

## Flow in the unsaturated zone to quantify Acid Mine Drainage: numerical and analytical approaches

MARTÍ BAYER-RAICH & SALVADOR JORDANA

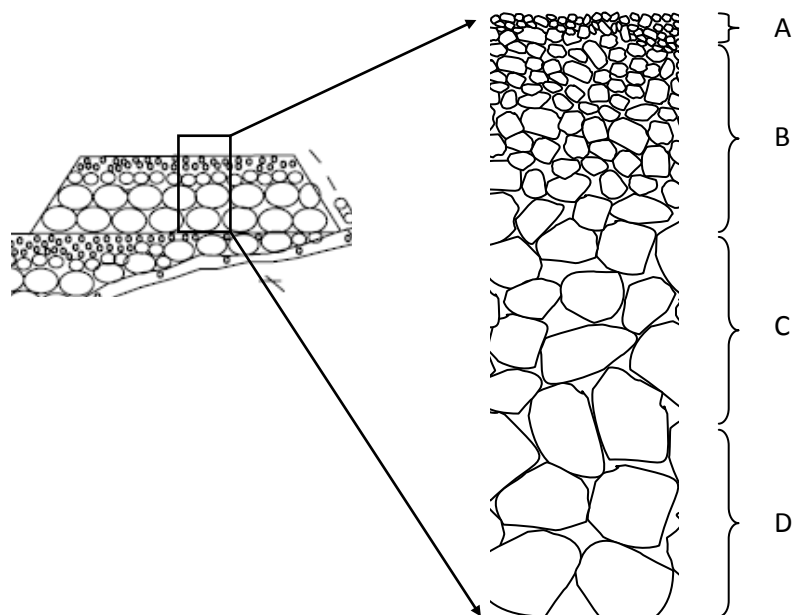
*Amphos21, Passeig de Garcia i Faria 49-51 1-1, 08019. Barcelona, Spain*  
[marti.bayer@amphos21.com](mailto:marti.bayer@amphos21.com)

**Abstract** We study flow conditions in the unsaturated zone to quantify Acid Mine Drainage (AMD) in mine-waste piles using the van Genuchten model. Both analytical and numerical methods are applied to estimate the fundamental variables governing flow in the unsaturated zone: water saturation, water velocity (i.e. travel times), Darcy velocity, water pressure and head. We use numerical solutions, obtained with the code MIN3P to estimate saturation and travel times in multi-layered systems, for three hypothetical scenarios. A simple, novel analytical approach is developed for vertical flow in homogeneous media to obtain order-of-magnitude estimates of saturation and water velocity. This solution is applied to the hypothetical scenarios and shows that variability of hydraulic conductivity (up to four orders of magnitude) has little effect on travel times.

**Key words** vadose zone; mine-waste piles; unsaturated flow; groundwater

### FLOW IN THE UNSATURATED ZONE THROUGH MINE-WASTE PILES

In order to quantify long term impacts of Acid Mine Drainage (AMD) it is necessary to characterize the water flow through mine-waste piles. Infiltration of rainfall through these materials, which may be more than 100 m thick, is modelled using van Genuchten parameterizations. Figure 1 shows a typical mine-waste pile, with changes of grain size (from a few centimeters to >1 m in diameter) along the vertical within each layer. Usually, layers are spread evenly on the final top sheet of the previously superposed layers, as shown in Fig. 1.



**Fig. 1** Conceptualization of mine-waste pile: layers with four materials (A, B, C and D) in each layer.

Typical values of infiltration rate due to precipitation (e.g. Lefebvre *et al.*, 2001; Mayer *et al.*, 2002) are around 100 mm/year; for both homogeneous and multi-layered environments, flow is 1-D-vertical. To estimate travel times, particle velocities are subject to:

$$v_w = \frac{q_w}{n_e S_w} \quad (1)$$

where  $q_w$  ( $L T^{-1}$ ) is water infiltration rate,  $n_e(-)$  is porosity and  $S_w(-)$  is water saturation. Using the formulation given in van Genuchten (1980),  $S_w$  is related to both water pressure  $P_w$  (or suction, defined as  $h_c = -P_w$ ) and relative hydraulic conductivity through the relations:

$$S_{ew} = \left[ 1 + (\alpha h_c)^{1/(1-m)} \right]^{-m} \quad (2)$$

$$K_{rw} = \sqrt{S_{ew}} \left[ 1 - (1 - S_{ew}^{1/m})^m \right]^2 \quad (3)$$

where  $S_{ew} = (S_w - S_{rw}) / (1 - S_{rw})$  is effective water saturation,  $S_{rw}(-)$  residual saturation,  $K_{rw} = K_{unsat} / K_{sat}$  relative hydraulic conductivity and  $\alpha$  ( $L^{-1}$ ) and  $m$  ( $-$ ) the van Genuchten parameters. Specific values of these parameters used in this work are taken from a field-scale study described in Lefebvre *et al.* (2001); Table 1 provides values used here.

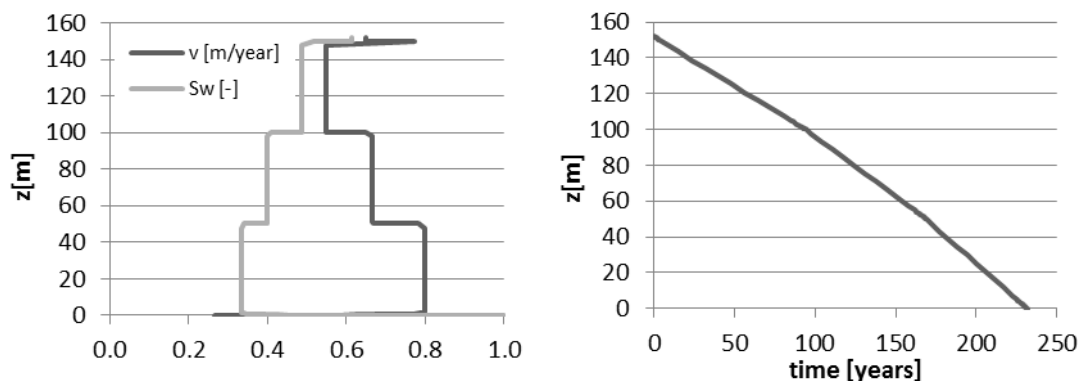
**Table 1** Parameter values used in this study for four materials (A, B, C and D) in each layer.

	A	B	C	D
Porosity $n_e$ ( $-$ )	0.2	0.3	0.3	0.3
Hydraulic conductivity $K_{sat}$ (m/day)	1	10	100	1000
van Genuchten parameter $\alpha$ ( $m^{-1}$ )	100	100	100	100
van Genuchten parameter $m$ ( $-$ )	0.23	0.23	0.23	0.23
residual water saturation $S_{rw}$ ( $-$ )	0.15	0.1	0.1	0.1

## NUMERICAL SOLUTIONS FOR MULTI-LAYERED MEDIA

We used the code MIN3P (Mayer *et al.*, 2002) to solve the flow equation in unsaturated zone steady-state conditions for infiltration rate  $q_w = 80$  mm/year. We assume that the water table is located at the base of the mine-waste pile and consider three different scenarios:

- CASE 1 (Fig. 2). Vertical infiltration through one layer with 4 materials A, B, C and D with thicknesses of 2 m (material A) and 50 m (materials B, C and D). Total thickness 152 m.
- CASE 2 (Fig. 3). Vertical infiltration through a (not fully developed) layer with 4 materials A, B, C and D with thicknesses of 2 m (material A) and 10 m (materials B, C and D). Total thickness 32 m.
- CASE 3 (Fig. 4). Vertical infiltration through two layers with four materials each: A, B, C and D with thicknesses of 2 m (material A) and 50 m (materials B, C and D). Total thickness 304 m.



**Fig. 2** Velocity and travel time for case 1. Average velocity 0.655 m/year.

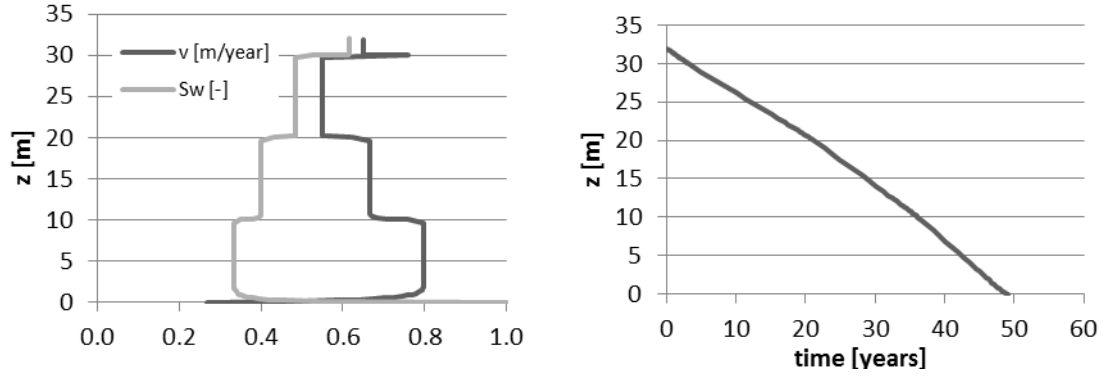


Fig. 3 Velocity and travel time for case 2. Average velocity 0.650 m/year.

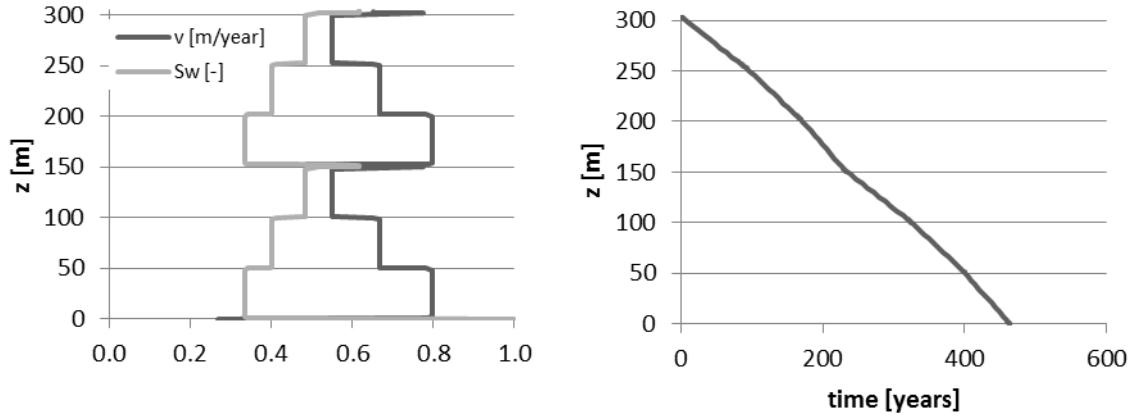


Fig. 4 Velocity and travel time for case 3. Average velocity 0.655 m/year.

Results show that, despite the variability in  $K_{sat}$ , travel times through different layers remain relatively constant, time vs  $z$  plots can be well approximated by straight lines. (Advective) Travel time is defined here as  $t(z) = \int_0^z \frac{d\hat{z}}{|v_w(\hat{z})|}$ . Water velocities vary from 0.55 to 0.8 m/year and water saturation is within the range 0.33–0.6.

## APPROXIMATE ANALYTICAL SOLUTION

### Analytical expressions to estimate saturation and average velocity

Considering a steady-state infiltration rate  $q_w$  at the top of the domain, under steady-state conditions, the same rate  $q_w$  applies to any location along the vertical. For homogeneous conditions, water saturation, pressure and pore velocity will be constant along the vertical. For  $P_w$  constant (i.e. independent of  $z$ ), using the definition of head  $h = z + P_w$ , the hydraulic gradient becomes  $\nabla h = 1$  and Darcy velocities for unsaturated conditions are:

$$q_w = K_{unsat} \nabla h = K_{unsat} \quad (4)$$

This expression, valid for gravitational flow in unsaturated conditions is also discussed in Roth (2007). Inserting  $S_{ew}$  from (2) into (3) and considering  $K_{rw} = q_w / K_{sat}$  we get:

$$\left[ 1 - \left( 1 - \frac{1}{H} \right)^m \right]^2 \frac{1}{H^{m/2}} - \frac{q_w}{K_{sat}} = 0 \quad (5)$$

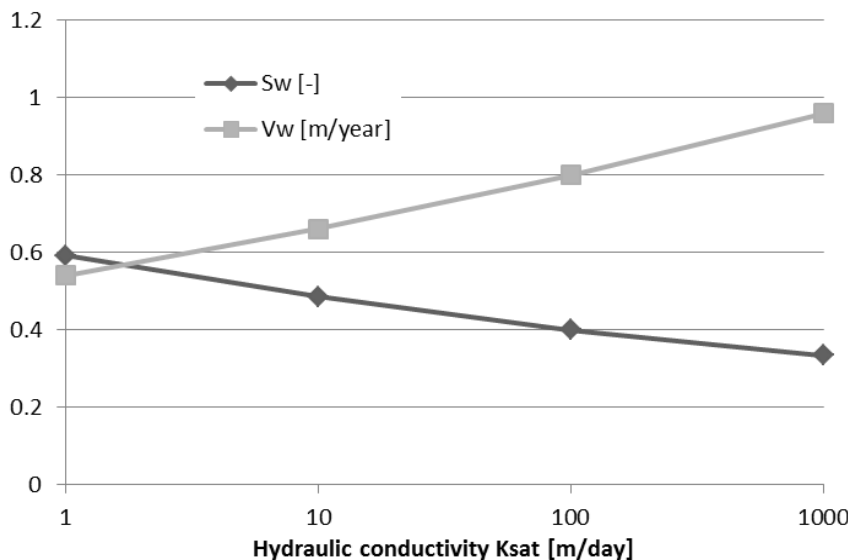
where  $H = 1 + (\alpha h_c)^{1/(1-m)}$ . Equation (5) is a nonlinear expression which has to be solved for  $H$  for a given infiltration rate  $q_w$ , hydraulic conductivity  $K_{sat}$  and the van Genuchten parameter  $m$ . Once we have the value for  $H$  we can easily obtain water saturation  $S_w$  using  $S_{ew} = H^{-m}$  with

$$S_w = H^{-m}(1 - S_{rw}) + S_{rw} \quad (6)$$

Note that saturation  $S_w$  in equation (6) and velocity  $v_w$  in equation (1) are uniquely determined by some (not all) parameters of the media:  $S_w = S_w(q_w, K_{sat}, m, S_{rw})$  and  $v_w = v_w(q_w, K_{sat}, m, S_{rw}, n_e)$ , i.e. both are independent of the van Genuchten parameter  $\alpha$ .

### Application to cases 1, 2 and 3

We can consider cases 1, 2 and 3 to have the same values for  $m = 0.23$ ,  $S_{rw} = 0.1$  and  $n_e = 0.3$ . The value to be used for hydraulic conductivity is not straightforward, since it varies from 1 to 1000 m/s, so we find values for  $H$  solving equation (5) for each value of  $K_{sat}$ ; then  $S_w$  and  $v_w$  are calculated for each  $K_{sat}$  using equations (6) and (1). As stated above, the solutions for saturation, water velocity and travel time are independent of the van Genuchten parameter  $\alpha$ . Figure 5 shows the values of  $S_w$  and  $v_w$  as a function of  $K_{sat}$ .



**Fig. 5** Water saturation and water velocity for varying hydraulic conductivity. Arithmetic average: velocity = 0.74 m/year; saturation = 0.45.

## CONCLUSIONS

We have developed a methodology for analytical modelling of flow in the unsaturated zone, which may be used to obtain order-of-magnitude estimates of water content and travel times in mine-waste piles. The approach is strictly valid for 1-D vertical unsaturated conditions assuming steady-state flow and homogeneous media.

Using several scenarios of multi-level aquifers with changes in hydraulic conductivity, we compared numerical solutions obtained with the code MIN3P (Mayer *et al.*, 2002) to the analytical approximation. Using an average value of  $K_{sat} = 100$  m/day, water velocity obtained analytically equals to 0.8 m/year while in the stratified media, velocities computed numerically vary from 0.5 to 0.8 m/year. Results show that changes in hydraulic conductivity have relatively small influence in travel times. The water velocity (and travel time) in unsaturated conditions depends on

infiltration rate, porosity and saturation. We found that travel time is mainly controlled by infiltration rates; in semi-arid regions (recharge rates around 100 mm/year) water velocities in the unsaturated zone are very low, often below 1 m/year.

## REFERENCES

- Lefebvre, R., Hockley, D., Smolensky, J. & Gélinas, P. (2001). Multiphase transfer processes in waste rock piles producing acid mine drainage I: Conceptual model and system characterization. *J. Contaminant Hydrol.* 52, 137–164.
- Mayer, K. U., Frind, E. O. & Blowes, D. W. (2002) Multicomponent reactive transport modeling in variably saturated porous media using a generalized formulation for kinetically controlled reactions. *Water Resour. Res.* 38(9).
- Roth, K. (2007) Soil Physics Lecture Notes, V1.2, University of Heidelberg, Institute of Environmental Physics, D-69120 Heidelberg, Germany.
- van Genuchten, M. Th. (1980) A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil. Sci. Soc. Am. J.* 44, 892–898.