet methods compared to the central difference, but further smoothing would be needed to be able to use the drawdown in any interpretation effort.

High order polynomials and splines were fitted to the drawdown data. Figure 2 shows the observed drawdown data at different observation points during pumping test TW2A. These results are representative of the other pumping tests. The plots show the drawdown data and best fit polynomials of order 6 and 9 and splines (with the smoothing parameter p = 0.8 and 0.95). Figure 2 shows that all of these polynomials provide a good fit with the real drawdown data, with R^2, the goodness of fit, exceeding 0.998 for all cases, indicating that the models are capable of capturing the majority of the variability in the data. The residual plots show that the error is consistently smaller than 0.02 m. Using a lower order polynomial produces over smoothing of the data, resulting in a larger residual.

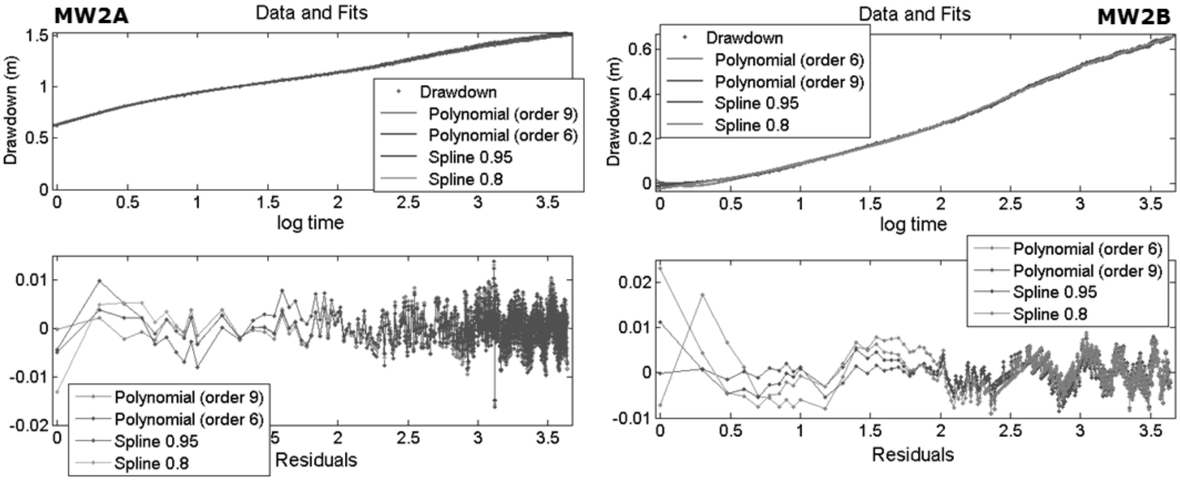

Fig. 2 Drawdown data from pumping test TW2A and best fit polynomials (of order 6 and 9) and splines (with p = 0.8 and 0.95), and residual drawdown.

Interpretation of pumping test data

Figure 3 shows the parameter $\gamma_L = \frac{2.3s}{s'}$ for different pumping tests. The drawdown derivative was estimated based on splines for a smoothing parameter p = 0.8. Because the derivative of drawdown is taken in terms of the log of time, the drawdown derivative has length units, and γ_L, is dimensionless. The shape of the curves in Fig. 3 may help in identifying the type of aquifer present. Figure 1, which was developed for the Hantush leaky aquifer system, shows that γ_L should increase with time. Depending on the value of *r/B*, γ_L increases rapidly at late times as the drawdown approaches steady state. In general, the γ_L curves depicted in Fig. 3 exhibit a relatively sharper increase in γ_L at late times suggesting that the aquifer system is slightly leaky.

Variations between different individual γ_L curves may reveal important information about the aquifer conditions. For example the response at MW2A and MW2B which are both less than 10 m away from TW2A are significantly different, demonstrating the complexity of the geologic conditions. MW2A exhibits a larger drawdown at early times, resulting in a larger initial γ_L value compared to MW2B. Because both monitoring wells are at the same distance from the test well, the smaller γ_L value of MW2B corresponds to a lower value of 1/*u* (Fig. 1).

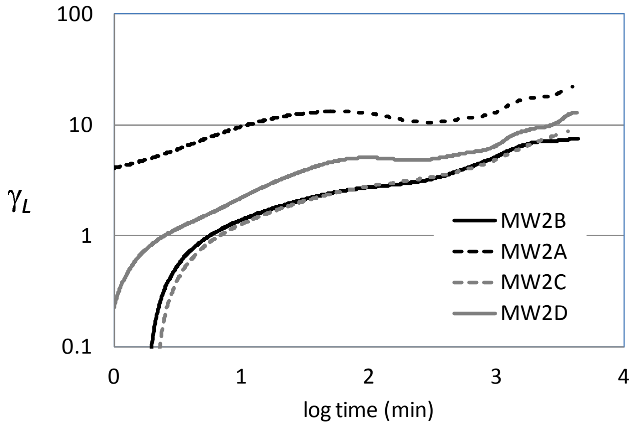

Fig. 3 $\gamma_L = \frac{2.3s}{s'}$ for TW2A estimated from the drawdown data and its time derivative.

This may be an indication of a higher “apparent” storativity value for MW2B caused by poorer hydraulic connectivity of MW2B to the pumping well. This is indeed consistent with the geology within the vicinity of TW2A. Monitoring well MW2A is completed at the same elevation and within the same sand/gravel layer as TW2A, and hence is well connected to the pumping well. MW2B is completed in a hydraulically connected sand/gravel above the elevation of the test well, with less connectivity to the pumping well. The lower connectivity to MW2B is reflected in the higher estimated storativity value.

Table 1 lists the flow parameter values estimated with the Walton curve-fitting method. The leakage factors are generally high (low *r/B* values) indicating, as suggested above, that the aquifer system is a slightly leaky aquifer. The estimated transmissivities from the different tests exhibit little variation with an average of about 0.01 m^2/s). It is important to emphasize that the Walton method uses all the drawdown data from a single pumping test to estimate single “representative” values of the transmissivity

Unlike the transmissivity, the estimated storativity varies significantly from about 4×10^{-5} to as high as 10^{-1} for TW2A-MW2B (which was predicted qualitatively based on the shape of the γ_L curves). The variation in the estimation of the storativity stems in part from its dependence on the point-to-point connectivity of the heterogeneous transmissivity field (e.g. Sanchez-Vila *et al*., 1999).

Table 1 Estimated Parameters using the Walton curve-fitting method.

Pumping well	Observation point	r (m)	T (m^2/s)	S	B (m)	r/B
TW2A	MW2A	9.45	0.0083	0.00004	4720	0.002
	MW2B	8.42	0.0096	0.11	8420	0.001
	MW2C	109	0.0096	0.00067	10900	0.001
	MW2D	113	0.0093	0.00012	113000	0.001

Figure 4 shows the estimated flow parameters obtained with the CD method. The calculations were repeated for the different derivative estimation techniques considered above. At each time t, the drawdown and its derivatives were first estimated and then used to compute γ_L and the flow parameters at that specific time. The estimation was repeated for all times until the end of the pumping test. The $T(t)$ and $S(t)$ plots were converted to radial relationships $T(r)$ and $S(r)$, respectively, according to equation (6). The average S value obtained from the Walton method was used for the conversion of the x-axis from time to radial distance.

The results obtained from the proposed CD method are consistent with the results obtained from the Walton method. For example, for pumping test TW2A, the Walton method predicted a transmissivity of about 0.01 m^2/s, and a storativity of about 0.0001 at all wells except for TW2A-MW2B (where the storativity was about 0.1). Similar storativity and transmissivity values were also obtained with the CD method (See Fig. 4). However, the estimated parameters obtained with the CD method exhibit some variation particularly at early distance (or time). At later times, the transmissivity estimate tends to stabilize suggesting that the aquifer system is behaving close to a homogeneous system. This corresponds to a characteristic length of about 600 m for monitoring well MW2A. For MW2B, which has a much higher apparent storativity value due to its lower connectivity to the test well, the aquifer system starts to behave as a homogeneous aquifer at scales greater than about 60 m. For applications with scales larger than the characteristic length of the transmissivity field, a conventional deterministic approach may be appropriate. However, for applications of smaller scales the results of this approach may be inaccurate; a stochastic approach would be required to quantify the uncertainty in any predictions of the response of the system to future changes and stresses.

Furthermore, the estimated transmissivity values at the very early times are mostly dependent on the transmissivity values in the immediate vicinity of the test well. Therefore, the variability of the transmissivity values at early times can be indicative of the transmissivity variance of the aquifer system, provided that a sufficient number of tests is available to satisfy the ergodicity requirement.

Inspection of the observed data also indicates that the different derivative estimation techniques do not yield significantly different estimates of the flow parameters, suggesting that the results of the pumping test analysis method are not too sensitive to the methods selected for derivative estimation. Another feature of the results is that the estimated transmissivity and storativity are negatively correlated. This has also been noted previously, but based on synthetic pumping test data (e.g. Trinchero *et al.*, 2008; Copty *et al.*, 2011). Moreover, these results again demonstrate the difficulty of estimating an accurate value of the storativity.

CONCLUSIONS

A novel method is proposed for the interpretation of pumping test data from leaky aquifer data. Unlike conventional methods which provide single “representative” estimates of the flow parameters, the proposed pumping test interpretation method, referred to as the continuous differentiation (CD) method, attempts to provide some information of the spatial variability of the flow parameters as a function of radial distance from the well.

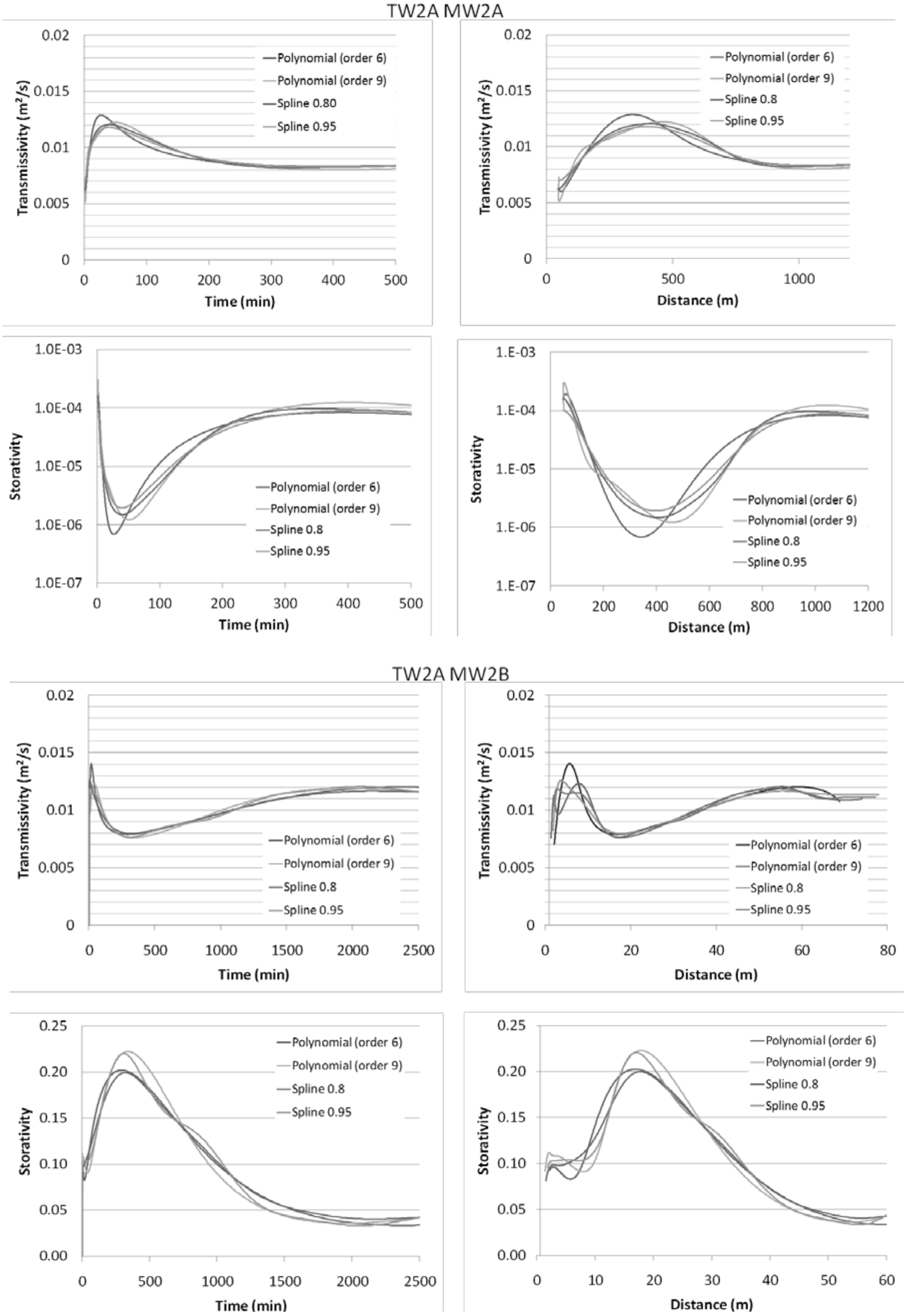

Fig. 4 Estimated transmissivity and storativity as a function of time and radial distance from the well for pumping tests TW2A and monitoring wells MW2A and MW2B.

An important feature of this method is that it requires the estimation of the time derivative of the drawdown. Because field drawdown data contain noise, different differentiation methods were considered for the estimation of the required derivatives. Results of this application suggest that fitting the drawdown data to high order polynomials (order 6–9) or splines and then differentiation of the fitted functions with respect to time was the most effective method for the estimation of reliable derivative of the drawdown.

Application of the CD method to pumping test data from an alluvial leaky aquifer system suggests that the described methodology is a useful tool for the identification of the type of aquifer present. Moreover, the results show that the method may be a viable pumping test analysis method that can yield some information about the spatial variability, specifically the characteristic length scale and variance of the transmissivity.

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REFERENCES

Bourdet D., Ayoub J. A. & Pirard, Y. M. (1989) Use of pressure derivative in well-test interpretation. *SPE Reprint Ser* 4, 293–302.

Cooper, H. & Jacob, C. (1946) A generalized graphical method for evaluating formation constants and summarizing well-field history. *Trans. Am. Geophys. Union* 27(4), 526–534.

Copty, N. K., Trinchero P. & Sanches-Vila, X. (2011) Inferring spatial distribution of radially integrated transmissivity from pumping tests in heterogeneous confined aquifers. *Water Resour. Res.* 47, doi: 10.1029/2010WR009877.

Hantush, M. & Jacob, C. (1955) Non-steady radial flow in an infinite leaky aquifer. *Trans. Am. Geophys. Union* 36(1), 95–100.

Hantush, M. (1956) Analysis of data from pumping tests in leaky aquifers. *Trans. Am. Geophys. Union* 37(6), 702–714.

Meier, P. M., Carrera, J. & Sanchez-Vila, X. (1998) An evaluation of Jacob’s method for the interpretation of pumping tests in heterogeneous formations. *Water Resour. Res.* 34(5), 1011–1025.

Neuman, S. P., Guadagnini, A. & Riva, M. (2004) Type-curve estimation of statistical heterogeneity. *Water Resour. Res.* 40, W04201, doi:10.1029/2003WR002405.

Renard, P., Glenz, D. & Mejias, M. (2009) Understanding diagnostic plots for well-test interpretation. *Hydrogeology J.* 17(3), 589–600.

Sanchez-Vila, X., Guadagnini, A. & Carrera, J. (2006) Representative hydraulic conductivities in saturated groundwater flow. *Reviews of Geophysics* 44(3), RG3002.

Sanchez-Vila, X., Meier, P. M. & Carrera, J. (1999) Pumping tests in heterogeneous aquifers: An analytical study of what can be obtained from their interpretation using Jacob’s method. *Water Resour. Res.* 35(4), 943–952.

Theis, C. V. (1935) The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage. *Trans. Am. Geophys. Union* 2, 519–524.

Trinchero, P., Sanchez-Vila, X., Copty, N. K. & Findikakis A. N. (2008a) A new method to interpret pumping tests in leaky aquifers. *Ground Water* 46(1), 133–143.

Walton, W. (1962) Selected analytical methods for well and aquifer evaluation. *Illinois State Water Survey Bulletin* 49.